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Some features of high-temperature superconductivity on flat bands

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In this letter, we examine how the presence of flat band leads to the formation of a high-temperature superconductor even in the case of repulsive pairing interactions. We also show that in the case of flat bands, the high-temperature superconducting state deforms the flat band, tilting it and making the effective mass finite. As a result, neither the superfluid weight nor the supercurrent disappear. Our results are in good agreement with experimental data.

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In standard the Bardeen–Cooper–Schrieffer (BCS) theory, attractive forces such as the electron-phonon interaction form Cooper pairs. However, in 1965, Kohn and Luttinger demonstrated that even if the pure interaction between fermions is strictly repulsive (e.g., Coulomb repulsion), quantum mechanical corrections and dynamical screening by the Fermi surface can change the sign of the interaction in certain angular momentum channels. Thus, higher-order quantum scattering processes lead to a change in the repulsive interaction at certain momentum transfers, such as $q \approx 2p_F$, where p_F is the Fermi momentum. This leads to an effective attraction, allowing fermions to form pairs, see, e.g., [1].

Flat bands formed by the topological fermion condensation quantum phase transition are fixed at the chemical potential μ, being shaped by strong electron-electron interactions within the region $p_f \geq p \geq p_i$ [2–4],

$$\varepsilon(\mathbf{p}) - \mu = 0; \;\text{ if }\; p_f \geq p \geq p_i, \tag{1}$$

where $\varepsilon(\mathbf{p})$ is single-particle energy and μ is the chemical potential. The occupation numbers of quasiparticles $n(\mathbf{p})$ become fractional within the flat band range given by Eq. (1)

$$1 > n(\mathbf{p}) > 0. \tag{2}$$

These flat bands have influenced the landscape of modern condensed matter physics, which studies strongly correlated Fermi systems [2–9]. Volovik demonstrated that the fermion condensation is essentially formed by topological phase transition in momentum space, leading to a new class of Fermi liquids [3]. In Landau's Fermi liquid theory, electronic properties were understood primarily in terms of the kinetic energy of quasiparticles: How quasiparticles move within a crystal. Flat bands have changed this paradigm. By reducing the kinetic energy to zero, flat bands make electron-electron interactions the absolutely dominant force determining the properties of strongly correlated Fermi systems.

As a result, the BCS theory has also undergone changes; for example, in the case of high-temperature (high-T_c) flat-band superconductors, the critical temperature T_c is strictly linear [2, 4, 5]

$$T_c \propto \Delta_1 \propto \lambda_0, \tag{3}$$

rather than being exponentially suppressed $T_c \propto \propto \exp(-1/\lambda_0 N(0))$. From Equation (3) it is clear that the flat band significantly increases the critical temperature T_c of high-temperature (high T_c) superconductors. Here λ_0 is the superconducting coupling constant, Δ_1 is the maximum value of the superconducting gap and $N(0)$ is the density of states at the Fermi surface. According to Eq. (3), flat bands are considered a pathway to high-temperature superconductivity, see, for example, [9, 10], while recent experimental data may confirm these long-standing predictions of [11].

In standard wide-band gap BCS superconductors, at zero temperature $T = 0$, all electrons participate in the formation of the superfluid density. The superfluid weight is described by:

$$D_s \simeq \frac{n_e e^2}{M^*}, \tag{4}$$

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where n_e is the electron carrier density, e is the electron charge, and M^* is the effective mass. As can be seen from Eq. (4), heavy electrons suppress the superfluid weight D_s. If the band narrows, M^* increases, which leads to a decrease in D_s. In strongly interacting, topologically nontrivial flat bands pinned at the chemical potential μ (for example, in twisted bilayer graphene), the energy dispersion $\varepsilon(\mathbf{p})$ is constant. Therefore, the effective mass becomes infinite, $M^* \to \infty$, which, according to common sense, means that quasiparticles cannot move, see, e.g., [12]. Surprisingly, D_s does not drop to zero. To explain this fact, it is usually assumed that the finite values of both D_s and T_c are completely determined by quantum-geometric origin, see, e.g., [10, 12, 13]. On the other hand, experimental data show that T_c is proportional to the velocity V_F of quasiparticles at the Fermi surface, $T_c \propto V_F$, [14–16]. In this case, the disappearance of the Fermi velocity ($V_F \propto 1/M^* \to 0$) corresponds to the presence of flat band, while $T_c \to 0$ and the superconducting state vanishes, not being restored by the quantum geometry approach. Thus, at present, an approach based solely on quantum geometry cannot explain the behavior of flat-band superconductivity.

To clarify this important point, we turn to the traditional equations of the BCS theory, applicable in the case of flat bands [16], linking the single-particle energy $\varepsilon(\mathbf{p})$ with the superconducting gap $\Delta(\mathbf{p})$

$$\varepsilon(\mathbf{p}) - \mu = \Delta(\mathbf{p})\frac{1-2v^2(\mathbf{p})}{2v(\mathbf{p})u(\mathbf{p})}. \qquad (5)$$

At $T = 0$ the coherence factors $v(\mathbf{p})$ and $u(\mathbf{p})$ are given by

$$n(\mathbf{p}) = v^2(\mathbf{p}),\ u^2(\mathbf{p}) + v^2(\mathbf{p}) = 1, \qquad (6)$$

and the order parameter $\kappa(\mathbf{p})$ becomes

$$\kappa(\mathbf{p}) = v(\mathbf{p})u(\mathbf{p}) = \sqrt{n(\mathbf{p})(1-n(\mathbf{p}))}. \qquad (7)$$

Here $n(\mathbf{p})$ is given by Eq. (2). The superconducting gap $\Delta(\mathbf{p})$ is given by

$$\Delta(\mathbf{p}) = -\frac{1}{2}\int \lambda_0 V(\mathbf{p},\mathbf{p}_1)\frac{\Delta(\mathbf{p}_1)}{E(\mathbf{p}_1)}\frac{d\mathbf{p}_1}{4\pi^2}. \qquad (8)$$

Here the excitation energy $E(\mathbf{p})$ is defined by the Bogoliubov quasiparticles

$$E(\mathbf{p}) = \sqrt{(\varepsilon(\mathbf{p})-\mu)^2 + \Delta^2(\mathbf{p})}. \qquad (9)$$

From Equation (5) it is clear that a high-T_c superconducting state operates within the well-known "self-help" paradigm. Indeed, one takes into account the "terrible" Eq. (4) and immediately eliminates the flat band and reduces the value of the effective mass M^* based on Eq. (5), and the effective mass voluntarily becomes $M^* \propto 1/\Delta_1$ [6, 16, 17], which invalidates the disappearance of superfluid weight. One then checks whether the "self-help" paradigm has done a good job: from Eq. (5) it can be seen that $\varepsilon(\mathbf{p}) - \mu \simeq \Delta$, one substitutes this relation into Eq. (8) and arrives at Eq. (3), concluding that the result is consistent with the "well done" principle. As a result, the "dangerous" flat band is cleverly eliminated, Eq. (3) continues to work, and the supercurrent persists. Then, it is seen from Eqs. (2) and (7) that $\kappa(\mathbf{p}) \neq 0$ even for $\lambda_0 = 0$, when $T_c \propto \Delta_1 \propto V_F = 0$. It shows that the pairing is initiated by strong repulsive interaction that forms flat bands [2, 4, 5]. From Eqs. (3) and (8) seen that a non-trivial solution $\Delta_1 \neq 0$ can exist even if λ_0 becomes repulsive [5]. Indeed, according to Eqs. (3) and (8), λ_0 is nothing more than a proportionality factor in the relationship between gap Δ_1 and the order parameter $\kappa(\mathbf{p})$. Thus, high-T_c superconductors with repulsive pair interaction must be based on flat-band compounds, since only in this case the superconductivity is ensured by Eq. (3). In the case of effective attraction, allowing fermions to form pairs, T_c is determined by Eq. (8), which allows only an exponentially small value of T_c in the absence of the corresponding flat band, see, e.g., [1]. Data show that the corresponding flat bands in twisted graphite are formed by electron correlations [18], while the long-awaited flat-band superconductivity at room temperature [9, 10] is likely observed in intercalated graphite [11].

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