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First law of de Sitter thermodynamics

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The de Sitter state has a special symmetry: it is homogeneous, and its curvature is constant in space. Since all the points in the de Sitter space are equivalent, this state is described by local thermodynamics. This state has the local temperature $T = H/\pi$ (which is twice the Gibbons–Hawking temperature), the local entropy density, the local energy density, and also the local gravitational degrees of freedom – the scalar curvature $\mathcal{R}$ and the effective gravitational coupling K. On the other hand, there is the cosmological horizon, which can be also characterized by the thermodynamic relations. We consider the connections between the local thermodynamics and the thermodynamics of the cosmological horizon. In particular, there is the holographic connection between the entropy density integrated over the Hubble volume and the Gibbons–Hawking entropy of the horizon, $S_{\rm volume} = S_{\rm horizon} = A/4G$. We also consider the first law of thermodynamics in these two approaches. In the local thermodynamics, on the one hand, the first law is valid for an arbitrary volume V of de Sitter space. On the other hand, the first law is also applicable to the thermodynamics of the horizon. In both cases, the temperature is the same. This consideration is extended to the contracting de Sitter with its negative entropy, $S_{\rm volume} = S_{\rm horizon} = -A/4G$.

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There are different approaches to thermodynamics and quantum theories of de Sitter space, see [1–8] and references therein. The reason is that, unlike a black hole, whose thermodynamics is more or less generally accepted, the de Sitter state is not a localized object. It cannot be considered as a region limited by the cosmological horizon. The de Sitter state is the unbounded symmetric state with constant scalar curvature $\mathcal{R}$. All the points of the de Sitter space are equivalent, that suggests that the temperature of this state is uniform being the same for all the local (co-moving) observers.

It is typically assumed that this temperature is associated with the temperature of the Hawking radiation from the cosmological horizon – the Gibbons–Hawking temperature $T_{\rm GH} = H/2\pi$, where H is the Hubble parameter. However, if we consider the behaviour of any object, for example an atom placed in a de Sitter medium, it turns out that this object perceives this medium as a heat bath with double the Gibbons–Hawking temperature, $T = 2T_{\rm GH} = H/\pi$. This coefficient of two provides the difference between two physical temperatures: the local temperature and temperature of Hawking radiation. This is one of the contradictions present in the construction of the thermodynamics of the de Sitter state. Other contradictions concern the first law of thermodynamics; the sign of entropy, since in some approaches the entropy of the cosmological horizon is negative; and also the value and sign of the energy of the static patch.

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Here we construct the consistent thermodynamics of the de Sitter state, based on the local temperature $T = H/\pi$, which determines the entropy density, and on the contribution of the gravitational degrees of freedom to the thermodynamic pressure. In this approach, the presence of a cosmological horizon does not play any role. However, it turns out that the entropy of the region inside the cosmological horizon (the entropy S_H of the Hubble volume) is positive and obeys the holographic bulk-surface correspondence: its value coincides with the Gibbons–Hawking entropy of the cosmological horizon. Also the holographic bulk-surface correspondence is obtained for the first law of thermodynamics.

We consider the de Sitter thermodynamics using the Painlevé–Gullstrand (PG) coordinates [9, 10], where the metric has no singularity at the cosmological horizon. This allows us to consider local thermodynamics on the same basis at any point in de Sitter space, both inside and outside the horizon: all the points are equivalent. In the PG metric the shift velocity is $\mathbf{v}(\mathbf{r}) = H\mathbf{r}$, where H is the Hubble parameter. The quantum tunneling processes in this metric show that any external object in the expanding de Sitter space radiates pairs of particles with the rate corresponding to the temperature T, which is twice the Gibbons–Hawking temperature, $T = 2T_{\rm GH} = H/\pi$ [11–14]. The process of radiation is local, it takes place well inside the horizon, and thus does not depend on the thermodynamics related to the horizon.

Thus, instead of acting as an inversely populated medium as suggested by Polyakov [15], the de Sitter vacuum looks as a thermal bath with temperature $T = H/\pi$. With the local temperature $T = H/\pi$, the entropy density of de Sitter state is:

$$s_{\rm dS} = -\frac{\partial F}{\partial T} = \frac{\partial \epsilon_{\rm vac}}{\partial T} = \frac{3\pi}{4G}T = \frac{3}{4G}H\,. \qquad (1)$$

Since the de Sitter state is homogeneous, i.e. its local thermodynamic variables do not depend on position, we can consider thermodynamic relations for an arbitrary volume V, regardless of whether it is smaller or larger than the volume of space inside the horizon. The entropy S_H of the part of the de Sitter space, which is surrounded by the cosmological horizon, i.e. the total entropy of the Hubble volume $V_H = (4\pi/3)\, r_H^3$, is $S_{\rm Hubble} = V_H s_{\rm dS} = A/4G$, where $A = 4\pi r_H^2$ is the horizon area and $r_H = 1/H$. This bulk entropy exactly coincides with the entropy of the cosmological horizon suggested by Gibbons and Hawking. However, this global entropy comes from the local entropy of the de Sitter state, rather than from the horizon degrees of freedom. This demonstrates the holographic bulk-surface correspondence.

The conventional Gibbs–Duhem law in the non-gravitating system is $Ts = \epsilon + p$. When gravity is included the thermodynamically conjugate variables are added, $\mathcal{R}$ and $K = 1/16\pi G$, which describe the gravitational degrees freedom [16, 17]. Then we have the modified thermodynamics law $Ts = \epsilon + p + K\mathcal{R}$. It is convenient to introduce the modified pressure $P = p + K\mathcal{R}$, then one returns to the conventional Gibbs–Duhem relation, but with pressure P, i.e. $Ts = \epsilon + P$. The equation of state in terms of the modified pressure is $P = w\epsilon_{\rm vac}$ with $w = 1$. This is equivalent to the equation of state for the Zel'dovich stiff matter [18], in which the speed of sound c_s is equal to the speed of light, $c_s^2 = dP/d\epsilon_{\rm vac} = 1$.

With the modified pressure, the first law of thermodynamics is $TdS = dE + PdV$. This first law of thermodynamics can be applied now to the part of the de Sitter space inside the cosmological horizon – to the Hubble volume V_H:

$$TdS_H = dE_H + PdV_H. \qquad (2)$$

Using the local temperature $T = H/\pi$ one has:

$$TdS_H = T\frac{dA}{4G} = \frac{H}{\pi}d\left(\frac{\pi}{H^2G}\right) = -2\frac{dH}{GH^2}, \qquad (3)$$

$$dE_H = d\left(\frac{4\pi}{3H^3}\frac{3H^2}{8\pi G}\right) = d\left(\frac{1}{2GH}\right) = -\frac{1}{2}\frac{dH}{GH^2}, \qquad (4)$$

$$PdV_H = \frac{3H^2}{8\pi G}d\left(\frac{4\pi}{3H^3}\right) = -\frac{3}{2}\frac{dH}{GH^2}, \qquad (5)$$

and Eq. (2) is satisfied.

This demonstrates that the local thermodynamics in the de Sitter space and the thermodynamics of the cosmological horizon are holographically connected both in relation to entropy and in relation to the first law of thermodynamics. This consideration is extended to the contracting de Sitter with its negative entropy, $S_{\rm volume} = S_{\rm horizon} = -A/4G$, and to black and white holes with the de Sitter core.

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