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Emergent long-lived Zitterbewegung in Su–Schrieffer–Heeger lattice with third-nearest-neighbor hopping

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With the Dirac equation as its core, relativistic quantum mechanics predicts that particles show a high-frequency Zitterbewegung (ZB) even in free motion [1, 2]. For a free electron, the Dirac equation predicts the ZB is so high in frequency ($\omega_{ZB} \sim 10^{21}$ Hz) and so small in amplitude ($A_{ZB} \sim 10^{-12}$ m) that it is impossible to observe it directly with current experimental techniques [3]. Though early studies on ZB were mainly focused on electrons, current research has proved that ZB phenomenon is actually not unique to electrons, but widely exists in condensed matter systems with linear dispersion such as graphene [4], Weyl semi-metal [5], topological insulator [6] and other systems [7]. A valid Dirac equation is shared by all these systems to describe the dynamical properties. With the progress of quantum simulation technology, the simulation of ZB has been realized on various quantum simulation platforms, such as phononic/photonic crystals [8], trapped ions [9], ultracold atoms [10].

Furthermore, J. A. Lock predicted in 1979 that the finite-sized initial wavepacket will lead to the damping of ZB, i.e., the corresponding amplitude will decay very fast over time [11]. His prediction has been verified by theories [12] and experiments [9] in succession. The fleeting existence of ZB makes it difficult to observe in experiments. Besides, recent studies have found that the introduction of the third-nearest-neighbor (N3) hopping can change the energy band structure, leading to Dirac point metamorphosis [13], a growing number of research concern relative systems with N3 hopping. It is natural to ask whether a quasi-flat band structure can be produced by Dirac point metamorphosis, thereby reducing the diffusion rate of the quasiparticles and ultimately producing the long-lived ZB phenomenon? To answer this question, we propose a N3 Su–Schrieffer–Heeger (SSH) model which features not only the nearest-neighbor hopping, but also the N3 hopping.

As for cold atomic lattice system, the standard SSH can be implemented by loading rubidium atoms [14]. By introducing the auxiliary laser, one can effectively control the N3 hopping. The low-energy effective Hamiltonian of the system can be obtained as

$$H_{\text{eff}} = v_x q_x \sigma_x + \left(\Delta + \frac{q_x^2}{2m^*}\right)\sigma_z, \qquad (1)$$

where v_x denotes the effective light velocity in x direction. Δ is the gap parameter and m^* is the effective mass. The corresponding dynamical properties of quasiparticles in a N3 SSH lattice can be described by the following equation of motion, i.e.,

$$i\hbar\frac{\partial}{\partial t}\Psi = H_{\text{eff}}\Psi. \qquad (2)$$

In order to study the wavepacket dynamics of quasiparticles in the N3 SSH lattice, we directly solve the equation of motion Eq. (2) with Hamiltonian Eq. (1). Throughout the calculation, a universal Gaussian wavepacket without initial velocity is considered as the initial state, i.e.,

$$|\Psi(x)\rangle = G(x)|\Phi\rangle. \qquad (3)$$

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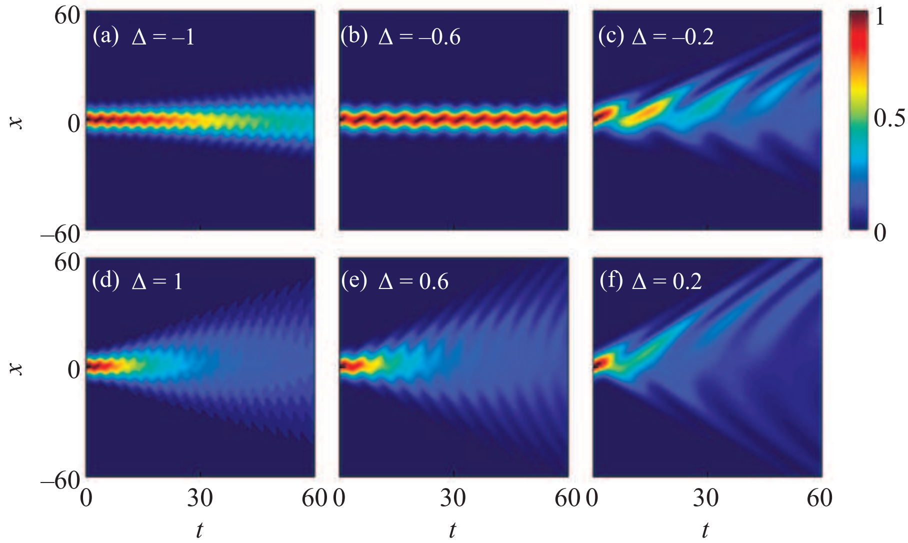

Fig. 1. (Color online) Numerically calculated probability distributions, $|\Psi(x,t)|^2$, for $\Delta = -1, -0.6, -0.2, 1, 0.6, 0.2$ (from panels (a) to (f)). Throughout the calculation, the width of the initial wavepacket $L = 5$, effective light velocity in x direction $v_x = 1$ and the effective mass $m^* = 0.5$

Visualized evolution of wavepacket is plotted in Fig. 1a–f. As shown in the Fig. 1, the centroid of the wavepacket exhibits periodic oscillations whenever $\Delta \neq 0$, which is the evidence of ZB effect. To be specific, when $\Delta < 0$ (see Fig. 1a–c), the attenuation of ZB effect decreases first and then increases with the decreasing Δ. Notably, one can see a stable ZB effect when $\Delta = -0.6$ (see Fig. 1b), where the width (amplitude) of wavepacket barely expands (decreases). On the other hand, when $\Delta > 0$, no matter how the parameter Δ changes, the attenuation of ZB effect is still very strong and its life-span is very short. Then, in order to further grasp the physical mechanism behind the long-lived ZB phenomenon, the long-time average inverse participation rate and momentum distribution of the system are calculated, and the corresponding energy band structure of the system is discussed. The results of the theoretical analysis show that the N3 SSH model has a unique advantage over the standard SSH model, i.e., the introduction of the N3 hopping can effectively modulate the energy band structure near the high symmetry point to the quasi-flat band, thus restraining the diffusion of the quasiparticle wave packets, which in turn prolongs the interference time of the positive and negative branches, and finally leads to the long-lived ZB phenomenon of the system. Furthermore, we propose an experimental scheme in cold atomic system. Due to its easy accessibility and high controllability, we hope the long-lived ZB effect predicted here can be verified experimentally in the near future.

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