SPINOR STRUCTURES IN THE SUPERSTRING THEORY [1)]

G.S.Danilov

Petersburg Nuclear Physics Institute,
Gatchina, 188350, St.-Petersburg, Russia

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Superconformal Schottky groups appropriate for the description of all the superstring spinor structures are built.

In the Neveu-Schwarz-Ramond [1] superstring theory the multiloop amplitudes are written usually [2,3,4] as sums over spin structures integrated over Riemann moduli. This form of the above amplitudes arises after the integrating over odd moduli that are performed in the accordance with the prescription given in [2,3]. However, in the above scheme [2,3,4] the multiloop amplitudes turn out to be depended on a choice of basis of the gravitino zero modes [2,4]. It means that the two-dimensional supersymmetry is lost in the scheme discussed. Indeed, in the superstring theory both the "vierbein" and the gravitino field are the gauge fields. Owing to the gauge invariance the "true" superstring amplitudes are independent of a choice of a gauge of the above gauge fields. Therefore, they have no dependence on a choice of basis of the gravitino zero modes.

The discussed dependence on a choice of basis of the gravitino zero modes appears to be a serious difficulty in the considered scheme. The above difficulty is absent in the formulation [5] possessing of the manifest two-dimensional supersymmetry. In this scheme the n-loop superstring amplitudes A_n turn out to be [6,7,8] the integral over $(3n-3|2n-2)$ complex moduli q_N and their complex conjugated $\overline{q}_N$, as well:

$$A_n = \int \prod_N dq_N d\overline{q}_N \sum_{L,L'} \hat{Z}^{(n)}_{L,L'} < V >_{L,L'} \tag{1}$$

where $\hat{Z}^{(n)}_{L,L'}$ are the partition functions and $< V >_{L,L'}$ denote the vacuum expectations of the vertex products. The index L (L') labels "superspin" structures of right (left) fields. The above superspin structures are defined for superfields living on the complex $(1|1)$ supermanifolds [5]. Being twisted about (A,B)-cycles, the superfields are changed by mappings that present superconformal versions of fractional linear transformations. Generally, every considered mapping depends on $(3|2)$ parameters [5]. For odd parameters to be arbitrary, the above mappings include, in addition, fermion-boson mixing. It differs the superspin structures from the ordinary spin ones. Indeed, the ordinary spin structures imply that boson fields are single-valued on Riemann surfaces and that under 2π-twists about (A,B)-cycles fermion fields can be multiplied by (-1). For all odd parameters to be equal to zero every genus-n superspin structure $L=(l_1,l_2)$ is reduced to the ordinary (l_1,l_2) spin one. Here l_1 and l_2 are the theta function characteristics: $l_i=\{l_{is}\}$ where $l_{is}=0,1/2$ with $i=1$ or 2 and $s=1,2,...n$.

In the discussed scheme the partition functions can be computed from equations [7,8] that are non other than Ward identities. These equations realize the

[1)] E-mail address: danilov@inpi.spb.su

requirement that the superstring amplitudes are independent of both the "vierbein" and the gravitino field. Therefore, the multiloop amplitudes being calculated in the terms of the superspin structures turn out to be consistent with the requirement of the gauge invariance of the superstring.

In above refs. [3,6,7,8] only superspin structures with all l_{1s} to be equal to zero were studied. For the description of these superspin structures superconformal versions of the Schottky groups [9,10] have been employed. The goal of this paper is to build the superconformal Schottky groups appropriate for all the superspin structures including those where $l_{1s} \neq 0$. In this formulation the superfields associated with the above superspin structures turn out to be branched on the complex z-plane where Riemann surfaces are mapped. It makes to be rather difficult the calculations for the superspin structures in question. Nevertheless, it is the only formulation allowing to perform the explicit calculation in the terms of even and odd moduli of both the partition functions and the vacuum superfield correlators. So the discussed formulation seems to be interesting.

Generally, every superspin structure given on a genus-n complex $(1|1)$ supermanifold is defined by the transformations $(\Gamma_{a,s}(l_{1s}), \Gamma_{b,s}(l_{2s}))$ that are associated with rounds about (A_s, B_s)-cycles, respectively. The above supermanifolds are mapped by the supercoordinate $t = (z|\theta)$ where z is a local complex coordinate and θ is its odd partner.

To build all the $\Gamma_{a,s}(l_{1s}), \Gamma_{b,s}(l_{2s})$ mappings one can note that for genus $n = 1$ there are no odd moduli. Indeed, the genus-1 amplitudes are obtained in the terms of ordinary spin structures [11]. Then, for every particular s, all the odd parameters in both $\Gamma_{a,s}(l_{1s})$ and $\Gamma_{b,s}(l_{2s})$ can be reduced to zero by a suitable transformation $\tilde{\Gamma}_s$, which is the same for both the above transformations:

$$\Gamma_{a,s}(l_{1s}) = \tilde{\Gamma}_s^{-1}\Gamma_{a,s}^{(o)}(l_{1s})\tilde{\Gamma}_s, \qquad \Gamma_{b,s}(l_{2s}) = \tilde{\Gamma}_s^{-1}\Gamma_{b,s}^{(o)}(l_{2s})\tilde{\Gamma}_s \tag{2}$$

where $(\Gamma_{a,s}^{(o)}(l_{1s}), \Gamma_{b,s}^{(o)}(l_{2s}))$ are equal to $(\Gamma_{a,s}(l_{1s}), \Gamma_{b,s}(l_{2s}))$ calculated at all the odd moduli to be equal to zero.

For the $\Gamma_{b,s}^{(o)}(l_{2s})$ mappings we employ Schottky transformations. Simultaneously the θ spinor receives the $(c_s z + d_s)^{-1}$ factor. Moreover, for $l_{2s} = 0$, the spinors receive the sign [3]. Therefore,

$$\Gamma_{b,s}^{(o)}(l_{2s}) = \{z \to (a_s z + b_s)(c_s z + d_s)^{-1}, \theta \to -\theta(-1)^{2l_2}(c_s z + d_s)^{-1}\} \tag{3}$$

where (a_s, b_s, c_s, d_s) are complex parameters and $a_s d_s - b_s c_s = 1$. The discussed $\Gamma_{b,s}(l_{2s})$ present superconformal versions of the above $\Gamma_{b,s}^{(o)}(l_{2s})$ transformations.

We take $\Gamma_{b,s}(l_{2s} = 1/2)$ to be the same as in [3,6,7,8]:

$$\Gamma_{b,s}(l_{2s} = 1/2) = \left\{ z \to \frac{a_s(z + \theta\varepsilon_s) + b_s}{c_s(z + \theta\varepsilon_s) + d_s}, \quad \theta \to \frac{\theta + \varepsilon_s}{c_s(z + \theta\varepsilon_s) + d_s} \right\}$$
$$\text{with} \quad \varepsilon_s = \alpha_s(c_s z + d_s) + \beta_s, \qquad a_s d_s - b_s c_s = 1 - \varepsilon_s \partial_z \varepsilon_s \; . \tag{4}$$

In (4) the even (a_s, b_s, c_s, d_s) and odd (α_s, β_s) parameters can be expressed [3,8] in the terms of two fixed points $(u_s|\mu_s)$ and $(v_s|\nu_s)$ on the complex $(1|1)$ supermanifold together with the multiplier k_s as

$$a = \frac{u - kv - \sqrt{k}\mu\nu}{\sqrt{k}(u - v - \mu\nu)}, \quad d = \frac{ku - v - \sqrt{k}\mu\nu}{\sqrt{k}(u - v - \mu\nu)}, \quad c = \frac{1 - k}{\sqrt{k}(u - v - \mu\nu)},$$
$$\alpha = (\mu + \sqrt{k}\nu)(1 + \sqrt{k})^{-1}, \quad \beta = -(\nu + \sqrt{k}\mu)(1 + \sqrt{k})^{-1}, \tag{5}$$

the index s being omitted. To obtain the above $\Gamma_{b,s}(l_{2s}=1/2)$ mappings in the form (4) we choose the $\tilde{\Gamma}_s$ mapping in (2) as

$$\begin{aligned}&\tilde{\Gamma}_s: \quad z \to z_s + \theta_s \tilde{\varepsilon}_s(z_s), \qquad \theta \to \theta_s(1+\tilde{\varepsilon}_s\tilde{\varepsilon}'_s/2) + \tilde{\varepsilon}_s(z_s);\\ &\tilde{\varepsilon}'_s = \partial_z \tilde{\varepsilon}_s(z), \qquad \tilde{\varepsilon}_s(z) = [\mu_s(z-v_s) - \nu_s(z-u_s)](u_s-v_s)^{-1}.\end{aligned} \tag{6}$$

Furthermore, we choose $(3|2)$ of the (u_s, v_s, μ_s, ν_s) parameters to be the same for all the genus-n supermanifolds, the rest of them together with the k_s multipliers being $(3n-3|2n-2)$ complex moduli q_N in (1). One can think that $|k_s| < 1$. Also, for the isomorphism between (4) and (5) to be, we fix the branch of $\sqrt{k_s}$, for example, as $|arg k_S| \leq \pi$. Then $\Gamma_{b,s}(l_{2s}=0)$ is obtained from (4) and (5) by the $\arg k_s \to \arg k_s + 2\pi$ replacement. The discussed $\Gamma_{b,s}(l_{2s}=0)$ appear to be slightly different from those in [3,6].

In fact, the above $\arg k_s \to \arg k_s + 2\pi$ replacement presents the (super)modular transformation turning $(l_{1s}=0, l_{2s}=1/2)$ into $(l_{1s}=0, l_{2s}=0)$.To prove this statement it is sufficient to check it for the genus $n=1$. For $n=1$ the period ω is given by [3,10] $\omega = (2\pi i)^{-1}\ln k$. So, we see that ω is turned into $\omega+1$ under the replacement discussed. Employing the explicit form of the theta functions, one can verify that this transformation of ω is accompanied by the replacement $(l_1=0, l_2=1/2) \to (l_1=0, l_2=0)$. Therefore, in our scheme the $|\arg k_s| \leq \pi$ condition provides in eq.(1) the separating of the $(l_{1s}=0, l_{2s}=1/2)$ and $(l_{1s}=0, l_{2s}=0)$ superspin structures from each other.

Every mapping (3) turns the circle $C_s^{(-)} = \{z : |c_s z + d_s| = 1\}$ into $C_s^{(+)} = \{z : |-c_s z + a_s| = 1\}$. The round about the $C_s^{(-)}$ or $C_s^{(+)}$ circle corresponds to 2π-twist about A_s-cycle. For $l_{1s}=1/2$ the spinors receive the sign under the above round [3]. Then the $\Gamma_{a,s}(l_{1s}=1/2)$ mappings appear to be

$$\Gamma_{a,s}(l_{1s}=1/2) = \{z \to z - 2\theta\tilde{\varepsilon}_s(z), \quad \theta \to -\theta(1+2\tilde{\varepsilon}_s\tilde{\varepsilon}'_s) + 2\tilde{\varepsilon}_s(z)\} \tag{7}$$

where $\tilde{\varepsilon}_s$ is defined by eq.(6). So in this case the cut $\tilde{C}_s$ appears on the considered z-plane. As far as $\Gamma_{a,s}(l_{1s}=1/2)^2 = I$, its endcut points are the square root branched ones. One of them is placed inside the $C_s^{(-)}$ circle and the other is placed inside $C_s^{(+)}$.

Superconformal p-tensors $F_p(t)$ being considered, $\Gamma_{a,s}(l_{1s}=1/2)$ relate $F_p(t)$ with its value $F_p^{(s)}(t)$ obtained from $F_p(t)$ by 2π-twist about $C_s^{(-)}$ or $C_s^{(+)}$-circle. So, $F_p(t)$ changes under the $\Gamma_{a,s}(l_{1s}) = \{t \to t_s^a\}$ and $\Gamma_{b,s} = \{t \to t_s^b\}$ mappings as

$$F_p(t_s^a) = F_p^{(s)}(t) Q^p_{\Gamma_{a,s}}(t), \qquad F_p(t_s^b) = F_p(t) Q^p_{\Gamma_{b,s}}(t)\,. \tag{8}$$

For $l_{1s} \neq 0$ the above $F_p(t)$ can not be obtained by a simple supersymmetrization of the conformal p-tensors in the boson string theory. The construction of the above superconformal p-tensors will be consider in an another paper.

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1. A.Neveu and J.H.Schwarz, Nucl. Phys. B321, 86 (1971); P.Ramond, Phys. Rev. D4, 1109 (1971).
2. E. Verlinde and H. Verlinde, Phys. Lett. B192, 95 (1987).

3. E. Martinec, Nucl. Phys. **B281**, 157 (1986).
4. J.Atick and A. Sen, Nucl. Phys. **B296**, 157 (1988); J.Atick and A. Sen, Nucl. Phys. **B299**, 279 (1988); G. Moore and A. Morozov, Nucl. Phys. **B306**, 387 (1988); A. Yu. Morozov, Teor. Mat. Fiz. **81**, 24 (1989);
5. M.A. Baranov and A.S. Schwarz, Pis'ma ZhETF **42**, 340 (1985) [JETP Lett. **49**, 419 (1986)]; D. Friedan, Proc. Santa Barbara Workshop on Unified string theories, eds. D. Gross and M. Green (World Scientific, Singapure, 1986).
6. P. Di Vecchia, L. Hornfeck, M. Frau, A. Ledra, and S. Sciuto, Phys. Lett. **B211**, 301 (1988).
7. G.S. Danilov, Phys. Lett. **B257**, 285 (1991).
8. G.S. Danilov, Jadernaya Fizika, **52**, 1143 (1990) [Sov.J.Nucl.Phys. **52**, 727 (1990)].
9. C. Lovelace, Phys. Lett. **B32**, 703 (1970); V. Alessandrini, Nuovo Cim. **2A**, 321 (1971); V. Alessandrini and D.Amati, Nuovo Cim. 2A, 793 (1971).
10. D. Friedan, E. Martinec, and S. Schenker, Nucl. Phys. **B271**, 93 (1986).
11. N. Seiberg and E. Witten, Nucl.Phys. **B276**, 272 (1986).