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Lyapunov exponents in fundamental models of nonlinear resonance

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The problem of analytical estimation of the Lyapunov exponents and Lyapunov timescales of the motion in multiplets of interacting nonlinear resonances is considered. To this end, we elaborate a unified framework, based on the separatrix map theory, which incorporates both an earlier approach for the first fundamental model of perturbed resonance (given by the perturbed pendulum Hamiltonian) and a new one for its second fundamental model (given by the perturbed Andoyer Hamiltonian). Within this framework, new accurate estimates for the Lyapunov timescales of the inner and outer subsystems of the Solar planetary system are presented and discussed.

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Estimations of the Lyapunov exponents provide insights in most fundamental properties of dynamical systems [1–3]. We consider the problem of analytical estimation of the Lyapunov exponents and Lyapunov timescales of the motion in multiplets of interacting nonlinear resonances.

To describe the interaction of resonances in multiplets, we take two fundamental models of the perturbed nonlinear resonance (the guiding one in the multiplet): the first one is the perturbed pendulum model (introduced in [1] as a "universal" one), and the second one is the perturbed Andoyer model (introduced in [4] to describe resonances in orbital dynamics).

We elaborate a unified framework, based on the separatrix map theory, which incorporates both an earlier approach [5–7] for the first fundamental model and a new one for the second fundamental model. We assume that the resonances in the multiplet have comparable strengths. This choice is inspired by the fact that, in realistic applications, the perturbations are usually not at all weak (otherwise the chaotic component of phase space can be simply unimportant); see examples in [5, 8].

Within the given framework, we perform new accurate estimates for the Lyapunov timescales of the inner and outer subsystems of the Solar planetary system, and discuss their conformity with earlier analytical and numerical-experimental estimates.

In particular, we show that the low-dimensional model of [9] provides adequate estimates of the Lyapunov time for the inner Solar system, which conforms with the known massive numerical-experimental estimates in full problem settings.

For the outer Solar system, the presented theory, as applied within the model elaborated in [10] for the 3J-5S-U7 three-body mean-motion resonance, also provides adequate analytical values of the Lyapunov time, in agreement with known numerical-experimental estimates.

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