Pis'ma v ZhETF, vol. 124, iss. 6, pp. 435 – 437

The Encke and Keeler Gaps as Chirikov–Wisdom effects

I. I. Shevchenko[+*1)], *S. A. Drobchik*[+]

[+] *Saint Petersburg State University, 199034 Saint Petersburg, Russia*

[*] *Institute of Applied Astronomy, Russian Academy of Sciences, 191187 Saint Petersburg, Russia*

Submitted 30 June 2026
Resubmitted 11 August 2026
Accepted 12 August 2026

We consider and explain the Encke and Keeler Gaps in the A ring of Saturn as Chirikov–Wisdom effects, namely, as due to accumulation and overlap of the first-order mean-motion resonances of the ring particles with satellites Pan and Daphnis. An analytical improvement of Wisdom's formula for the radial width of the "planetary" (here, satellite's) chaotic zone is provided. It accounts for specific properties of the system: large number of interacting resonances in the multiplet, and their large (similar to each other, by the order of magnitude) amplitudes. Using the improved theory, we show that the width of the Encke Gap is accurately predicted, while the theoretically estimated Keeler Gap width is two times greater than the actual one. The disparity in the latter case is tentatively explained by the relatively small mass of Daphnis (two orders of magnitude less than that of Pan); when mass is so small, one has to take into account the central regular (horseshoe-orbit) part of the Gap.

DOI: 10.7868/S3034576626090131

The Encke and Keeler Gaps in Saturn's ring A are "maintained" by the small satellites Pan and Daphnis, respectively. Pan, discovered by Showalter [1], orbits within the Encke Gap (≈ 325 km wide) at 133 583 km from Saturn's center. Daphnis, discovered by Porco et al. [2], resides in the much narrower Keeler Gap (≈ 42 km wide) at 136 505 km. Both satellites gravitationally perturb the surrounding ring material, obviously maintaining the gaps and creating edge waves [3–5].

Known theoretical estimates for the radial sizes of gaps in planetary rings [4, 6, 7] are purely heuristic, leading to Hill-type scalings ($\Delta a \propto \mu^{1/3}$, μ being the satellite-to-planet mass ratio) with rather indefinite pre-law coefficients.

We consider and explain the Encke and Keeler Gaps as Chirikov–Wisdom effects, namely, as due to accumulation and overlap of the first-order mean-motion resonances of the ring particles with the satellites, and argue that the actual scaling for the gaps' radial sizes is $\propto \mu^{2/7}$, with a well-defined pre-law coefficient.

The theoretical framework for understanding "planetary chaotic zones" was developed by Wisdom [8], who applied Chirikov's resonance overlap criterion [9, 10] to describe close-to-coorbital motion in the restricted three-body problem: the overlap of first-order mean-motion resonances $(p+1){:}p$ between a test particle and a planet (here, a ring particle and a satellite) creates a chaotic zone of half-width $\Delta a_{\rm cr} \propto \mu^{2/7} a_{\rm pl}$ around the planet's orbit. Equating the resonance width $w_{\rm res} = 3.16\,\mu^{2/3} p^{1/3}$ [8] to the inter-resonance separation $r_{\rm sep} = 2/(3p^2)$ gives the critical resonance degree $p_{\rm cr} = 0.513\,\mu^{-2/7}$ and the pre-law coefficient $C = 1.31$ in the formula $\varepsilon_{\rm cr} \equiv \Delta a_{\rm cr}/a_{\rm pl} \approx C\,\mu^{2/7}$. Massive numerical simulations [11] showed the actual zone to be $\sim 20\,\%$ broader ($C = 1.57$), and a theoretical value $C = 1.62$ was later derived by Shevchenko [12, 13] using a specially designed "tokamap" (a kind of the encounter map).

In this work, adapting the Chirikov criterion in the standard-map description of a multiplet of comparable-amplitude first-order resonances, we obtain a new theoretical estimate of the half-width of the planetary chaotic zone; namely,

$$\Delta a_{\rm ch} = \frac{2}{3p_{\rm cr}} \approx 1.586\,\mu^{2/7} a_{\rm pl}. \qquad (1)$$

The derived pre-law coefficient, $C = 1.586$, agrees closely with both the numerical-experimental law of [11] ($C = 1.57$) and the tokamap-based estimate of [12] ($C = 1.62$).

Applying Eq. (1) to Pan ($\mu_{\rm Pan} = m_{\rm Pan}/M_{\rm Saturn} = 8.80 \times 10^{-12}$, using $m_{\rm Pan} = 5.0 \times 10^{15}$ kg [3]) gives $\Delta a_{\rm ch} \approx 148.5$ km for the Encke Gap, in close agreement with the observed value $\Delta a_{\rm obs} \approx 162.5$ km [2, 4] (ratio 1.09), confirming that the Encke Gap is a classical

1) e-mail: i.shevchenko@spbu.ru

Chirikov–Wisdom resonance-overlap effect, observable directly in astronomical images rather than only in analytical or numerical-experimental diagrams.

For Daphnis ($\mu_{\rm Daph} = 1.20 \times 10^{-13}$, using $m_{\rm Daph} = 6.8 \times 10^{13}$ kg derived from ring edge waves [4]), the same formula gives $\Delta a_{\rm ch} \approx 44.0$ km for the Keeler Gap, about twice the observed half-width $\Delta a_{\rm obs} \approx 21$ km [2, 4].

The exposure age of Saturn's rings is estimated to be 100–400 million years [14]. Comparing this age with theoretical timescales for chaotic clearing, we find that the Keeler Gap has apparently had quite enough time to be completely cleared, and the reasons for its observed narrowness should be mostly dynamical.

The central zone of stable horseshoe orbits has a radial extent of the order of one or two Hill radii (see [11]), while the width of the chaotic zone in the Daphnis–Keeler system is equal to approximately five or even four Hill radii, – that is, these two zones in the Daphnis–Keeler system are comparable in width; this is a significant difference from the case of the Pan–Encke system. Since the first-order mean-motion resonances accumulate towards the boundary of the zone of horseshoe librations (and all the latter correspond to the 1/1 resonance), it is necessary, when analyzing the overlap of the resonances, to take into account the width of this central stable zone. This, as we see, is especially important in estimates for the Daphnis–Keeler system. Therefore, the observed significant (twofold) discrepancy for the existing theory (which does not take into account the presence of the central stable zone) with observations of the Keeler Gap is quite expected at a qualitative level. However, accurate quantitative estimates require significant theoretical development, which has not yet been undertaken. Possible avenues for such development and obtaining more accurate estimates will be considered in a separate work. In particular, one should take into account that the first-order resonances monotonously rise in size, according to Wisdom's theory [8], with increasing p (i.e., on approaching the 1/1 resonance) up to infinity, and this is paradoxical; this paradox matters much more in the Daphnis–Keeler case than in the Pan–Encke one, because $p_{\rm cr}$ is much greater.

We also note that the Hamiltonian model that is used here does not include viscous diffusion of ring matter across the gaps. Viscous replenishment of the ring with matter from the gap's vicinity would narrow the observed gap, and this is qualitatively consistent with the fact that the observed width of the Keeler Gap is smaller than our theoretical value. However, quantitative estimates of this effect require specialized modelling, which is beyond the scope of this study. Furthermore, the good agreement between our model and observations for the Encke Gap suggests that, at least in the Encke Gap case, the viscosity effect does not play a significant role.

Concluding, we have presented a systematic analytical application of the Chirikov–Wisdom resonance overlap theory to the Encke and Keeler Gaps in Saturn's ring A. Our main conclusions are as follows.

For the first time ever we have explained the Encke and Keeler Gaps in the A ring of Saturn as Chirikov–Wisdom effects, namely, as due to accumulation and overlap of the first-order mean-motion resonances of the ring particles with satellites (Pan and Daphnis), which thus produce and maintain the Gaps.

We have improved Wisdom's formula for the radial width of the "planetary" (here, satellite's) chaotic zone, by accounting for system's specific properties, namely, the large number of interacting resonances in the multiplet and their large (similar to each other, by the order of magnitude) amplitudes.

Using the improved theory, we have shown that the width of the Encke Gap is accurately predicted, confirming that the Encke Gap is a classical Chirikov's resonance overlap effect.

The theoretically estimated Keeler Gap width is two times greater than the actual (observed) one. This disparity can be due to reasons either dynamical (closeness of the Gap edges to the inner region of regular horseshoe orbits), or cosmogonical (insufficient age of the system as compared to the complete clearing timescale). We argue that the reasons are mostly dynamical.

Our results provide a unified physical and dynamical picture of two iconic features in Saturn's rings and demonstrate that, when using Chirikov's resonance overlap criterion, it is important to include modifications allowing for the considered system specific properties.

Finally, let us note that usually outcomes of chaotic clearings in celestial-mechanical systems are visualized in specific diagrams, histograms, etc., constructed on a basis of either observational data (as in the case of Kirkwood Gaps, see [11, 15]), or numerical-experimental data (as in the case of planetary chaotic zones, see [16–18]). It is remarkable that the Encke and Keeler Gaps provide us with a rare, if not unique, opportunity to see the outcome of resonance overlap directly, in astronomical photo images.

Funding. This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

Conflict of interest. The authors of this work declare that they have no conflicts of interest.

1. M. R. Showalter, "Visual detection of 1981S13, Saturn's eighteenth satellite, and its role in the Encke gap", Nature **351**, 709 (1991).
2. C. C. Porco and The Cassini Imaging Team, "S/2005 S 1", IAU Circular **8524**, 1 (2005).
3. C. C. Porco, P. C. Thomas, J. W. Weiss, and D. C. Richardson, "Saturn's small inner satellites: clues to their origins", Science **318**, 1602 (2007).
4. J. W. Weiss, C. C. Porco, and M. S. Tiscareno, "Ring edge waves and the masses of nearby satellites", Astron. J. **138**, 272 (2009).
5. R. A. Jacobson, J. Spitale, C. C. Porco, K. Beurle, N. J. Cooper, M. W. Evans, and C. D. Murray, "Revised orbits of Saturn's small inner satellites", Astron. J. **135**, 261 (2008).
6. P. Goldreich and S. Tremaine, "The dynamics of planetary rings", Annu. Rev. Astron. Astrophys. **20**, 249 (1982).
7. N. Torii, S. Ida, E. Kokubo, and S. Michikoshi, "Global N-body simulation of gap edge structures created by perturbations from a small satellite embedded in Saturn's rings", Icarus **415**, 116029 (2024).
8. J. Wisdom, "The resonance overlap criterion and the onset of stochastic behavior in the restricted three-body problem", Astron. J. **85**, 1122 (1980).
9. B. V. Chirikov, "Resonance processes in magnetic traps", Journal of Nuclear Energy **1**, 253 (1960).
10. B. V. Chirikov, "A universal instability of many-dimensional oscillator systems", Phys. Rep. **52**, 263 (1979).
11. C. D. Murray and S. F. Dermott, *Solar System Dynamics* (Cambridge University Press, Cambridge, 1999).
12. I. I. Shevchenko, "Lyapunov and clearing timescales in planetary chaotic zones", Astron. J. **160**, 212 (2020).
13. I. I. Shevchenko, *Dynamical Chaos in Planetary Systems* (Springer Nature, Cham, 2020).
14. S. Kempf, N. Altobelli, J. Schmidt, J. N. Cuzzi, P. R. Estrada, and R. Srama, "Micrometeoroid infall onto Saturn's rings constrains their age to no more than a few hundred million years", Sci. Adv. **9**, eadf8537 (2023).
15. A. Morbidelli, *Modern Celestial Mechanics: Aspects of Solar System Dynamics* (Taylor and Francis, London, 2002).
16. T. V. Demidova and I. I. Shevchenko, "Three-lane and multi-lane signatures of planets in planetesimal disks", Mon. Not. R. Astron. Soc. **463**, L22 (2016).
17. T. V. Demidova and I. I. Shevchenko, "Long-term dynamics of planetesimals in planetary chaotic zones", Astron. Lett. **46**, 774 (2020).
18. T. V. Demidova and I. I. Shevchenko, "Evolution of planetary chaotic zones in planetesimal disks", Astron. Lett. **47**, 771 (2021).