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Tensor train optimization of parameterized quantum circuits

G. Paradezhenko[*1)], *A. Pervishko*[*+], *D. Yudin*[*+]

[*] *Skolkovo Institute of Science and Technology, 121205 Moscow, Russia*

[+] *Laboratory of Spin-Orbitronics, Institute of High Technologies and Advanced Materials, Far Eastern Federal University, 690950 Vladivostok, Russia*

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Significant progress towards stable operation of multi-qubit quantum systems with relatively short decoherence times allows nowadays to address simple optimization tasks [1–4]. The wider use of these noisy intermediate-scale quantum processors is hampered by the noise inevitably present in quantum gates, which severely limits the possible depth of a quantum circuit. The problem can nevertheless be partially relaxed in the approach of variational quantum computing, widely accepted as the most viable way to achieve quantum supremacy [5, 6].

As a rule, in most variational quantum algorithms, with the variational quantum eigensolver (VQE) being the most notable example, one looks for the ground state of a given interacting quantum system [7]. In this case, a quantum processor is used to prepare a family of probe states as implemented by a parametrized quantum circuit, as well as to estimate the energy for that family of state representing thus a multi- parameter cost function. By virtue of standard optimization methods on a classical computer one minimizes the cost function to determine the optimal parameters of the quantum circuit that approximate the ground state of a given Hamiltonian. The main advantage of this methodology is in the fact that one does not need to design a deep quantum circuit [8–10].

It is believed that derivative-free methodology to optimization to be more noise-resilient, including but not limited to Nelder–Mead algorithm and Powell's conjugate direction method. We herein propose a derivative-free optimization technique based on tensor train optimizer (TTOpt) [11]. Particularly, we are to make use of the transverse field Ising model (TFIM) with open boundary conditions as the VQE algorithm in this case faces the convergence issues when utilizing shallow circuits [12]. In our analysis, we rely on the use of both the hardware efficient ansatz (HEA) [9] and the Hamiltonian variational ansatz (HVA) [12]. We also address the effect of the depolarizing error channel that represents a completely positive trace-preserving map and transforms a given quantum state into the linear combination of this state and maximally mixed state.

Our numerical findings are shown in Fig. 1 for TFIM of $n = 4$, 6, and 8 qubits. A close inspection of Fig. 1a reveals that the HVA optimization with Broyden–Fletcher–Goldfarb–Shanno algorithm (BFGS) steadily improves with the citcuit depth L providing a proper approximation to the ground state starting from $L = 4{-}8$ layers. However, for extremely shallow circuits down to $L = 2$ layers, it is not robust to random initialization of variational parameters. As opposed, the TTOpt outperforms the BFGS optimizer for $L = 2{-}4$ layers. In the presence of the depolarizing noise specified the BFGS optimizer completely fails in achieving convergence. On the contrary, the results of TTOpt do not change much with noise providing a reasonable accuracy in comparison to the BFGS optimizer. Thus, the TTOpt seems to be noise-resilient at least for the case of shallow circuits. Finally, if we switch to the HEA-type ansatz the results of TTOpt are even more impressive as illustrated in Fig. 1b. The TTOpt outperforms the BFGS optimizer in the range of $L = 1{-}3$ layers for both pure and noisy simulations. Meanwhile, the larger number of qubits to be involved the deeper circuits have to be utilized, making TTOpt more computationally demanding.

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[1)] e-mail: g.paradezhenko@skoltech.ru

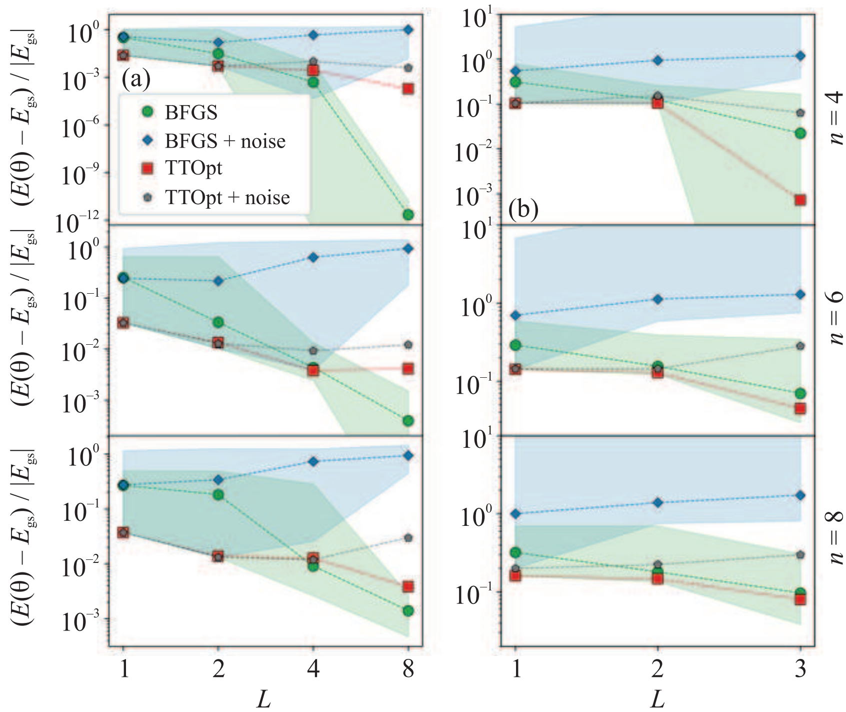

Fig. 1. (Color online) Optimized cost function for TFIM $E(\boldsymbol{\theta})$ relative to its exact ground-state energy $E_{\rm gs}$ plotted versus the ansatz depth L, where the optimization is performed as based on the BFGS and TTOpt optimizers. The VQE simulations are implemented for TFIM of $n = 4$, 6, and 8 qubits under open boundary conditions with: (a) – HVA and (b) – HEA being used as variational quantum circuits. The BFGS results are averaged over 100 random initial guesses for the variational parameters $\boldsymbol{\theta}$. The green and blue shaded areas depict the standard deviation of the optimized values for $E(\boldsymbol{\theta})$. The results with noise are obtained by applying the depolarizing quantum channel for one-and two-qubit gates in quantum circuits, with the depolarizing parameter being equal to 0.005

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1. M. P. Harrigan, K. J. Sung, M. Neeleyet et al. (Collaboration), Nat. Phys. **17**, 332 (2021).
2. S. Ebadi, A. Keesling, M. Cain et al. (Collaboration), Science **376**, 1209 (2022).
3. S. Yarkoni, E. Raponi, T. Bäck, and S. Schmitt, Rep. Prog. Phys. **85**, 104001 (2022).
4. M.-T. Nguyen, J.-G. Liu, J. Wurtz, M. D. Lukin, S.-T. Wang, and H. Pichler, PRX Quantum **4**, 010316 (2023).
5. J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik, New J. Phys. **18**, 023023 (2016).
6. R. Babbush, D. W. Berry, I. D. Kivlichan, A. Y. Wei, P. J. Love, and A. Aspuru-Guzik, New J. Phys. **18**, 033032 (2016).
7. A. Peruzzo, J. McClean, P. Shadbolt, M.-H. Yung, X.-Q. Zhou, P. J. Love, A. Aspuru-Guzik, and J. L. O'Brien, Nat. Commun. **5**, 4213 (2014).
8. P. J. J. O'Malley, R. Babbush, I. D. Kivlichan et al. (Collaboration), Phys. Rev. X **6**, 031007 (2016).
9. A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J. M. Chow, and J. M. Gambetta, Nature **549**, 242 (2017).
10. J. R. McClean, S. Boixo, V. N. Smelyanskiy, R. Babbush, and H. Neven, Nat. Commun. **9**, 4812 (2018).
11. K. Sozykin, A. Chertkov, R. Schutski, A.-H. Phan, A. Cichocki, and I. Oseledets, arXiv:2205.00293 (2022).
12. M. Larocca, N. Ju, D. García-Martín, P. J. Coles, and M. Cerezo, arXiv:2109.11676 (2021).