Pis'ma v ZhETF, vol. 120, iss. 4, pp. 245 – 246

Modified Bridgman formula for the thermal conductivity of complex (dusty) plasma fluids

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Submitted 17 May 2024
Resubmitted 11 July 2024
Accepted 19 July 2024

A simple and popular Bridgman's formula predicts a linear correlation between the thermal conductivity coefficient and the sound velocity of dense liquids. Unfortunately, it cannot be applied to strongly coupled plasma-related fluids, because the sound velocity can greatly increase as screening weakens. We propose a modification of the Bridgman formula by correlating the thermal conductivity coefficient with the transverse (shear) sound velocity. This approach is demonstrated to work reasonably well in screened Coulomb (Yukawa) fluids and can be useful in the context of complex (dusty) plasmas.

DOI: 10.31857/S0370274X24080146, EDN: LLMCVJ

About a century ago Bridgman proposed a simple formula relating the coefficient of thermal conductivity with the sound velocity and density of dense liquids [1],

$$\Lambda = \frac{2c_s}{\Delta^2}, \tag{1}$$

where Λ is the coefficient of thermal conductivity, c_s is the sound velocity, $\Delta = \rho^{-1/3}$ is the mean separation between the molecules, and ρ is the liquid number density. Bridgman's expression was apparently the first theoretical formula applied to explain the thermal conductivity of dense fluids, but it still remains a useful simple reference correlation often discussed in contemporary literature [2–5]. One of the present authors has recently performed a systematic analysis of correlations between the thermal conductivity coefficients and sound velocities in several model and real liquids [6]. It has been demonstrated that linear correlations are well reproduced for model fluids as well as real monatomic and diatomic liquids, but seem less convincing in polyatomic molecular liquids. Even if not truly universal, Bridgman's formula remains quite appealing, because the information about the sound velocity in various liquids is either easily accessible or can be relatively easily measured experimentally.

However, there is a serious deficiency in the Bridgman's approach. It cannot be applied to strongly coupled plasma-related fluids with soft screened Coulomb interactions, because the sound velocity can greatly increase as screening weakens. To be concrete, let us concentrate on complex (dusty) plasmas – systems of charged particles immersed in a neutralizing plasma medium [7]. In the first approximation, the particles in complex plasmas are interacting with the screened Coulomb (Debye–Hückel or Yukawa) potential of the form

$$\phi(r) = \frac{Q^2}{r}\exp\left(-\frac{r}{\lambda}\right), \tag{2}$$

where Q is the particle charge and λ is the screening length. The properties of Yukawa systems are described by the two dimensionless parameters: the Coulomb coupling parameter $\Gamma = Q^2/aT$ and the screening parameter $\kappa = a/\lambda$, where $a = (4\pi\rho/3)^{-1/3}$ is the Wigner–Seitz radius.

To explain why the original Bridgman's formula cannot be applied to plasma-related systems, let us consider the weakly screened regime $\kappa \lesssim 1$. In this regime the sound velocity is virtually independent of the coupling strength and tends to the conventional dust acoustic wave (DAW) velocity [8]. Using the definition of the DAW velocity [9] we get

$$c_s \simeq c_{\rm DAW} = \omega_p\lambda \propto \kappa^{-1}, \tag{3}$$

where $\omega_p = \sqrt{4\pi Q^2\rho/m}$ is the plasma frequency and m is the particle mass. The sound velocity increases indefinitely when κ approaches zero. Therefore, Eq. (1) is clearly irrelevant.

The purpose of this Letter is to modify Eq. (1) in such a way that it becomes applicable to plasma-related systems with soft pairwise interactions. The idea is very simple, we propose to substitute the conventional sound velocity in Eq. (1) by the *transverse sound velocity* c_t. The transverse sound velocity does not exhibit divergence for soft interaction potentials, as c_s

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does. The reduced transverse sound velocity increases monotonously with the coupling parameter and reaches a quasi-universal value at the fluid-solid phase transition [10]. So does the reduced thermal conductivity coefficient of simple fluids in the high density regime [11, 12]. There are several physical grounds to believe that this approach can be successful.

To verify a correlation between the thermal conductivity coefficient and the transverse sound velocity, it is convenient to use appropriate system-independent reduced units. For the thermal conductivity coefficient we use

$$\Lambda_{\rm R} = \Lambda \frac{\Delta^2}{v_{\rm T}}, \quad (4)$$

where $v_{\rm T} = \sqrt{T/m}$ is the thermal velocity. This normalization is essential in the Rosenfeld's excess entropy scaling approach [13], and this is reflected by the subscript "R". The sound velocity is naturally expressed in units of the thermal velocity $v_{\rm T}$. For the Yukawa fluid, the reduced transverse sound velocity is a quasi-universal function of the coupling parameter Γ divided by its value at freezing $\Gamma_{\rm fr}$ [14]:

$$\frac{c_t}{v_{\rm T}} \simeq \left(1 + 22.3\frac{\Gamma}{\Gamma_{\rm fr}}\right)^{1/2}. \quad (5)$$

The values of $\Gamma_{\rm fr}$ for various values of κ (≤ 5) can be found in [15], or estimated from a simple analytical fit of [16].

The dependence of the reduced thermal conductivity coefficient of the Yukawa fluid with different κ values on the reduced transverse sound velocity is shown in Fig. 1. The thermal conductivity coefficient diverges at $c_t/v_{\rm T} = 1$ and drops quickly as $c_t/v_{\rm T}$ increases. The minimum is reached at $c_t/v_{\rm T} \simeq 1.4$. Then the thermal conductivity $\Lambda_{\rm R}$ increases monotonously with $c_t/v_{\rm T}$. At the freezing point we expect quite generally $c_t/v_{\rm T} \sim 5$ [10, 14] and $\Lambda_{\rm R} \sim 10$ for monatomic fluids [6]. The OCP fluid data from [17] clearly support this. In the dense fluid regime a linear correlation between $\Lambda_{\rm R}$ and c_t can be described by a simple linear fit

$$\Lambda_{\rm R} \simeq 2.8\left(\frac{c_t}{v_{\rm T}} - 1\right). \quad (6)$$

To conclude, we propose a modification of the Bridgman formula by correlating the thermal conductivity coefficient with the transverse (shear) sound velocity. This approach is demonstrated to work reasonably well in the screened Coulomb (Yukawa) fluid and can be useful in the context of complex (dusty) plasmas.

Funding. This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

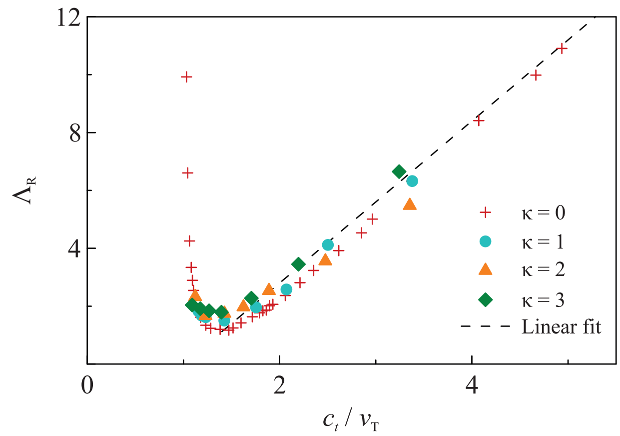

Fig. 1. (Color online) Reduced thermal conductivity coefficient $\Lambda_{\rm R}$ versus the reduced transverse sound velocity $c_t/v_{\rm T}$ in the Yukawa fluid. The symbols correspond to numerical results from [17, 18]. The dashed curve is a linear fit of Eq. (6)

Conflict of interest. The authors of this work declare that they have no conflicts of interest.

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