Письма в ЖЭТФ, том 56, вып.1, стр.40 - 43 10 июля 1992 г.

MACROSCOPIC JOSEPHSON EFFECT IN SUPERFLUID ^{3}He-B

T. Sh. Misirpashaev, and G.E. Volovik

Landau Institute for Theoretical Physics,
117334, Moscow, Russia
Low Temperature Laboratory, Helsinki University of Technology,
02150, Espoo, Finland

Submitted 9 June 1992

Recently an experiment with a single vortex line of macroscopic length in superfluid ^{3}He-B has been reported, in which the periodic motion of the vortex has been observed with a period of several minutes [1]. We discuss this periodic process in terms of the macroscopic ac Josephson effect and derive an expression for the period T, which is defined by the geometry and by the circulation quantum $\kappa = h/2m_3$.

The vibrating wire technique, used to measure quantized circulation of superfluid velocity in superfluid ^{4}He, [2] has been recently applied to ^{3}He-B. As a result the circulation quantum $\kappa = h/2m_3$ trapped by the wire, when the wire absorbs one vortex filament, has been measured [3]. In further experiments the transient process of untrapping of circulation from the wire has been observed [1], in which a segment of quantized vortex line "unzips" from the wire. This allowed to investigate the dynamics of the individual vortex. It appeared that the unzipped vortex segment performs a precessing motion around the wire with the stable period of precession $T = 253$ s. We interpret this periodic motion as the macroscopic manifestation of the ac Josephson effect, in which the corresponding voltage is produced by the hydrodynamic energy of the trapped circulation, while the phase slip occurs due to the precession of the vortex segment. We show that the period T can be obtained from the Josephson relation and is defined by the geometry and the circulation quantum. For given geometry of the experiment we obtain $T = 253.2$ s.

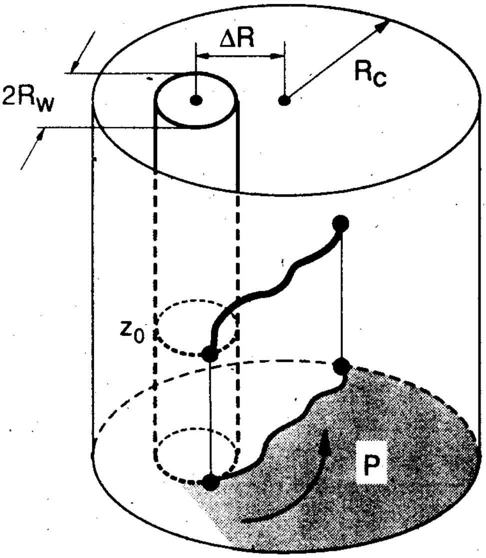
The geometry of the experiment [1] is shown in Fig.1: a wire of radius $R_w = 8$ μm is inside a cylindrical cell of radius $R_c = 1.48$ mm at a distance $\Delta R = 0.35$ mm from the axis of the cylinder. In the transient process one quantum of circulation κ of velocity is trapped by the wire at $z < z_0$, while at $z = z_0$ a vortex filament is unzipped off the wire and terminates at the wall of the vessel. We consider here the general case of the vortex segment with $N_2 - N_1$ quanta, whose termination point z_0 on the wire separates the upper part of the wire with N_2 trapped quanta from the lower part with N_1.

First we consider the dynamics of the vortex segment. Since the temperature is quite low $\sim 0.2T_c$, we neglect the dissipation. To find the precession rate of the segment around the wire one can use Newton's second law applied to the whole vortex segment:

$$dP/dt = F \quad . \tag{1}$$

Here P is the z projection of the linear momentum of the vortex line. It is a multivalued quantity expressed through the variation of the area swept by the projection of the vortex line on the cross-sectional plane: $\delta P = (N_2 - N_1)\rho\kappa\ \delta S$ [4].

The external force F comes from the difference in the hydrodynamic energy of superflow around the wire, induced by the difference in trapped circulation above and below the vortex segment: $F = E(N_2) - E(N_1)$. In the case when both R_c



Sketch of the vortex (heavy solid line) precessing around the wire. The vortex momentum P increases under the hydrodynamic force arising from the trapped flux. As a result the projection of the vortex on the transverse plane sweeps across the plane, producing a periodic phase-slip

and R_w are larger then the coherence length, the energy $E(N)$ is given by the classical expression $(1/2)\int dS\ \rho\vec{v}_S^2$. The integral is over the cross section of the vessel far above or far below the vortex segment, where the superfluid velocity field $\vec{v}_S = (\kappa/2\pi)\vec{\nabla}\Phi$ around the wire is not disturbed by the presence of the vortex line. Under these conditions the calculation of $E(N)$ corresponds to the calculation of the electric capacity of two noncoaxial cylinders, which gives:

$$E(N) = \rho N^2 \frac{\kappa^2}{4\pi} \cosh^{-1} \frac{R_w^2 + R_c^2 - (\Delta R)^2}{2R_w R_c} \quad . \tag{2}$$

Under the force F the momentum P increases with time, which means that the vortex segment is forced to precess, sweeping the whole cross sectional area $S = \pi(R_c^2 - R_w^2)$ during one period T. The change in the momentum during the precession period, $P(t+T) - P(t) = (N_2 - N_1)\rho\kappa S$, should be equal FT, which gives for T the following equation:

$$T = \frac{(N_2 - N_1)\rho\kappa S}{E(N_2) - E(N_1)} \quad . \tag{3}$$

The period is thus completely determined by the circulation quantum κ, the numbers N_1 and N_2 of trapped flux, and by the geometry. It does not depend on any details of the vortex precession, i.e. on the shape of the vortex line or on its core structure. Inserting the values of R_c, R_w, and ΔR, one has $T = |N_1 + N_2|^{-1} \cdot 253.2$ s which coincides with the experimental $T = 253 \pm 1$ s, obtained for the case $N_1 = 1$, $N_2 = 0$. The theoretical result presented in [1] is slightly different: it was assumed in [1] that the vortex segment moved as a solid body, which is valid only if the wire is not shifted from the axis of the vessel, i.e. $\Delta R = 0$. We used here the less restrictive assumption, that the motion is periodic.

This process bears all the features of the ac Josephson effect. The constant force F is applied to the liquid, which results in the time-dependent (periodic) motion of the vortex line. One can represent this force in the familiar form of the difference in the chemical potentials, which play the role of voltage in the electrically neutral liquid. The force per unit area of the vessel F/S plays the part of the pressure difference applied between the top and bottom walls of the vessel, and divided by the particle density ρ/m_3 it gives the effective difference in the chemical potentials,

$$\mu_2 - \mu_1 = \frac{Fm_3}{\rho S} = \frac{m_3(E(N_2) - E(N_1))}{\rho S} \quad . \tag{4}$$

According to Eq.(3) the frequency of the periodic process, $\omega = 2\pi/T$, is related to this voltage by the Josephson relation

$$\omega = \frac{2}{\hbar}(\mu_2 - \mu_1) \quad . \tag{5}$$

This is not surprising since the vortex, which sweeps the cross section of the vessel, realizes the periodic phase-slip process: after each period T the phase difference $\Phi_2 - \Phi_1$ between the top and bottom walls changes by 2π due to vortex motion, which compensates the change caused by the difference in the chemical potentials. The kinematic phase slip equation is [5]

$$\partial_t \vec{v}_S + \nabla\mu = \kappa(N_2 - N_1) \oint d\sigma \;\; \vec{r}_\sigma \times \dot{\vec{r}} \;\; \delta^3(\vec{r} - \vec{r}(\sigma, t)) \quad , \quad \mu = \frac{1}{2}\vec{v}_S^2 + \frac{\partial \varepsilon(\rho)}{\partial \rho} \tag{6}$$

The integration of this equation over the volume of the vessel and over period T under condition that there is no difference in the mass density ρ between upper and lower parts of the container gives:

$$\int_0^T dt \int d^3r \partial_t \vec{v}_S = 0 \quad , \tag{7}$$

$$\int_0^T dt \int d^3r \nabla\mu = T\hat{z}\frac{E(N_2) - E(N_1)}{\rho} \quad , \tag{8}$$

$$\int_0^T dt \int d^3r \kappa(N_2 - N_1) \oint d\sigma \;\; \vec{r}_\sigma \times \dot{\vec{r}} \;\; \delta^3(\vec{r} - \vec{r}(\sigma, t)) = \kappa(N_2 - N_1) \int d\sigma \;\; dt \;\; \vec{r}_\sigma \times \dot{\vec{r}} =$$

$$= \kappa(N_2 - N_1)S\hat{z} \quad , \tag{9}$$

Here $\vec{r}(\sigma, t)$ denotes the position of the vortex line, and σ is the coordinate along the line with

$$\vec{r}_\sigma = \partial_\sigma \vec{r}(\sigma, t) \quad , \quad \dot{\vec{r}} = \partial_t \vec{r}(\sigma, t) \quad ,$$

S is again the cross sectional area of the vessel, which is swept by the vortex during one period. From Eq.(6) it follows that Eq.(8)=Eq.(9), as a result one obtains the Eq.(3) for the period of precession.

Hence the precession frequency can be derived directly from the Josephson equation (5) and (4) without using the motion equation for the vortex, Eq.(1). The Josephson relation is of the kinematic origin and does not depend on the details of vortex dynamics. In particular it is not important if the vortex is frozen into the liquid, i.e. moves with the local velocity, or if it has its own inertial mass. The Josephson equation is so general that can be applied even when the hydrodynamic equations, which define the vortex motion, do not hold, for example in the case when R_w is of order or even less than the coherence length. Even in this case the Eqs. (5) and (4) do hold, but the energy $E(N)$ of trapped flux is given not only by the hydrodynamic energy of superflow: one should also add the energy of the distortion of the superfluid state in the layer of the coherence length size near the wire. Note that the precession frequency ω will change if in addition to the internal "voltage" caused by the trapped circulation, an external pressure or temperature difference is applied.

In conclusion we have shown that the experiment reported in Ref.[1] represents the observation of the quantum Josephson effect on the macroscopic scale, where the phase slip process is realized by the extremely slow precessing motion of the single vortex line.

We thank M. Krusius, R.E. Packard, E.B. Sonin and K.W. Schwarz for discussions. This work was supported through the ROTA co-operation plan of the Finnish Academy and the USSR Academy of Sciences.

---

1. R.J.Zieve, Yu.Mukharsky, J.D.Close, J.C.Davis, and R.E.Packard, Phys. Rev. Lett. **68**, 1327 (1992).
2. W.F.Vinen, Prc. R. Soc. London **A 260**, 218 (1961).
3. J.C.Davis, J.D.Close, R.J. Zieve, and R.E.Packard, Phys. Rev. Lett. **66**, 329 (1991).
4. M.Rasetti, and T.Regge, Physica **A 80**, 217 (1975).
5. E.B.Sonin, Rev. Mod. Phys. **59**, 87 (1987).