

On series and integral representations of some NRQCD master integrals

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At present we have a lot of techniques for calculating multiloop Feynman diagrams, see [1–3] for recent reviews. A particular useful method is the solution of some system of differential equations [4–10]. In many cases the results for Feynman diagrams can be written in terms of multiple polylogarithms (MPLs) [11–13], which is well studied class of functions at the moment. In this particular case the corresponding system of differential equations can be reduced to the so-called ϵ -form [9, 10, 14]. When it is not possible we are required to introduce both new more general classes of functions and new solution techniques.

In the present short note, we use an example a set of two-loop master integrals arising in the process of matching of QCD to NRQCD. We are considering master integrals for a family of Feynman integrals studied previously in [15, 16]. The latter is defined as

$$J_{b_1 b_2 b_3 a_1 a_2}^{mM} = \int \frac{d^d k d^d l}{\pi^2} \frac{1}{D_1^{b_1} D_2^{b_2} D_3^{b_3} D_4^{a_1} D_5^{a_2}}, \quad (1)$$

where the propagators are $D_1 = (k + q_1)^2 - m^2$, $D_2 = (k - q_2)^2 - m^2$, $D_3 = k^2 - m^2$, $D_4 = (l + \frac{q_1 - q_2}{2})^2 - M^2$, $D_5 = (l - k)^2 - M^2$ and the kinematics is given by $q_1^2 = q_2^2 = 0$ and $q_1 \cdot q_2 = 2m^2$. A graphical representation of this family of integrals can be found in Fig. 1, where we defined $p = \frac{1}{2}(q_1 + q_2)$. Using integration by parts (IBP) relations [17, 18] all integrals in this family can be reduced to the set of 9 master integrals. The latter can be chosen as

$$(J_1, \dots, J_9)^\top = \left\{ J_{00111}^{mM}, J_{00101}^{mM}, J_{00111}^{mM}, J_{00211}^{mM}, J_{00121}^{mM}, J_{01011}^{mM}, J_{02011}^{mM}, J_{01111}^{mM}, J_{11101}^{mM} \right\}^\top.$$

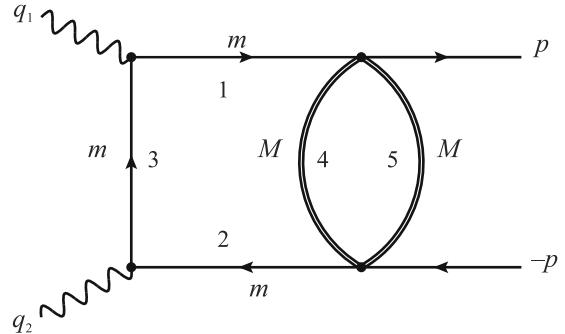


Fig. 1. Graphical representation for the family of integrals in Eq. (1)

In the first part of the work we following [27] consider analytical Frobenius²⁾ solutions of our master integrals for general values of space-time dimension. It turns out, that if we look for solutions of the system of differential equations in the form of Frobenius power series in the square of mass ratio m^2/M^2 , then the recurrence relations for the series coefficients can be reduced to first-order difference equations. The solution of the latter does not cause any difficulties. The final result can then be rewritten in terms of generalized hypergeometric functions ${}_3F_2$, ${}_4F_3$ and ${}_5F_4$. Next, we discuss the use of Feynman parameter trick to reduce the problem of evaluation of two-loop master integrals to effective one-loop problem. The latter can then be solved with the use of differential equations method with respect to mentioned Feynman parameter [15, 16, 25, 26]. Note, that similar Feynman parameter trick was used before in [28–31] under the name of effective mass approach [4, 6, 32]. In the last part we show how the exact Frobenius results in terms of hypergeometric ${}_pF_q$ -functions

²⁾For previous applications of Frobenius method in the context of Feynman diagrams see for example [16, 19–24].

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can be transformed into corresponding integral representations exact in space-time dimension.

The obtained results agree with those obtained previously when either exact results exists or up to available expansion order in ϵ . The presented techniques for obtaining Frobenius power series and integral representations are both simple and powerful enough with a great potential for their extension to other problems.

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