Письма в ЖЭТФ, том 58, вып.6, стр.457 - 461


# SUPERCONDUCTIVITY WITH LINES OF GAP NODES: DENSITY OF STATES IN THE VORTEX

*G.E. Volovik*

*Landau Institute for Theoretical Physics*

*117334, Moscow, Russia*

Submitted 23 August 1993

The density of states (DOS) produced by the vortices in the superconductors with the lines of zeros in electronic energy spectrum is calculated with application to high temperature superconductors. DOS of the isolated vortex is $\propto N_F\xi\cdot\min\{R,\lambda\}$, where $N_F$ is the DOS of the normal metal, $\xi$ is the coherence length, $\lambda$ is the penetration length, and $R$ is the distance between the vortices. Only a small part of the DOS results from the fermions, localized in the vortex core. Symmetry of the isolated vortex line in superconductors with the gap corresponding to $\Gamma_3$ representation is also discussed.

## 1. Introduction

Recent experiments with angle-resolved photoemission [1] revealed the existence of lines of the gap nodes in high temperature superconductor $Bi_2Sr_2CaCu_2O_8$. The position of the gap nodes allows us to identify the symmetry class of superconductivity. The pairing occurs into the spin-singlet state described by the one-dimensional $\Gamma_3$ representation of the (approximate) tetragonal symmetry group $D_4$ of the $CuO_2$ planes. This superconducting state corresponds to the symmetry class $D_4(D_2)\times T$ in the classification scheme of Ref.[2], where $T$ is the time inversion symmetry. The gap function of such unconventional superconductivity has the general form

$$\Delta(\mathbf{k},\mathbf{r}) = ((\mathbf{k}\cdot\hat{a})^2 - (\mathbf{k}\cdot\hat{b})^2)f(\mathbf{k})\Psi(\mathbf{r}) \quad , \tag{1.1}$$

where the real function $f(\mathbf{k})$ has the symmetry $D_4$ of the normal metal; $\hat{a}$ and $\hat{b}$ are unit vectors along the Cu–O bond directions ($x$ and $y$) with $z$ being along the 4-fold symmetry axis. The complex scalar $\Psi(\mathbf{r})$ is the order parameter, which depends on the coordinate $\mathbf{r}$ of the center of mass of Cooper pair. Though the complete form of the gap function can be found only from the microscopic theory, the important property of its symmetry - the position of the gap nodes - does not depend on details of the system. The energy of the Bogoliubov excitations,

$$E(\mathbf{k}) = \sqrt{\varepsilon^2(\mathbf{k}) + |\Delta(\mathbf{k})|^2} \quad , \tag{1.2}$$

is zero on the four lines $\mathbf{k}_n(k_z)$ $(n = 1,2,3,4)$ defined by equations $\mathbf{k}\cdot\hat{a} = \pm\mathbf{k}\cdot\hat{b}$, $\varepsilon(\mathbf{k}) = 0$.

The presence of the lines of nodes influences the low-temperature properties of superconductor, in particular this produces the power-law temperature dependence of the penetration length, observed in $YBa_2Cu_3O_7$ – another superconductor with $CuO_2$ planes [3]. Here we consider the effect of nodes on the electronic properties of the vortex lines in $Bi_2Sr_2CaCu_2O_8$ and calculate the DOS in a mixed state of superconductor under magnetic field.

## 2. DOS of the fermions localized in the vortex core

In conventional superconductors (without gap nodes) the DOS, $N(0) \propto N_F \xi^2$, comes from the branch of localized fermions with low energy [4]. According to general topological property, such anomalous branch which crosses zero as a function of impact parameter should exist also for vortices in case of unconventional pairing [5]. So let us start with the DOS which comes from the localized fermions occupying the anomalous branch.

The Bogoliubov Hamiltonian for the fermions with given spin projection is $2 \times 2$ matrix

$$\mathbf{H} = \hat{\tau}_3 \varepsilon(\mathbf{k}) + \hat{\tau}_1 Re\Delta(\mathbf{k}, \mathbf{r}) - \hat{\tau}_2 Im\Delta(\mathbf{k}, \mathbf{r}) \ . \tag{2.1}$$

As will be seen below the main contribution into the DOS comes from the vicinity of the gap nodes and outside the vortex core. If the azimuthal angle $\alpha$ of $\mathbf{k}$ is close to $\alpha_n = \frac{\pi}{4}(1+2n)$, the gap function outside the core of the vortex with winding number 1 has the following general form:

$$\Delta(\mathbf{k}, \mathbf{r}) \approx \vec{\gamma}_n(k_z) \cdot (\mathbf{k} - \mathbf{k}_n(k_z)) e^{i\phi} \quad , \tag{2.2}$$

with $\vec{\gamma}_n(k_z) \parallel \hat{z} \times \mathbf{k}_n$ and $\phi$ being the azimuthal angle of $\mathbf{r}$. The energy spectrum of electrons in the normal metal is

$$\varepsilon(\mathbf{k}) \approx -i\mathbf{v}_F(\mathbf{k}_n) \cdot \vec{\nabla} = (-i) v(k_z)(\cos\alpha_n \nabla_x + \sin\alpha_n \nabla_y) \tag{2.3}$$

Here only the gap slope at the node, $\gamma(k_z) = |\vec{\gamma}_n(k_z)|$, and the transverse component of the Fermi-velocity at the nodes, $v(k_z) = |\mathbf{v}_{F\perp}(\mathbf{k}_n)|$, are defined by the details of the system and can be considered as phenomenological parameters.

Let us introduce new coordinate and momentum variables

$$\tilde{x} = \mathbf{r} \cdot \hat{k}_{n\perp} \quad , \quad \tilde{y} = \mathbf{r} \cdot \hat{\gamma}_n \quad , \quad \tilde{k}_x = (\mathbf{k} - \mathbf{k}_n) \cdot \hat{k}_{n\perp} \quad , \quad \tilde{k}_y = (\mathbf{k} - \mathbf{k}_n) \cdot \hat{\gamma}_n \quad ,$$

where $\hat{k}_{n\perp} = \mathbf{k}_{n\perp}/|\mathbf{k}_{n\perp}|$ are the unit vectors in the direction of the gap nodes in $a-b$ plane; $\hat{\gamma}_n = \vec{\gamma}_n/|\gamma|$ are the unit vectors in the perpendicular directions, $\tilde{y}$ is thus the impact parameter. Then for small impact parameter

$$e^{i\phi} \approx e^{i\alpha_n}(\mathrm{sign}(\tilde{x}) - i\frac{\tilde{y}}{|\tilde{x}|}) \quad , \tag{2.4}$$

and the Hamiltonian is:

$$\mathbf{H} = \mathbf{H}^{(0)} + \mathbf{H}^{(1)} \quad ,$$

$$\mathbf{H}^{(0)} = -i\hat{\tau}_3 v(k_z)\nabla_{\tilde{x}} + \tilde{k}_y \gamma(k_z)(\cos\alpha_n \hat{\tau}_1 + \sin\alpha_n \hat{\tau}_2)\mathrm{sign}(\tilde{x}) \quad ,$$

$$\mathbf{H}^{(1)} = \tilde{k}_y \gamma(k_z)(\cos\alpha_n \hat{\tau}_2 - \sin\alpha_n \hat{\tau}_1)\frac{\tilde{y}}{|\tilde{x}|} \quad . \tag{2.5}$$

The Hamiltonian $\mathbf{H}^{(0)}$ has a zero eigen value with the localized eigen function:

$$\Psi^{(0)}(\tilde{x}) = \sqrt{|\tilde{k}_y|\frac{\gamma(k_z)}{2v(k_z)}}[1 - \mathrm{sign}(\tilde{k}_y)(\cos\alpha_n\hat{\tau}_2 - \sin\alpha_n\hat{\tau}_1)]\exp\{-\frac{\gamma(k_z)}{v(k_z)}|\tilde{x}\tilde{k}_y|\} \quad , \quad (2.6)$$

and from the first order in perturbation $\mathbf{H}^{(1)}$ one obtains the spectrum of the anomalous branch of localized fermions:

$$E(\tilde{y},\tilde{k}_y,k_z) = 2\tilde{y}\tilde{k}_y^2\frac{\gamma^2(k_z)}{v(k_z)}\int_\xi^\infty \frac{dr}{r}\exp\{-\frac{2\gamma(k_z)}{v(k_z)}|\tilde{k}_y r\} \approx 2\tilde{y}\tilde{k}_y^2\frac{\gamma^2(k_z)}{v(k_z)}\ln\frac{k_F}{|\tilde{k}_y|} \quad . \quad (2.7)$$

Here we used an estimate $\gamma(k_z)/v(k_z) \sim (\xi k_F)^{-1}$. The spectrum crosses zero at $\tilde{y}=0$ and in addition it touches zero at $\tilde{k}_y=0$. However it should be taken into account that the Eq.(2.7) is valid if the effective range of integration in Eq.(2.7) does not exceed the intervotex distance $R$ or the penetration length $\lambda$. This gives the condition for the angles at which the spectrum (2.7) does hold:

$$k_F \gg |\tilde{k}_y| \gg \frac{v(k_z)}{\gamma(k_z)\min\{R,\lambda\}} \sim k_F\frac{\xi}{\min\{R,\lambda\}} \quad . \quad (2.8)$$

Below the limit $v(k_z)/(\gamma(k_z)\min\{R,\lambda\})$ the states are delocalized.

The anomalous branch of localized fermions gives the following contribution to the DOS:

$$N_{\mathrm{loc}}(0) = \int\frac{d\tilde{k}_y\, d\tilde{y}}{2\pi}\int\frac{dk_z}{2\pi}\;\delta(E(\tilde{y},\tilde{k}_y,k_z)) = \int\frac{dk_z}{2\pi}\frac{v(k_z)}{\gamma^2(k_z)}\int\frac{d\tilde{k}_y}{2\pi\tilde{k}_y^2\ln(k_F/|\tilde{k}_y|)} \quad . \quad (2.9)$$

The integral over $\tilde{k}_y$ diverges at small $\tilde{k}_y$ and the constraints in Eq.(2.8) give the following esimate for contribution of localized states:

$$N_{\mathrm{loc}}(0) \approx \frac{\min\{R,\lambda\}}{\ln(\min\{R,\lambda\}/\xi)}\int\frac{dk_z}{2\pi\gamma(k_z)} \sim N_F\xi\cdot\min\{R,\lambda\}/\ln\frac{\min\{R,\lambda\}}{\xi} \quad . \quad (2.10)$$

## 3. DOS of delocalized fermions

The divergency of the density of the localized states at small angles, i.e. at the edge of the continuum, means that the main contribution into DOS comes from the delocalized states. For these states one can use the semiclassical approach in which the local energy is Doppler shifted by the local superfluid velocity $\mathbf{v}_s$:

$$N_{\mathrm{deloc}}(0) = 2\int\frac{d^3k}{(2\pi)^3}\int d^2r\;\delta(E(\mathbf{k},\mathbf{r}) + m_e\mathbf{v}_F\cdot\mathbf{v}_s) \quad , \quad (3.1)$$

where $m_e$ is the mass of electron. The main contribution into the DOS again comes from the vicinity of the gap nodes in the momentum space and from the region far outside the vortex core in the real space:

$$N_{\mathrm{deloc}}(0) = \frac{1}{4\pi^3}\sum_n\int d\tilde{k}_y dk_z\;\frac{d\varepsilon}{v(k_z)}d^2r\;\delta(\sqrt{\varepsilon^2 + \tilde{k}_y^2\gamma^2(k_z)} + m_e\mathbf{v}_F(\mathbf{k}_n)\cdot\mathbf{v}_s) =$$

$$= \int \frac{dk_z}{2\pi^2\gamma(k_z)} \sum_n \int d^2r |m_e \mathbf{v}_s \cdot \hat{k}_{n\perp}| \quad . \tag{3.2}$$

For the isolated vortex the superfluid velocity at the distance $\xi \ll r \ll \min\{R, \lambda\}$ is $\mathbf{v}_s = (\hbar/2m_e)(\hat{\phi}/r)$, as a result the space integral is divergent at large distances:

$$N_{\rm deloc}(0) = 2\int \frac{dk_z}{\pi^2\gamma(k_z)} \int_\xi^{\min\{R,\lambda\}} dr \sim N_F\xi \cdot \min\{R, \lambda\} \quad . \tag{3.3}$$

This term does not contain the logarithm in denominator and thus exceeds the density of localized states in Eq.(2.10).

For the vortex lattice in the magnetic field region $H_{c2} \gg H \gg H_{c1}$ the intervortex distance is $R \sim \xi\sqrt{H_{c2}/H} < \lambda$. The DOS averaged over the vortices is thus

$$N(0) = K N_F \sqrt{\frac{H}{H_{c2}}} \quad . \tag{3.4}$$

where the factor $K$ is of order unity and, according to Eq.(3.2), is defined by the vortex lattice structure in the coordinate space and by the slope of the gap near the gap node in the momentum space.

## 4. Symmetry of the vortex line in $D_4(D_2) \times T$ superconductor

Finally we discuss the symmetry of the isolated vortex in the superconductor of class $D_4(D_2) \times T$. According to Ref.[6] the elements of the maximal symmetry group of the vortex line can be found from the asymptote of the gap function far from the vortex core

$$\Delta(\mathbf{k}, \mathbf{r}) = (k_x^2 - k_y^2) f(\mathbf{k}) e^{i\phi} \quad . \tag{4.1}$$

8 elements of symmetry of this function form the group $D_4(E)$:

$$D_4(E) =$$
$$= (E,\ C_z^\pi e^{i\pi},\ C_z^{\pi/2} e^{i\pi/2},\ C_z^{-\pi/2} e^{-i\pi/2},\ C_x^\pi T,\ C_y^\pi e^{i\pi} T,\ C_{x+y}^\pi e^{i\pi/2} T,\ C_{x-y}^\pi e^{-i\pi/2} T), \tag{4.2}$$

where $C_i^\alpha$ is the rotation about axis $i$ by angle $\alpha$, This group is isomorphic to the $D_4$ group; for the physical quantities, such as $|\Delta(\mathbf{k}, \mathbf{r})|^2$, which are gauge invariant and invariant under time inversion, this group coincides with initial $D_4$ group of $CuO_2$ planes.

In conventional superconductors with $D_4$ group the symmetry of the vortex with asymptote $f(\mathbf{k})e^{i\phi}$ is similar but not the same:

$$D_4'(E) =$$
$$= (E,\ C_z^\pi e^{i\pi},\ C_z^{\pi/2} e^{-i\pi/2},\ C_z^{-\pi/2} e^{i\pi/2},\ C_x^\pi T,\ C_y^\pi e^{i\pi} T,\ C_{x+y}^\pi e^{-i\pi/2} T,\ C_{x-y}^\pi e^{i\pi/2} T), \tag{4.3}$$

and Eq.(4.2) characterizes the symmetry of the vortex with an opposite winding number, i.e. with asymptote $f(\mathbf{k})e^{-i\phi}$. This leads to two consequences.

1) The core of the vortex in $D_4(D_2) \times T$ state should contain all the possible terms consistent with Eq.(4.2), and in particular it contains the amplitude of conventional ($s$-wave) pairing with inverse circulation of superfluid velocity, such as:

$$\Delta_s(\mathbf{k},\mathbf{r}) = f(\mathbf{k})|\Psi_s(\mathbf{r})|e^{-i\phi} \quad . \tag{4.4}$$

Due to this correction the total gap function has no lines of gap nodes within the core. This is not surprising since as distinct from the point zeros the lines of zeroes are topologically unstable [7]. This however does not change the result for DOS, since it comes mostly from the region outside the core.

2) There is incompatibility between the symmetry and topology when the vortex line punctures the interface between the conventional and unconventional superconductors. The topology requires that the winding number of the vortex is the same on both sides of the interface. On the other hand if the symmetry $D_4(E)$ is conserved, the vortices on two sides of interface should have opposite winding numbers. The topology is however more important and therefore the symmetry should be broken. In particular the rotation $C_z^{\pi/2}$, which is combined with different phase factors in Eqs.(4.2) and (4.3), is no more the element of symmetry. Thus the rotational symmetry about 4-fold axis is broken near the interface.

## 5. Conclusion

While in the mixed state of conventional superconductors the electronic DOS, $N(0) \propto N_F H/H_{c2}$, comes from the low-energy states localized in the vortex cores, in the superconductors with lines of gap nodes the DOS, $N(0) \propto N_F\sqrt{H/H_{c2}}$, comes mostly from the continuous spectrum, concentrated in the vicinity of the gap nodes and outside the vortex core region.

I thank M.E. Zhitomirsky for discussions.

---


1. Z.-X.Shen, D.S.Dessau, B.O.Wells, et al, Phys. Rev. Lett., **70** , 1553 (1993).
2. G.E.Volovik and L.P. Gor'kov, Zh. Eksp. Teor. Fiz. **88** , 1412 (1985). [Sov. Phys. - JETP **61**, 843 (1985)].
3. W.N.Hardy, D.A.Bonn, D.C.Morgan, et al, Phys. Rev. Lett., **70** , 2806 (1993).
4. C.Caroli, P.G. de Gennes, and J. Matricon, Phys. Lett., **9**, 307 (1964).
5. G.E.Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **57**, 233 (1993); [JETP Lett. **57**, 244 (1993) ].
6. M.M.Salomaa and G.E. Volovik, Rev. Mod. Phys. **59**, 533 (1987).
7. P.G.Grinevich and G.E. Volovik, J. Low Temp. Phys. **72**, 371 (1988).