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Thermodynamics of Kerr black hole: Tsallis–Cirto composition law and entropy quantization

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The processes of splitting and merging of black holes obey the composition law generated by the Tsallis–Cirto $\delta = 2$ statistics. The same composition law expresses the full entropy of the Reissner–Nordström black hole via the entropies of its outer and inner horizons. Here we apply this composition law to the thermodynamics of the Kerr black hole. As distinct from Reissner–Nordström black hole, where the full entropy depends only on mass M and does not depend on its charge Q, the entropy of Kerr black hole is the sum of contributions from its mass M and angular momentum J, i.e. $S(M, J) = S(M, 0) + 4\pi\sqrt{J(J + 1)}$. Here $S(M, 0)$ is the entropy of the Schwarzschild black hole. This demonstrates that when the Kerr black hole with $J \gg 1$ absorbs or emits a massless particle with spin $s_z = \pm 1/2$, its entropy changes by $|\Delta S| = 2\pi$. We also considered the quantization of entropy suggested by the toy model, in which the black hole thermodynamics is represented by the ensemble of the Planck-scale black holes – Planckons. The Tsallis–Cirto composition law is also extended to the thermodynamics of Kerr–Newman black hole and Schwarzschild–de Sitter black hole.

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The processes of splitting and merging of black holes obey the composition law generated by the Tsallis–Cirto $\delta = 2$ statistics. The same composition law expresses the full entropy of the Reissner–Nordström (RN) black hole via the entropies of its outer and inner horizons. Here we apply this composition law to the thermodynamics of the Kerr black hole. As distinct from RN black hole, where the full entropy depends only on mass M and does not depend on its charge Q, the entropy of Kerr black hole is the sum of contributions from its mass M and angular momentum J, i.e. $S(M, J) = S(M, 0) + 4\pi\sqrt{J(J + 1)}$. Here $S(M, 0)$ is the entropy of the Schwarzschild black hole. This demonstrates that when the Kerr black hole with $J \gg 1$ absorbs or emits a massless particle with spin $s_z = \pm 1/2$, its entropy changes by $|\Delta S| = 2\pi$. We also considered the quantization of entropy suggested by the toy model, in which the black hole thermodynamics is represented by the ensemble of the Planck-scale black holes. The Tsallis–Cirto composition law is also extended to the thermodynamics of Kerr–Newman black hole and to the Schwarzschild–de Sitter (SdS) black hole.

Hawking radiation from and Bekenstein entropy of the black holes with multiple horizons and the corresponding thermodynamics can be different from that which is obtained by consideration of only the effects of the outer horizon, see, e.g., the recent paper [1] and references therein. For the spherically symmetric RN black hole with two horizons, several different approaches have been used [2, 3]. This includes: (i) coherent Hawking radiations from two horizons; (ii) macroscopic quantum tunneling; (iii) adiabatic transformations; (iv) singular coordinate transformations; and also (v) the Tsallis–Cirto non-extensive statistics with $\delta = 2$ [4, 5]. Here, we consider the thermodynamics of rotating black holes using the Tsallis–Cirto entropy $S_{\delta=2} = \sum_i p_i \left(\ln \frac{1}{p_i}\right)^2$. Application of this statistics to the composition law for the inner and outer horizons of the Kerr black hole gives the total entropy of the Kerr black hole:

$$\sqrt{S_{\rm Kerr}} = \sqrt{S_+} + \sqrt{|S_-|}, \tag{1}$$

where the entropies of the outer and inner horizons are

$$S_\pm = \pm 2\pi G M^2 \left(1 \pm \sqrt{1 - J^2/M^4G^2}\right). \tag{2}$$

From Eq. (1) one obtains that the total entropy of the Kerr black hole contains two separate terms, the contribution from the mass M and quantized contribution from rotation with angular momentum J:

$$S_{\rm Kerr}(M, J) = 4\pi G M^2 + 4\pi J = S_{\rm Schwarzschild}(M) + 4\pi J. \tag{3}$$

If the Kerr black hole with $J \gg 1$ absorbs or emits a massless particle with spin $s_z = \pm 1/2$, then the mass

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M of the black hole remains the same, while its entropy changes by $|\Delta S| = 2\pi$. Such stepwise behaviour of the black hole entropy is discussed in many papers, starting with Bekenstein who argued that entropy should be quantized in equidistant steps [6, 7], see, e.g., the recent paper [8] and references therein. But in Equation (3) only the rotational part of the entropy of RN black hole is quantized.

The entropy of the extremal Kerr black hole with $J = GM^2$ is

$$S_{\rm Kerr\,extreme} = S(M, J = GM^2) = 2S(M,0) = 8\pi J\,. \tag{4}$$

It is four times the traditionally discussed value $S_0 = 2\pi J$. Also, if Equation (4) is written in the form

$$S_{\rm Kerr\,extreme} = 8\pi\sqrt{J(J+1)}\,, \tag{5}$$

this corresponds to Eq. (11) in [9] with an additional factor of 4. Instead of being two times smaller than the entropy of a Schwarzschild black hole of the same mass M, it is two times larger.

For the Kerr–Newman black hole with charge Q in terms of the electron charge one obtains:

$$S(M,J,Q) = S(M,0,0) + 2\pi\left(\sqrt{\alpha^2Q^4 + 4J^2} - \alpha Q^2\right), \tag{6}$$

where α is the fine structure constant. For $Q = 0$ it becomes the Eq. (3) for the Kerr black hole, for $J = 0$ this transforms into the entropy of the RN black hole.

Planckon model is the toy model, in which the black hole is considered as an ensemble of Planck-scale quanta, Planckons [10]. The Tsallis–Cirto $\delta = 2$ statistics with the non-additive composition law suggests that the entropy of the ensemble is determined by the correlations between Planckons that experience long-range gravitational interaction. It is proportional to the number of the Planckon pairs, i.e. the number of ways to select two Planckons from an ensemble of N Planckons:

$$S_{\rm BH}(N) = C\binom{N}{2} = C\frac{N!}{2!(N-2)!} = C\frac{N(N-1)}{2}\,. \tag{7}$$

Here the dimensionless parameter C depends on the choice of the Planckon mass quantum. In [10] the mass of Planckon was chosen as the reduced Planck mass, $m_{\rm P} = 1/\sqrt{8\pi G}$, which corresponds to $C = 1$. If we choose the Planck mass $M_{\rm P} = 1/\sqrt{G}$ as the mass quantum, i.e. $M = NM_{\rm P}$, one has $C = 8\pi$. Then for the Kerr black hole with $M = NM_{\rm P}$ one would have:

$$S_{\rm Kerr}(M,J) = 4\pi\left(N(N-1) + \sqrt{J(J+1)}\right). \tag{8}$$

For the extremal RN black hole, in the semiclassical limit $N \gg 1$ one has $J = N^2$. This toy model produces the results, which are similar to that of the $SU(N)$ matrix model discussed in [11, 12], where the horizon of Schwarzschild black hole is identified with the fuzzy sphere and N is the dimension of the Hilbert space of quantum gravity. Also, the N^2 entropy in terms of the $N\times N$ Hermitian matrices was considered for the de Sitter cosmological horizon – the dS-matrix theory [13].

Thermodynamic relation $dM = TdS + \mathbf{\Omega}\cdot d\mathbf{J}$ gives temperature and angular velocity of the Kerr hole:

$$T = \frac{\partial M}{\partial S}\bigg|_{\mathbf{J}} = \frac{1}{8\pi GM}\,,\ \mathbf{\Omega} = \frac{\partial M}{\partial \mathbf{J}}\bigg|_{S} = -\frac{\mathbf{J}}{2GMJ}\,. \tag{9}$$

The angular velocity demonstrates the negative moment of inertia and it has the following connection with the temperature: $\Omega \equiv |\mathbf{\Omega}| = 4\pi T$. This suggests the possible modification of the Zel'dovich–Starobinsky rotational superradiance [14–17]. At external temperatures, spontaneous emission can occur even at $\omega > m\Omega$ [18]. The black hole temperature can play a similar role.

Finally, the non-additive Tsallis–Cirto $\delta = 2$ statistics was applied to the Schwarzschild–de Sitter (SdS) black hole with two horizons: the black hole horizon at $r = r_-$ and the cosmological horizon at $r = r_+ > r_-$. In the Susskind dS-matrix theory approach, the discrepancy of a factor of 2 between the composition law and the calculated entropy deficit was found [13]. Here we use the approach based on the singular coordinate transformations [19]. The coordinate transformation $r \to -r$ leads to the state with a single horizon at $r_0 = r_+ + r_-$, which corresponds to the negative mass of black hole. The entropies of the initial and final states obey the composition law $\sqrt{S(r_0)} = \sqrt{S(r_-)} + \sqrt{S(r_+)}$. This corresponds to the Tsallis–Cirto $\delta = 2$ statistics and justifies the dS-matrix theory by Susskind [13].

Whether negative-mass objects exist in the sky is an open question [20], but this does not affect the composition law. The same composition law has been suggested for the entanglement entropy between the geometric and dual sectors of doubled spacetime [21].

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