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THE COHERENT RESONANCES IN SPECTRA OF e^+-e^- PAIRS CREATED BY HARD γ-QUANTA IN ALIGNED SINGLE CRYSTALS

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New pronounced coherent effects of a resonant type are predicted theoretically in e^+-e^- pair production spectra under conditions of transition from the axial channeling regime to a planar one.

1. The space-energy density of e^+-e^- pairs created per unit time by a non-polarized beam of γ-quanta in an averaged potential of atomic chains in a single crystal aligned near an axial direction is expressed in the quasi-classical Baier-Katkov method [1] as an integral in time of a definite function depending on transverse (relative to the singled-out axial family) classical coordinates $\boldsymbol{r}_\perp^\pm$ and velocities $\boldsymbol{v}_\perp^\pm$ of one of the constituents (a positron (+) or an electron (-)) of the arising pair:

$$\frac{d^2N_p}{d\xi_\pm dS_\perp} = \frac{e^2m^2c^3}{S_\perp^0\pi\hbar^3\omega}\left\{\int_0^\infty\left[\frac{\xi_+^2+\xi_-^2}{4\xi_+\xi_-}\frac{\gamma_\pm^2}{c^2}\left(\boldsymbol{v}_\perp^\pm(\tau)-\boldsymbol{v}_\perp^\pm(-\tau)\right)^2-1\right]\frac{\sin f_\pm(\tau)}{\tau}\,d\tau+\frac{\pi}{2}\right\}, \tag{1}$$

$$f_\pm(\tau) = \frac{c}{\ell_c(\xi_\pm,\omega)}\left\{2\tau+\frac{\gamma_\pm^2}{c^2}\left[\int_{-\tau}^{\tau}\left(v_\perp^\pm(\tau')\right)^2d\tau'-\frac{1}{2\tau}\left(\boldsymbol{r}_\perp^\pm(\tau)-\boldsymbol{r}_\perp^\pm(-\tau)\right)^2\right]\right\}. \tag{2}$$

Here $\xi_\pm=\epsilon_\pm/\hbar\omega$ is the ratio of the pair-particle energy to the γ-quantum energy, $\gamma_\pm=\epsilon_\pm/mc^2$, $S_\perp^0$ is the unit-cell area for the two-dimensional lattice corresponding to the crystal axis family under consideration, ℓ_c is the coherence length for the pair-production process:

$$\ell_c(\xi_\pm,\omega) = 2\xi_+\xi_-\lambda\!\!\!^{-}_c\,\hbar\omega/mc^2. \tag{3}$$

Electronic charge, rest mass and Compton wavelength, speed of light and .Planck constant have standard notation.

Dependence of the pair-distribution density (1) upon the coordinate $\boldsymbol{r}_\perp$ of the "creation act" and upon the γ-quantum incidence angle is determined by initial conditions for coordinates and velocities standing in Eqs.(1), (2):

$$\boldsymbol{r}_\perp^\pm(0)=\boldsymbol{r}_\perp,\quad \boldsymbol{v}_\perp^\pm(0)=c\boldsymbol{k}_\perp/k,\quad k_\perp=k\sin\theta, \tag{4}$$

where $\boldsymbol{k}$ is the γ-quantum momentum. The integral of Eq.(1) over the two-dimensional unit-cell area ($dS_\perp=d^2r_\perp$) gives the spectrum $dN_p/d\xi_\pm$ of e^+-e^- pairs created by one incident γ-quantum per unit time.

2. Simple quantitative evaluations can be obtained for the pair-production rate in the field of a separate atomic chain in the region of fairly large angles θ, where the integral in τ of the expression proportional to $[\boldsymbol{v}_\perp^\pm(\tau)-\boldsymbol{v}_\perp^\pm(-\tau)]^2$ gives the main contribution to (1) and additional corrections to 2τ in the functions $f_\pm(\tau)$ may be disregarded.

Taking into account that the difference $v_\perp(\tau)-v_\perp(-\tau)$ is appreciably different from zero only for those $\tau's$ which correspond to the particle being found at distances $\lesssim a_{T\text{-}F}$ (atomic Tomas-Fermi screening radius) from the atomic-chain axis and assuming the condition

$$2a_{T\text{-}F}/\ell_c\,\theta \ll 1 \tag{5}$$

to be valid, in the main region of coordinates $r_\perp$ we find easily

$$\frac{d^2N_p}{d\xi_\pm dS_\perp} \cong \frac{e^2}{S_\perp^0\pi\hbar^3\omega}\,\frac{\xi_+^2+\xi_-^2}{4\xi_+\xi_-}\,\frac{\sin\left[2x/\ell_c(\xi_\pm,\omega)\,\theta\right]}{x\,\theta}\int_0^\infty [U(\tau)-U(-\tau)]^2\,d\tau$$

$$\cong \frac{e^2}{S_\perp^0\pi\hbar^3 c\,\omega}\,\frac{\xi_+^2+\xi_-^2}{4\xi_+\xi_-}\,\frac{\sin\left[2x/\ell_c(\xi_\pm,\omega)\,\theta\right]}{x\,\theta^2}\,\Delta U^2(y)\,a(y)\,. \tag{6}$$

Here $r_\perp \equiv (x,y)$, $x = r_\perp k_\perp/k_\perp$; $U(\tau)$ is the particle's potential energy at the time instant τ; $\Delta U^2(y)$ and $a(y)$ are, respectively, a characteristic magnitude of the potentiual energy squared and a spacial potential width on the trajectory with impact parameter $|y|$ relative to the chain axis ($r_\perp = 0$).

Eq.(6) shows that under the conditions examined the quantity $d^2N_p/d\xi_\pm dS_\perp$, as a function of the coordinate $r_\perp$:

(i) is appreciably different from zero in a narrow strip, $\sim 2a_{T\text{-}F}$ wide, which contains the atomic chain and is oriented along the vector $k_\perp$.

(ii) oscillates with increasing $|x|$, taking both positive and negative values and having a decreasing amplitude. These oscillations are of a purely quantum-mechanical nature, and their existence means, among other things, that $d^2N_p/d\xi_\pm dS_\perp$ may be interpreted as a spacial density of created pairs only after averaging over the oscillation period in space.

By choosing the $k_\perp$ direction in a single crystal one can achieve the conditions when the mentioned coordinate strip does not touch the nearby atomic chains. For the pair spectrum in a monoatomic crystal in this "single-string" situation we find from Eqs.(6), (5):

$$\frac{dN_p}{d\xi} \cong A\,\frac{e^2}{\hbar c}\,\frac{\pi a_{T\text{-}F}^2}{8S_\perp^1}\,\frac{U_0^2}{\hbar^2\omega\,\theta^2}\,\frac{\xi^2+(1-\xi)^2}{4\xi(1-\xi)} \quad \text{at } |\xi-\frac{1}{2}| \lesssim \frac{1}{2}\,(1-\theta_m/\theta)^{\frac{1}{2}}\,, \tag{7}$$

where U_0 is the single-atomic-chain potential-well depth, $S_\perp^1$ is the transverse area per one chain, A is a constant of the order of 1 and

$$\theta_m = \theta_m(\omega) = \frac{4a_{T\text{-}F}}{\lambda\!\!\!^{-}_c}\,\frac{mc^2}{\hbar\omega}\,. \tag{8}$$

Simple physical considerations bound up with the existence of the length of effective interaction of pair's particles with the atomic chain

$$\ell_{int} = \ell_\theta\,\ell_{cfa}/(\ell_\theta+\ell_{cfa})\,,\quad \ell_\theta = 2a_{T\text{-}F}/\theta,\quad \ell_{cfa} = a_{T\text{-}F}mc^2/|U_0|\,, \tag{9}$$

bring us to the conclusion that the pair-density maximum in the spectrum centre ($\xi=\frac{1}{2}$) and, together with it, the maximum of the total pair yield are achieved at $\ell_c(\frac{1}{2},\omega) \cong \ell_{int}$. For not very large frequences, which satisfy the condition $\theta_m(\omega) \gtrsim |U_0|/mc^2$, this means that the integral pair yield, as a function of the

angle θ, has a maximum at $\theta \cong \theta_m(\omega)$, the magnitude of which may be evaluated with the help of Eq.(7):

$$N_p(\theta_m) \cong \frac{e^2}{\hbar c} \frac{\pi a_{T-F}^2}{4S_\perp^1} \frac{U_0^2}{\hbar^2 \omega \theta_m^2} = \frac{e^2}{\hbar c} \frac{\pi \lambda\!\!\!^{-2}_c}{64 S_\perp^1} \frac{\omega U_0^2}{m^2 c^4} . \tag{10}$$

The obtained relations solve the problem of dependence of both the angular position of the pair-creation rate maximum and its height upon the crystal parameters and the γ-quantum frequency[1]. They are in a good agreement with the available experimental data [2].

3. When the transverse photon momentum $\boldsymbol{k}_\perp$ is directed along one of the crystallographic planes formed by the singled-out family of atomic chains, the coherent effects arise in the pair-creation processes. They are caused by the interference of probability waves from neighbouring chains transpierced by the vector $\boldsymbol{k}$.

For the interference leading to an increase of the e^+-e^- pair yield, the pair-creation probability amplitude phase difference for neighbouring atomic chains

$$\Delta\phi \cong \boldsymbol{q}\,\Delta\boldsymbol{r} \tag{11}$$

bounds to be divisible by 2π. Here $\hbar\boldsymbol{q}$ is the momentum transferred to the crystal in the pair-creation process

$$\hbar\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{p}_+ - \boldsymbol{p}_- \cong \hbar \boldsymbol{n}_k / \ell_c(\xi_\pm, \omega), \qquad \boldsymbol{n}_k = \boldsymbol{k}/k, \tag{12}$$

and $\Delta\boldsymbol{r}$ is the coordinate difference for the points of intersection of the γ-quantum "trajectory" with the neighbouring chain axes, the distance between which is equal to d,

$$\Delta r \cong n_k\, d/\theta . \tag{13}$$

Thus, the condition of the interference intensification of the pair-creation rate has the form

$$\ell_c(\xi_\pm, \omega)\,\theta \cong \frac{d}{2\pi i} , \tag{14}$$

where i is an integer.

Since the "physical" transverse size of an atomic chain is $2a_{T-F}$, the changes in the values of $\xi_\pm$, ω and θ, characterizing the width of the "coherent resonance" (14), are determined by the relation

$$\Delta\left[\ell_c(\xi_\pm, \omega)\,\theta\right] / \left[\ell_c(\xi_\pm, \omega)\,\theta\right]_{res} \sim 4a_{T-F}/d . \tag{15}$$

4. It is worth noting that the theories of coherent pair production, both standard and modified [1], are unable to provide a true description of e^+-e^- pair spectrum properties in the region of action of the coherent resonance mechanism even at relatively moderate γ-quantum frequences. Indeed, as was shown in Ref.[1], the results of these theories follow from Eqs.(1), (2) under the condition of validity of the straight-line trajectory approximation. The criterion of small deviations of

[1] Efficiency of the single-string approximation in a wide region of angles θ was first demonstrated in numerical calculations Ref.[2] in connection with results of pioneer experiments carried out in this work.

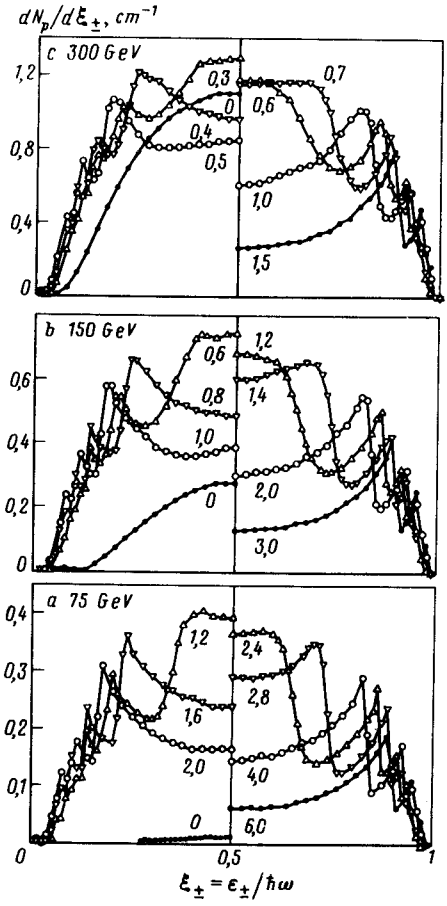

Fig.1. Energy spectra of e^+-e^- pairs in a <110> Si single crystal at temperature T= 300 K for different angles θ between the momentum k of a non-polarized γ-quantum and the <110> axis, $k \| \{001\}$. Each curve represents some simple approximation through the shown calculated points. The numbers on the curves denote the values of θ in units of mrad. The γ-quantum energy values are: *a*) $\hbar\omega$ = 75 GeV, *b*) $\hbar\omega$ = 150 GeV, *c*) $\hbar\omega$ = 300 GeV.

the $e^\pm$-projectile trajectory from a stright line, after passing the distance jd (j is an integer) in the transverse plane, is

$$\theta \gg j\,\theta_L(\epsilon_\pm)\,(2d/a_{T-F})^{1/2}, \tag{16}$$

where $\theta_L(\epsilon_\pm) = (2|U_0|/\epsilon_\pm)^{1/2}$ is the critical Lindhard angle [3]. Using the coherent-resonance angle value

$$\theta_{res}^{(i)}(\xi,\omega) \cong \frac{mc^2 d}{4\pi i \xi\,(1-\xi)\,\hbar\omega\lambda_c} \tag{17}$$

shows that at $\theta = \theta_{res}^{(i)}(\frac{1}{2},\omega)$ and $\epsilon = \hbar\omega/2$ the condition (16) is satisfied the harder, the larger are ω, i and j, and the smaller is the crystal atom nucleus charge. In the calculations, the results of which are described below, it fails already at $j = 2 \div 3$.

5. An effective algorithm, worked out within the framework of the quasi-classical operator method, allowed us to carry out unified-scheme calculations of

the e^+-e^- pair creation probabilities for hard non-polarized γ-quanta ($\hbar\omega = 75 \div 300$ GeV) in Si single crystals in a wide region of angles between the γ-quantum momentum and the <110> axial direction, beginning at $\theta = 0$, where the constant-field approximation is valid (see, e.g., Ref.[1] and references therein), and ending up with $\theta \sim 10^2\theta_L$, where the theory of coherent pair production holds true.

Coherent resonances in the e^+-e^- pair spectra were investigated under conditions when the incident vector k was parallel to the {001} plane where the <110> atomic chains are disposed with the period $d = 3.84$ Å. The potential of this chain system was described by the superposition of the Doyle-Turner potentials [4]. The crystal temperature was chosen to be $T = 300$ K. The incoherent Bethe-Heitler contribution was neglected as unessential.

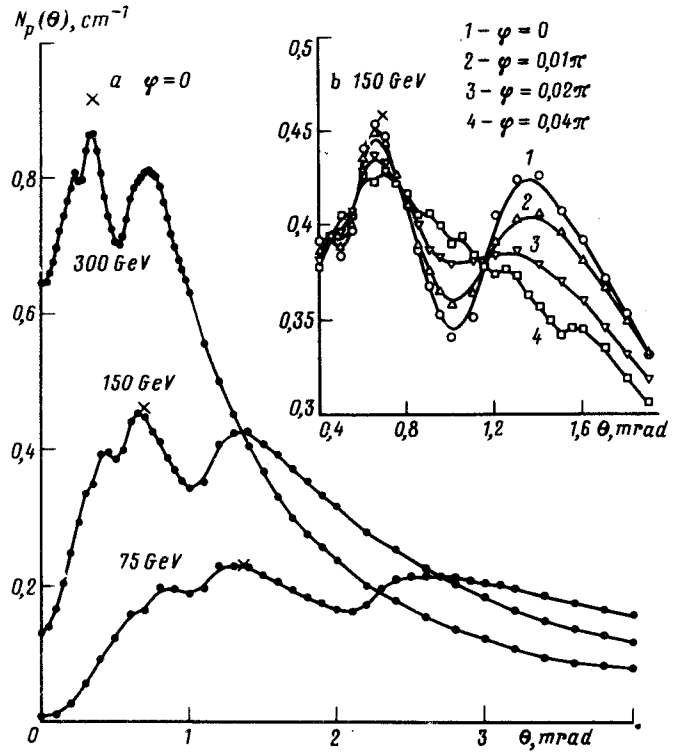

Fig.2. *a*) Angular dependences of the integral e^+-e^- pair yield (the curves represent some simple approximations through the shown calculated points) in a <110> Si single crystal at temperature $T = 300$ K for the three energy values of a non-polarized creating γ-quantum: $\hbar\omega = 75$, 150 and 300 GeV. θ is the angle between the γ-quantum momentum k and the <110> axis, $k \| \{001\}$. Symbols × mark the coordinates of the "single-string" maxima of the integral yield, calculated in accordance with Eqs.(8), (10).

b) Same as plots (*a*) but at fixed $\hbar\omega = 150$ GeV for different values of the angle φ between the vector $k_\perp$ and the {001} plane: 1 - $\varphi = 0$, 2 - $\varphi = 0.01\pi$, 3 - $\varphi = 0.02\pi$, 4 - $\varphi = 0.04\pi$.

Calculated e^+-e^- pair spectra for $\hbar\omega = 75$, 150 and 300 GeV are shown in Fig.1. The most pronounced resonance effects take place for $i = 1$, 2 ($\theta_{res}^{(1)}(\frac{1}{2}, 150$ GeV$) = 1.08$ mrad, $\theta_{res}^{(2)}(\frac{1}{2}, 150$ GeV$) = 0.54$ mrad). There are impressive powerful peaks and dips in the spectrum centre, one replacing another when θ increases, with nearly twofold change in intensity. The angular positions and the angular

widths of the central maxima, the narrowness of the intensity decrease fronts when ξ moves off from the spectrum centre, the positions of the cusp-like peaks (the resonances answering the larger values of i) near the spectrum edges — all these features are described well by the relations (17) and (15). The resonance effects manifest themselves relatively the stronger, the smaller is the γ-quantum frequency, but the absolute changes in intensity increase with increasing ω. Remarkable, that in the situations under consideration the resonance maximum with $i=2$ in the specrum centre is superimposed over the "single-string" maximum ($\theta_m(150$ GeV$) = 0.69$ mrad) and proves to be singled-out in intensity.

Accuracy of our calculations is $1 \div 2\%$. Within this accuracy the calculated spectra are symmetric with respect to $\xi = \frac{1}{2}$ and the replacement of electron variables by positron ones in Eqs.(1), (2) does not lead to changes in the spectra.

Orientation dependences of the total pair yield are presented in Fig.2*a*. The resonance variations of the integral intensity are seen to be appreciably smaller than those in the differential spectra, and the corresponding maximum positions are displaced slightly towards the larger angles as compared with $\theta_{res}^{(i)}(\frac{1}{2}, \omega)$. Fig.2*b* shows how quickly the resonance effects on the orientational curves (and, respectively, in the differential spectra) disappear with increasing the angle φ between the vector $\boldsymbol{k}_\perp$ and the {001} plane. Smoothing the resonance variations occurs at $\varphi \gtrsim 2a_{T-F}/d$. In this region of φ a "single-string" situation comes true with a small fine structure on the curves, which is caused by the influence of distant atomic chains.

It can be shown that at $\theta \gg \theta_m$, independently of the value of φ, the results of the theory of coherent pair production in the main spectrum region coincide with the results of the single-string approximation. This explains the fact that at large θ's, in accordance with Eq.(7), the inversion of integral-yield curves happens in Fig.2*a*: the curves with the larger ω's drop down lower than those with the smaller ω's.

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