Pis'ma v ZhETF, vol. 116, iss. 9, pp. 577 – 578

Particle creation: Schwinger + Unruh + Hawking

G. E. Volovik[1)]

Low Temperature Laboratory, Aalto University, P.O. Box 15100, FI-00076 Aalto, Finland

Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia

Submitted 13 August 2022
Resubmitted 15 September 2022
Accepted 20 September 2022

DOI: 10.31857/S1234567822210017, EDN: lggbpm

In this paper we discuss the connection of Schwinger particle creation in the constant electric field [1, 2] with the particle production in the Unruh [3] and Hawking [4] effects. For that we consider the combined effects, which involve simultaneously the Schwinger particle production and the other effects. These combined effects demonstrate that the entropy and temperature can be associated not only with the event horizons, as it was suggested by Gary Gibbons and Stephen Hawking [5], but also can be extended to the Schwinger effect.

The rate of the Schwinger pair creation is:

$$\Gamma^{\rm Schw}(M) = \frac{dW^{\rm Schw}}{dt} = \frac{q^2\mathcal{E}^2}{(2\pi)^3}\exp\left(-\frac{\pi M^2}{q\mathcal{E}}\right). \quad (1)$$

The comparison of Eq. (1) with the Unruh radiation in the accelerated frame reveals the factor of 2 problem. The Unruh temperature is $T_{\rm U} = a/2\pi$, where a is acceleration. In the Schwinger case the acceleration in the electric field is $a = q\mathcal{E}/M$, and $\Gamma^{\rm Schw}(M) = \exp(-M/2T_{\rm U})$. The factor of 2 problem arises also in the consideration of the Hawking radiation in the de Sitter expansion, see [6] and references therein. Subtleties in the tunneling approach to Hawking and Unruh radiation see in [7–12]. Different scenarios of resolving the above discrepancy between the Schwinger and Unruh mechanisms see in [13–15] and references therein. We consider the scenario somewhat similar to that in [14].

From Equation (1) it follows that the probability of creation of particle with mass $M+m$ and charge q in the limit $m \ll M$ can be expressed in terms of the probability of creation of particle with mass M and the extra term:

$$\Gamma^{\rm Schw}(M+m) = \Gamma^{\rm Schw}(M)\exp\left(-\frac{2\pi Mm}{q\mathcal{E}}\right). \quad (2)$$

The extra term can be described in terms of the Unruh radiation in the accelerated frame:

$$\Gamma^{\rm Schw}(M+m) = \Gamma^{\rm Schw}(M)\,\Gamma^{\rm Unruh}(m), \quad (3)$$

$$\Gamma^{\rm Unruh}(m) = \exp\left(-\frac{2\pi Mm}{q\mathcal{E}}\right) = \exp\left(-\frac{m}{T_{\rm U}}\right). \quad (4)$$

This is the coherent combined process in which the charged particle with mass M is created by Schwinger process. It moves with acceleration $a = q\mathcal{E}/M$ and plays the role of the detector, which experiences the emission of a neutral particles – the Unruh radiation.

The coherent combination of the several processes is similar to the phenomenon of cotunneling in the electronic systems, where electron experiences the coherent sequence of tunneling events: from an initial to the virtual intermediate state and then to the final state [16]. In our case the virtual state is the state of the created charged particle. Its motion with acceleration triggers the creation of the neutral particle.

The combination of two processes in Eq. (3) – the Schwinger creation of mass M and the Unruh creation of mass $m \ll M$ – is similar to the combination of two processes in the creation of pairs of Reissner–Nordström (RN) monopole black holes in magnetic field [17]:

$$\Gamma^{\rm BH,Monopole} = \Gamma^{\rm Monopole}\,\Gamma^{\rm BH},\ \Gamma^{\rm BH} = \exp\left(S_{\rm BH}\right). \quad (5)$$

According to [17], the instanton amplitude contains an explicit factor corresponding to the black hole entropy, $S_{\rm BH} = A/4$ in Eq. (5), where A is the horizon area. The monopole pair creation is modified by the black hole entropy and thus by the Hawking temperature.

The Hawking temperature can be derived by comparing the tunneling rate with the Boltzmann factor [18–20]. The black hole entropy can be found [21, 22] by comparing the tunneling rate with the thermodynamic fluctuations [23]. The tunneling process is considered as fluctuation, with the probability of fluctuation being determined by the entropy difference between the initial and final states, $w \propto \exp\left(S_{\rm final} - S_{\rm initial}\right)$. In the Schwinger process, the initial state is the vacuum with zero entropy is zero. Thus the Schwinger entropy is negative, $S_{\rm final} \equiv S_{\rm Schw}(M) = -\frac{\pi M^2}{q\mathcal{E}} < 0$. The negative entropy comes from the quantum entanglement between the radiated particles [24, 25] and also between particles and quantum fields in the vacuum. Then the corre-

[1)]e-mail: grigori.volovik@aalto.fi

sponding Schwinger temperature is also negative and is opposite to the Unruh temperature:

$$T_{\rm Schw}^{-1} = \frac{dS_{\rm Schw}}{dM} = -\frac{2\pi M}{q\mathcal{E}} = -\frac{2\pi}{a} = -T_{\rm U}^{-1}. \quad (6)$$

The negative temperature is a well defined quantity, especially in condensed matter systems [26].

The description of the quantum tunneling in terms of the thermodynamic fluctuation can be also applied to the creation of two charged RN black holes discussed in [17]. This is the triple cotunneling – the coherent sequence of three tunneling events. Adding the entropy of two created black holes one obtains for the full entropy, which determines the probability of creation:

$$S_{\rm Schw}(M,q) + S_{\rm RN\,BH}(M,q) + S_{\rm RN\,BH}(M,q) = \quad (7)$$

$$= -\frac{\pi M^2}{q\mathcal{E}} + 4\pi G M^2 + 4\pi G M^2\,. \quad (8)$$

Here it is assumed that $q\mathcal{E} \ll G^{-1} \sim E^2_{\rm Planck}$. As a result, the total contribution to Eq. (8) is negative. We also take into account that the entropy of the RN black hole does not depend on charge q, which is due to the correlations between the inner and outer horizons [22]. As in [17] (see also [27] and references in [28]), due to the positive entropy terms in Eq. (8) the creation of pairs of RN black holes is enhanced compared to pair production of particles or magnetic monopoles with the same masses and charges.

The sum rule for entropy in Eqs. (7) is also valid for the inverse temperatures. The corresponding temperature $1/T = dS_{\rm final}/dM$ describing the above three processes of creation is:

$$\frac{1}{T} = \frac{1}{T_{\rm Schw}} + \frac{1}{T_{\rm BH}} + \frac{1}{T_{\rm BH}},\; T_{\rm BH} = \frac{1}{8\pi G M}. \quad (9)$$

The summation law for inverse temperature is similar to that for the black hole with several horizons [22], where the temperature is expressed in terms of the Hawking temperatures on different horizons.

In conclusion, it appears that the Schwinger pair creation obeys the same thermodynamic laws as in the black hole thermodynamics. The thermodynamics naturally connects the Schwinger pair creation in electric field with the Hawking radiation and with the particle creation in the Unruh effect.

I thank D. Kharzeev and K. Tuchin for correspondence.

This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement # 694248).

This is an excerpt of the article "Particle creation: Schwinger + Unruh + Hawking". Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364022601968

---

1. J. Schwinger, Phys. Rev. **82**, 664 (1951).
2. J. Schwinger, Phys. Rev. **93**, 616 (1953).
3. W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
4. S. W. Hawking, Nature **248**, 30 (1974).
5. G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
6. G. E. Volovik, arXiv:2205.06585 [gr-qc].
7. E. T. Akhmedov, V. Akhmedova, and D. Singleton, Phys. Lett. B **642**, 124 (2006).
8. E. T. Akhmedov, V. Akhmedova, T. Pilling, and D. Singleton, Int. J. Mod. Phys. A **22**, 1705 (2007).
9. E. T. Akhmedov, T. Pilling, and D. Singleton, Int. J. Mod. Phys. D **17**, 2453 (2008).
10. V. Akhmedova, T. Pilling, A. de Gill, and D. Singleton, Phys. Lett. B **666**, 269 (2008).
11. V. Akhmedova, T. Pilling, A. de Gill, and D. Singleton, Phys. Lett. B **673**, 227 (2009).
12. A. de Gill, Am. J. Phys. **78**, 685 (2010).
13. R. Parentani and S. Massar, Phys. Rev. D **55**, 3603 (1997).
14. D. Kharzeev and K. Tuchin, Nucl. Phys. A **753**, 316 (2005).
15. M. Teslyk, O. Teslyk, L. Zadorozhna, L. Bravina, and E. Zabrodin, Particles **5**, 157 (2022).
16. M. V. Feigel'man and A. S. Ioselevich, JETP Lett. **81**, 277 (2005).
17. D. Garfinkle, S. B. Giddings, and A. Strominger, Phys. Rev. **49**, 958 (1994).
18. G. E. Volovik, JETP Lett. **69**, 705 (1999).
19. K. Srinivasan and T. Padmanabhan, Phys. Rev. D **60**, 024007 (1999).
20. M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000).
21. M. K. Parikh, *Energy Conservation and Hawking Radiation*, in: *The Tenth Marcel Grossmann Meeting, Proceedings of the MG10 Meeting held at Brazilian Center for Research in Physics (CBPF)*, ed. by M. Novello, S. P. Bergliffa, and R. Runi, Rio de Janeiro (2006), p. 1585; hep-th/0402166.
22. G. E. Volovik, ZhETF **162**, 449 (2022).
23. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics, Volume 5, Statistical Physics*, Pergamon Press, Oxford (1993).
24. B. Zhang, Q.-y. Cai, L. You, and M.-sh. Zhan, Phys. Lett. B **675**, 98 (2009).
25. A. Florio and D. E. Kharzeev, Phys. Rev. D **104**, 056021 (2021).
26. M. Baldovin, S. Iubini, R. Livi, and A. Vulpiani, Phys. Reports **923**, 1 (2021).
27. S. W. Hawking, G. T. Horowitz, and S. F. Ross, Phys. Rev. D **51**, 4302 (1995).
28. M. Astorino, Phys. Rev. D **89**, 044022 (2014).