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Analog Sommerfeld law in quantum vacuum

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The vacuum of the de Sitter (dS) spacetime is characterized by the local temperature $T = H/\pi$, where H is the Hubble parameter, see [1] and references therein. This temperature describes the thermal processes of decay of the composite particles and the other activation processes, which are energetically forbidden in the Minkowski spacetime, but are allowed in the dS background, see also Refs. [2–4]. In particular, this temperature determines the probability of the ionization of an atom in the dS environment, $\exp(-E/T)$, where E is the ionization potential. This activation temperature is twice the Gibbons–Hawking [5] temperature $T_{\rm GH} = H/2\pi$ of the cosmological horizon, $T = 2T_{\rm GH}$. As distinct from the $T_{\rm GH}$, the activation temperature has no relation to the cosmological horizon. It describes the local processes, which take place far away from the horizon. If $T = H/\pi$ is the local temperature in dS spacetime, the natural question is: does it determine the local thermodynamics of the dS vacuum? In this paper we discuss this thermodynamics.

In the Painlevé–Gullstrand form the metric in the dS expansion is

$$ds^2 = -dt^2 + (dr - Hrdt)^2 + r^2 d\Omega^2 \,, \qquad (1)$$

where H is the Hubble parameter.

From Friedmann equations of general relativity it follows that the vacuum energy density (which is the cosmological constant Λ) expressed in terms of the activation temperature $T = H/\pi$ is:

$$\epsilon_{\rm vac} = \Lambda = \frac{3}{8\pi G} H^2 = \frac{3\pi}{8G} T^2. \qquad (2)$$

If $T = H/\pi$ is the local temperature of the dS vacuum, we can determine the free energy density, $F = \epsilon_{\rm vac} - T d\epsilon_{\rm vac}/dT$, and thus the entropy density $s_{\rm vac}$:

$$s_{\rm vac} = -\frac{\partial F}{\partial T} = \frac{3\pi}{4G} T = \frac{3}{4G} H. \qquad (3)$$

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The quadratic dependence of vacuum energy on temperature is also important for consideration of the thermodynamic Gibbs–Duhem relation for quantum vacuum. It leads to the reformulation of the vacuum pressure. The conventional vacuum pressure $P_{\rm vac}$ obeys the equation of state $w = -1$ and enters the energy momentum tensor of the vacuum medium in the form:

$$T^{\mu\nu} = \Lambda g^{\mu\nu} = {\rm diag}(\epsilon_{\rm vac}, P_{\rm vac}, P_{\rm vac}, P_{\rm vac}), \; P_{\rm vac} = -\epsilon_{\rm vac}. \qquad (4)$$

In dS state the vacuum pressure is negative, $P_{\rm vac} < 0$.

This pressure $P_{\rm vac}$ does not satisfy the thermodynamic Gibbs–Duhem relation, $Ts_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac}$, because the right hand side of this equation is zero. The reason for that is that in this equation we did not take into account the gravitational degrees of freedom. Earlier it was shown, that gravity contributes to thermodynamics with the pair of the thermodynamically conjugate variables: the gravitational coupling $K = \frac{1}{16\pi G}$ and the Riemann curvature $\mathcal{R}$, see [6–8]. This is because the Einstein–Hilbert action contains the gravitational term $K\mathcal{R}$, and its contribution to thermodynamics is somewhat similar to the work density [9–12]. The gravitational thermodynamic variables allow us to write the modified Gibbs–Duhem relation:

$$Ts_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac} - K\mathcal{R}. \qquad (5)$$

This equation is obeyed, since $\epsilon_{\rm vac} + P_{\rm vac} = 0$ and $\mathcal{R} = -12H^2$. Equation (5) suggests that one may introduce the effective pressure, which is modified by gravitational degrees of freedom:

$$P = P_{\rm vac} - K\mathcal{R}. \qquad (6)$$

Then the conventional Gibbs–Duhem relation is satisfied:

$$Ts_{\rm vac} = \epsilon_{\rm vac} + P. \qquad (7)$$

The effective dS pressure P is positive, $P = \epsilon_{\rm vac} > 0$, and satisfies equation of state $w = 1$, which is similar to matter with the same equation of state. As a result, due to the gravitational degrees of freedom, the dS state has many common properties with the non-relativistic Fermi liquid, where the thermal energy is proportional to T^2, and with the relativistic matter with $w = 1$. This means that in thermodynamics the dS vacuum behaves as the stiff matter introduced by Zel'dovich [13], where the speed of sound is equal to the speed of light, $s^2 = c^2 dP/d\epsilon_{\rm vac} = c^2$.

In this vacuum thermodynamics, the total entropy in the volume V_H surrounded by the cosmological horizon with radius $R = 1/H$ is

$$s_{\rm vac} V_H = \frac{4\pi R^3}{3} s_{\rm vac} = \frac{\pi}{GH^2} = \frac{A}{4G}, \tag{8}$$

where A is the horizon area. This corresponds to the Gibbons–Hawking entropy of the cosmological horizon. However, it is the thermodynamic entropy coming from the local entropy of the dS quantum vacuum, rather than the entropy of the horizon degrees of freedom.

Since the thermodynamics of the dS state with the thermal energy $\epsilon_{\rm vac} \propto T^2$ is similar to the thermodynamics of the Fermi liquid, there is the analog of the Sommerfeld law, which states that the entropy per one atom of Fermi liquid is $S \propto T/E_F$, where E_F is Fermi energy. We do not know what are the "atoms of the vacuum", but from Eq. (3) it follows that the entropy density of the vacuum $s_{\rm vac} \sim (T/E_{\rm P})/l_{\rm P}^3$, where $l_{\rm P} = \sqrt{G}$ is Planck length and $E_{\rm P}$ is Planck energy. This suggests that $n_{\rm P} \sim 1/l_{\rm P}^3$ is the density of "atoms of the vacuum", and the entropy per "atom" is:

$$S = \frac{s_{\rm vac}}{n_{\rm P}} \sim s_{\rm vac} l_{\rm P}^3 \sim \frac{T}{E_{\rm P}}. \tag{9}$$

Eq. (9) is the full analog of the Sommerfeld law for Fermi liquid. This analogy also suggests that the corresponding density of states in the quantum vacuum (the analog of density of states at the Fermi level $N_F \sim m p_F$ in Fermi liquids) is $N_P \sim E_{\rm P}^2$. This huge density of states leads to a very large entropy of the dS vacuum even for very small temperature of the vacuum.

In conclusion, the quantum vacuum of de Sitter spacetime looks as specific form of the relativistic Fermi liquid with local temperature, local entropy and local Gibbs–Duhem relation. The entropy density in Eq. (3) is linear in the local temperature T. The local de Sitter temperature is determined by the action of the expanding Universe on the matter degrees of freedom: it describes the processes of activation, such as the thermal process of the ionization of atoms in the de Sitter environment. This activation temperature has no relation to the cosmological horizon, and is twice larger than the Hawking temperature related to the horizon. Nevertheless, the total entropy in the region inside the cosmological horizon is exactly the horizon entropy $A/4G$. This is a kind of the bulk-boundary correspondence.

In the quasi-equilibrium states with matter the system can be characterized by two temperatures: the temperature of the vacuum component and the temperature of matter degrees of freedom [14]. The present temperature of the vacuum component is much smaller than the temperature of matter degrees of freedom. For example, compared with the temperature of Cosmic Microwave Background (CMB) radiation it is $T_{\rm vac} \sim 10^{-30} T_{\rm CMB}$. But the entropy of the vacuum highly exceeds the entropy of matter due to large density of states in the quantum vacuum, $s_{\rm vac} \sim 10^{30} s_{\rm CMB}$.

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