BOUND STATES NEAR A SHORT RANGE IMPURITY IN CROSSED MAGNETIC AND ELECTRIC FIELDS

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The problem of quantum scattering of an electron by a short range impurity in a two-dimensional electron gas in the presence of a perpendicular magnetic and a longitudinal electric field is solved. It is shown that additionally to the bound state which exists also in the case of zero electric field, in the presence of an electric field N novel nondegenerate quasi bound states appear with energies close to the N-th Landau level, irrespective whether the impurity is attractive or repulsive.

In analytical and numerical papers, which adress magnetokinetic phenomena in bulk semiconductors [1], the Quantum Hall Effect [2,3], the conductance of a microconstriction [4], and Variable-range-hopping magnetoresistance [5], it was important to investigate the electron scattering process by an individual impurity in a magnetic field. In these works, the scatterer is of short range type. In paper [2], the scattering on an impurity of the drifting electron is considered for the case of crossed electric and magnetic fields, but the bound states near the lowest Landau level (LL) was mainly considered. In our paper we consider more carefully the possible bound states around the higher Landau levels, too. In the paper [3] the intersubband tunneling of drifting electrons via an impurity is studied and the scattering potential is taken to be a δ-function. On the incorrectness of the use of the $2D-\delta-$function as a scattering potential is written in the works [4-6]. In the works [7-9] we used a method,which allows to circumvent this difficulty. Additionally our method turned out to be very efficient for the investigation of the structure of bound states. Hence, the further concise calculations lie in the context of this method.

Let the magnetic field H is uniform and directed perpendicularly to the plane of the two-dimensional electron gas (2DEG), and the uniform electric field E lies in the plane of the 2DEG, being directed along the y axis. It is well-known that the electron under such circumstances drifts along the x axis with the velocity $v=cE/H$. In the Landau gauge, the wavefunctions of that motion, normalized to unity $\int d^2r\Psi_{nk}(r)\Psi^*_{n'k'}(r)=\delta(k-k')\delta_{nn'}$, have the form

$$\Psi_{nk}(r)=\frac{1}{(2\pi)^{3/4}\sqrt{n!}}\cdot e^{ik\xi}\cdot D_n(s-2k+2\alpha). \qquad (1)$$

Here, D_n are the functions of the parabolic cylinder with integer indices, $n=0,1,2,..$ [10]. Here we introduced the dimensionless coordinate $r=(\xi,s)$ with $\xi=\sqrt{2}/a_H\cdot x$, $s=\sqrt{2}/a_H\cdot y$; the momentum $k=a_H/\sqrt{2}\cdot p_x/\hbar$, the drift velocity $2\alpha=\sqrt{2}/a_H\omega_H\cdot v$, where the magnetic length is $a_H=\sqrt{\hbar/m\omega_H}$, and the cyclotron frequency $\omega_H=|e|H/mc$. The energy of the drifting electron in units of $\hbar\omega_H$ reads $\varepsilon_{nk}=n+\frac{1}{2}+2\alpha k-\alpha^2$ and is composed from the kinetic energy of the cyclotron motion $n+\frac{1}{2}$, the kinetic energy of the drifting motion α^2, and the potential energy $2\alpha(k-\alpha)$.

At a fixed energy ε there is an infinite set of drifting states which are separated in space if $\alpha \ll 1/\sqrt{n}$. Let be, additionally, that such a drifting state with a wavefunction $\Psi_{Nk_N}(r)$ and $k_N=(\varepsilon-N-\frac{1}{2}+\alpha^2)/2\alpha$ is scattered by a short range impurity (which is an s-scatterer) sitting in r_0. The scattered field for such an impurity is [8]

$$\psi'(r) = -2\pi\Psi_{Nk_N}(r_0)\cdot\frac{G_\varepsilon(r,r_0)}{D_\varepsilon(r_0)}. \tag{2}$$

Here, $G_\varepsilon(r,r_0)$ is the Green's function of outgoing waves of the Schrödinger's equation without scattering potential. $G_\varepsilon(r,r_0)$ can be expanded in eigenfunctions (1) (see [11]). Additionally, in a further expansion of the small parameter α the N-th LL gives the main contribution:

$$G_\varepsilon(r,r_0)|_{r_0=0} = \frac{1}{(2\pi)^{3/2}N!\cdot\alpha}\cdot\int_{-\infty}^{+\infty} dk \;\; e^{ik\xi}\cdot\frac{D_N(s-2k)D_N(-2k)}{2k-\varepsilon_N/\alpha-i\cdot 0}. \tag{3}$$

Here, we set for convenience $r_0=0$ and omit the gauge factor $e^{i\xi\alpha}$ (we can eliminate it by slightly varying the gauge), and $\varepsilon_N=\varepsilon-N-\frac{1}{2}-\alpha^2$. The expression (3) is valid everywhere except the region near the impurity where the Green's function diverges:

$$G_\varepsilon(r,r_0)|_{r\to r_0} = \frac{1}{2\pi}\left[\ln\left(\frac{1}{|r-r_0|}\right)+K_\varepsilon(r_0)\right]. \tag{4}$$

The quantity $K_\varepsilon(r_0)$ plays an important role for the analysis of the structure of bound states because it appears in the denominator of the scattering amplitude in (2):

$$D_\varepsilon(r_0)=\Lambda+K_\varepsilon(r_0), \qquad \Lambda=\ln\left(\frac{a_H}{\sqrt{2}\cdot a}\right), \tag{5}$$

where a is the 2D scattering length of the impurity scattering potential. The exact formula for $K_\varepsilon(r_0)$ takes the form [1)].

$$K_\varepsilon(r_0)|_{r_0=0} = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{1}{\alpha\cdot i^{n-1}}\cdot D_n(\varepsilon_n/\alpha)D_{-n-1}(-i\varepsilon_n/\alpha)+\frac{1}{\varepsilon_n}\right]+K_{\varepsilon,\alpha=0}\equiv K_\varepsilon, \tag{6}$$

where

$$K_{\varepsilon,\alpha=0} = -\frac{1}{2}\psi(1/2-\varepsilon)-C+\ln 2, \tag{7}$$

and $\psi(1/2-\varepsilon)$ is the digamma function [10], $C=0.577..$ is Euler's constant . Eq.(7) can be obtained using [10,12] from the well-known Green's function of an electron in a magnetic field [13]. The plots of the functions K_ε and $K_{\varepsilon,\alpha=0}$ are given in Fig.1. The presence of the electric field leads to a non-vanishing imaginary part of the Green's function, and to oscillations of both real and imaginary part of K_ε . These oscillations are localized close to each LL and do not overlap with those of neighboured levels if $\alpha \ll 1/\sqrt{N}$. The amplitude of the oscillations of K_ε increase

[1)] The principal way how to obtain (6) and a similar result are given in [9] with the decreasing of the electric field as $1/\alpha$. In this case it is convenient to rewrite (6) in the form

$$K_\epsilon = \frac{1}{2i^{N-1}\cdot\alpha}\cdot D_N(\varepsilon_N/\alpha)D_{-N-1}(-i\varepsilon_N/\alpha) + P, \tag{8}$$

where P is a quantity in the order of unity which weakly depends on N and on the energy in the vicinity of the N-th LL. With the condition $\alpha \ll 1/N$ the imaginary part of P is exponentially small. Thus we can take P as a real constant.

Bound states are defined as scattering amplitude poles $\bar{\varepsilon} - i\Gamma$, located near the real energy axis. These poles are solutions of the equation

$$D_\varepsilon = \tilde{\Lambda} + \frac{1}{2i^{N-1}\cdot\alpha}\cdot D_N(\varepsilon_N/\alpha)D_{-N-1}(-i\varepsilon_N/\alpha) = 0, \qquad \tilde{\Lambda} = P + \Lambda. \tag{9}$$

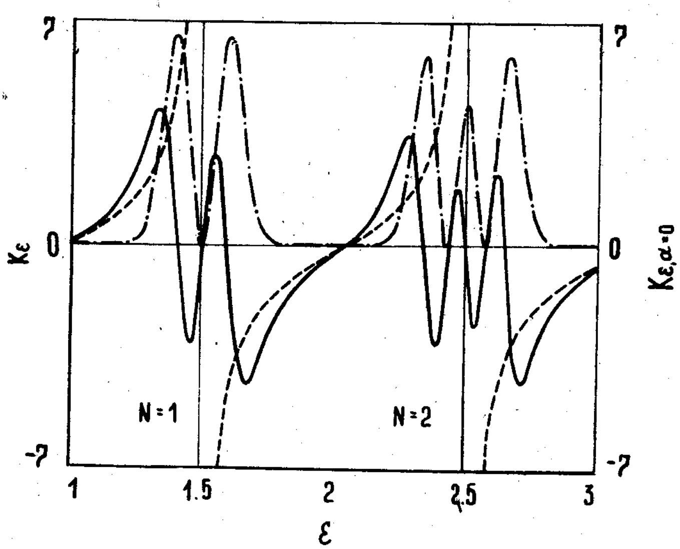
The assumption of the smallness of Γ allows us to write it in the form

$$\Gamma = \left[\frac{\mathrm{Im}K_\epsilon}{d/d\epsilon \mathrm{Re}K_\epsilon}\right]_{\epsilon=\bar{\epsilon}}. \tag{10}$$

From Fig. 1 and the general form of equation (9) immediately follows the first important result: bound states exist only if the condition

$$|\tilde{\Lambda}|\alpha \ll 1 \tag{11}$$

is fulfilled, whereas in the absence of an electric field ($\alpha = 0$) bound states always and for each impurity exist.



Plots of $\mathrm{Re}K_\epsilon$ (——) and $\mathrm{Im}K_\epsilon$ (–·–) as well as $K_{\epsilon,\alpha=0}$ (- - -), for N=1 and N=2. α is equal to 0.07. The separation of oscillations belonging to neighboured Landau levels is clearly to see.

If the condition (11) is satisfied, there is at least one bound state, fully equivalent to that at $\alpha = 0$. Using the asymptotic form [12] of the functions D_N and D_{-N-1} in the limit $|\varepsilon_N/\alpha| \to \infty$, we get from [10] the bound state energy relative to the N-th LL

$$\varepsilon_N = \frac{2}{\tilde{\Lambda}} \tag{12}$$

and their width

$$\Gamma_N \sim \frac{\alpha}{(\alpha\tilde{\Lambda})^{2N+2}} \cdot \exp\left[-\frac{2}{(\alpha\tilde{\Lambda})^2}\right]. \tag{13}$$

As it was to be expected, the width of this bound states rapidly vanishes for $\alpha \to 0$ and the states become non-decaying. This bound state can occur below a LL (if $\tilde{\Lambda} < 0$) as well as above (if $\tilde{\Lambda} > 0$).

The presence of the electric field gives a set of new nondegenerate states near each LL with $N > 0$. Their number for the N-th LL is equal to N in correspondence with the number of zeros in the imaginary part of K_ε

$$\mathrm{Im}K_\varepsilon = \sqrt{\frac{\pi}{8}} \cdot \frac{[D_N(\varepsilon_N/\alpha)]^2}{N! \cdot \alpha}. \tag{14}$$

The twofold degenerated zeros of $\mathrm{Im}K_\varepsilon$ lie very close to simple zeros of the real part of K_ε

$$\mathrm{Re}K_\varepsilon = \frac{1}{2\cdot\alpha} \cdot D_N(\varepsilon_N/\alpha) \cdot \left[i^{1-N} D_{-N-1}(-i\varepsilon_N/\alpha) - \frac{\sqrt{\pi/2}}{N!} \cdot D_N(\varepsilon_N/\alpha)\right] + P. \tag{15}$$

It is this fact that leads to the existence of a set of narrow bound states. Using (14), (15) and the Wronskian relation $W[D_N(z), D_{-N-1}(-iz)] = i^{N+1}$, we get the energies of the bound states up to the second order in α

$$\varepsilon_N^m = \alpha\sigma_N^m - 2\alpha^2 \cdot \tilde{\Lambda}, \qquad m = 1, 2, .., N. \tag{16}$$

Their widths are

$$\Gamma_N^m \sim \alpha^3 \tilde{\Lambda}^2. \tag{17}$$

Here, σ_N^m are the zeros of the N-th Hermite polynomial.

From (17) and (9) follows that the existence of non-decaying bound states ($\Gamma_N^m = 0$) is possible for a moderate strength of the impurity, $|\Lambda| = |P| \sim 1$. The wavefunction of these bound states is proportional to the scattered field (2), and hence to the Green's function (3). As it was to be expected, the wavefunction of these bound states is localized. Indeed, if $\tilde{\Lambda} = 0$, then $\varepsilon_N^m/\alpha = \sigma_N^m$ and the zero of the denominator of the integrand in (3) cancels the zero of $D_N(-2k)$. In this case, the right hand side of (3) is a Fourier integral of a smooth function and becomes exponentially small for $|\xi| \to \infty$.

It is obvious that these bound states exist not only in a uniform electric field but they are characteristic for any smooth potential in which drifting states are present (for example, in a parabolic confinement potential [9]).

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