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On the global temperature of the Schwarzschild–de Sitter spacetime

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The issue of the stability of the de Sitter (dS) vacuum is still an unsolved problem, see, e.g., [1, 2] and references therein. It is not excluded that even if the particle creation by Gibbons–Hawking (GH) radiation takes place, the dS expansion may immediately dilute the produced particles preventing the vacuum decay. One may think that study of the black hole in the dS environment – the Schwarzschild–de Sitter (SdS) spacetime with the presence of two horizons simultaneously – can be even more difficult task. Each horizon is characterized by its own temperature, see, e.g., [3–6]. It is not clear, whether such configuration allows a kind of global temperature [7].

Here we discuss the global temperatures $T_{\rm dS}$ and $T_{\rm SdS}$, which characterize the thermal activation processes in the dS and SdS environments. These processes are not directly related to the horizons and to the corresponding Hawking radiation. The global temperatures $T_{\rm dS}$ and $T_{\rm SdS}$ are obtained when one considers for example the process of ionization of atoms in the dS and SdS spacetimes. We find that in spite of existence of two horizons, the temperature $T_{\rm SdS}$ does not depend on the mass of the black hole. It is solely determined by the Hubble parameter H, but is by factor $2\sqrt{3}$ larger than the GH temperature $T_{\rm GH} = H/2\pi$ of Hawking radiation from the cosmological horizon in the dS spacetime.

We use the extension of Painlevé–Gullstrand (PG) coordinates, which describes the SdS metric in the whole range of radial coordinates [8]. In this coordinate system there is the point r_0 at which the shift function changes sign. At this point the observer is at rest, observers at $r < r_0$ are free falling to the black hole and observers at $r > r_0$ are free falling towards the cosmological horizon, see also [9]. The temperatures T_b and T_c of Hawking radiation and the activation temperature $T_{\rm SdS}$ are measured by the stationary observer.

The modified PG coordinates for SdS spacetime are:

$$ds^2 = -N^2 dt^2 + \frac{1}{N^2}(dr - v dt)^2 + r^2 d\Omega^2, \qquad (1)$$

where the lapse function N and shift function v in the Arnowitt–Deser–Misner formalism are:

$$N^2 = 1-C,\ v(r) = \sqrt{C(1-C)}\sqrt{\frac{r+2r_0}{3rr_0^2}}\,(r-r_0), \quad (2)$$

where C and the stationary point r_0 are:

$$C = 3(GMH)^{2/3},\ r_0^3 = \frac{GM}{H^2}, \qquad (3)$$

and M is the mass of black hole.

For the static observer at $r = r_0$ the measured frequency is red-shifted by the factor $N = (1-C)^{1/2}$. As a result, the Hawking temperatures of black hole and cosmological horizons are renormalized [3], $T_b = T_{b0}/N$ and $T_c = T_{c0}/N$, where T_{b0} and T_{c0} are determined by gravity at horizons. The global temperature of Hawking radiation exists only in the limit $C \to 1$, when the black hole horizon approaches the cosmological horizon. Two temperatures approach the Bousso–Hawking value [3]:

$$T_{\rm BH} = T_b = T_c = \sqrt{3}\frac{H}{2\pi} = \frac{1}{6\pi GM}. \qquad (4)$$

In our approach, the global temperature $T_{\rm SdS}$ is not the temperature of the Hawking radiation. The $T_{\rm SdS}$ describes the processes of thermal activation measured by the stationary observer at $r = r_0$. We consider these processes on example of the ionization of an atom, which is not possible in Minkowski vacuum, but is possible in SdS. We follow the semiclassical tunneling approach [10–12] applied in [13] to the ionization in the dS backgound. For small C, when $T_c \ll T_b$, the imaginary momentum on the tunneling trajectory of electron is:

$$\operatorname{Im} p_r(r) = \sqrt{2m\epsilon_0 - m^2 v^2(r)}, \qquad (5)$$

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where m is electron mass and ϵ_0 is the ionization energy. Near the stationary point $v(r) \approx \sqrt{3}H(r - r_0)$, and the probability of ionization is:

$$w \propto \exp\left(-2\mathrm{Im}\int dr\, p_r(r)\right) = \exp\left(-\frac{\pi\epsilon_0}{\sqrt{3}H}\right). \quad (6)$$

The ionization looks as thermal with temperature:

$$T = \frac{\sqrt{3}H}{\pi} \equiv T_{\mathrm{SdS}}. \quad (7)$$

The same result (7) is valid for the general case of arbitrary $C < 1$. This suggests that T_{SdS} is the universal temperature in the SdS environment, which does not depend on the mass of black hole M and is fully determined by the Hubble parameter.

Equation (7) is also valid in the limit when the two horizons merge, and we can compare the activation temperature T_{SdS} with the Bousso–Hawking temperature in Eq. (4). The activation temperature is twice larger, $T_{\mathrm{SdS}} = 2T_{\mathrm{BH}}$. The same takes place for the dS, where the activation temperature T_{dS} measured by stationary observer at $r = 0$ is double the GH temperature, $T_{\mathrm{dS}} = 2T_{\mathrm{GH}} = H/\pi$ [13–17]. The discussion of the possible origin of the doubling of the Hawking temperature in different arrangements see in [14]. See also [18, 19] for dS spacetime.

The activation temperature in dS is not related to the cosmological horizon, so one may suggest that $T = T_{\mathrm{dS}} = H/\pi$ plays the role of the local thermodynamic temperature. All the points in the dS space are equivalent, and thus this temperature is the same for all static observers. Then the energy density at any point of the dS vacuum can be viewed as the thermal energy:

$$\epsilon_{\mathrm{vac}} = \frac{3}{8\pi G}H^2 = \frac{3\pi}{8G}T^2, \quad (8)$$

and the local entropy density is

$$s_{\mathrm{vac}} = -\frac{\partial F}{\partial T} = \frac{3\pi}{4G}T = \frac{3}{4G}H, \quad (9)$$

where the free energy density $F = \epsilon_{\mathrm{vac}} - T d\epsilon_{\mathrm{vac}}/dT$. The total entropy in the volume surrounded by cosmological horizon corresponds to the GH entropy:

$$S_{\mathrm{vac}} = \frac{4\pi}{3H^3}s_{\mathrm{vac}} = \frac{\pi}{GH^2} = \frac{A}{4G}\,. \quad (10)$$

But it is the thermodynamic entropy coming from the local entropy density of the dS quantum vacuum.

Different temperatures for vacuum and for matter have been suggested in [20]. The present temperature H/π of the vacuum is much smaller than the temperature of matter degrees of freedom, $T_{\mathrm{dS}} \sim 10^{-30}T_{\mathrm{CMB}}$. But the entropy of the vacuum highly exceeds the entropy of matter due to large density of states in the quantum vacuum, $s_{\mathrm{vac}} \sim 10^{30}s_{\mathrm{CMB}}$.

In conclusion, we found that the activation temperature in SdS background, which is measured by the static observer, is universal. It does not depend on the black hole mass M and is fully determined by the Hubble parameter H of the dS expansion, although with different prefactor, $T_{\mathrm{SdS}} = \sqrt{3}H/\pi$. This universal temperature characterizes different processes including the process of ionization of atoms, the decay of the composite particles in the SdS, and all the other scattering or radiation processes, which are not possible in the Minkowski spacetime. We also considered the local thermodynamics of the dS vacuum.

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1. A. Yu. Kamenshchik, A. A. Starobinsky, and T. Vardanyan, Eur. Phys. J. C **82**, 345 (2022).
2. A. M. Polyakov, arXiv:1209.4135 [hep-th].
3. R. Bousso and S. W. Hawking, Phys. Rev. D **54**, 6312 (1996).
4. S. Shankaranarayanan, Phys. Rev. D **67**, 084026 (2003).
5. T. R. Choudhury and T. Padmanabhan, Gen. Rel. Grav. **39**, 1789 (2007).
6. Y. Qiu and J. Traschen, Class. Quantum Gravity **37**, 135012 (2020).
7. E. T. Akhmedov and K. V. Bazarov, arXiv:2212.06433.
8. G. E. Volovik, Ann. Phys. **449**, 169219 (2023).
9. A. V. Toporensky and O. B. Zaslavskii, Eur. Phys. J. C **83**, 225 (2023).
10. M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000).
11. K. Srinivasan and T. Padmanabhan, Phys. Rev. D **60**, 024007 (1999).
12. G. E. Volovik, JETP Lett. **69**, 705 (1999).
13. G. E. Volovik, JETP Lett. **90**, 1 (2009).
14. G. E. Volovik, Universe **8**, 639 (2022).
15. J. Bros, H. Epstein, and U. Moschella, JCAP **0802**, 003 (2008).
16. J. Bros, H. Epstein, M. Gaudin, U. Moschella, and V. Pasquier, Commun. Math. Phys. **295**, 261 (2010).
17. D. P. Jatkar, L. Leblond, and A. Rajaraman, Phys. Rev. D **85**, 024047 (2012).
18. N. Arkani-Hamed and J. Maldacena, arXiv:1503.08043.
19. P. R. Anderson, E. Mottola, and D. H. Sanders, Phys. Rev. D **97**, 065016 (2018).
20. S. N. Vergeles, arXiv:2301.01692.