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TWO-DIMENSIONAL VORTICES IN A STACK OF THIN ANISOTROPIC SUPERCONDUCTING FILMS

A. Yu. Martynovich

Physicotechnical Institute AS Ukraine

340114 Donetsk, Ukraine

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The solution for the problem of a two-dimensional ($2D-$) vortex in a stack of thin anisotropic superconducting films is presented. The influence of an anisotropic $2D$-vortex structure on $2D$- and $3D$- vortex interaction and on general peculiarities of current-voltage characteristics near the Kosterlitz - Thouless transition is discussed.

1. Magnetic anisotropy of the new high-T_c superconductors is stipulaparallel to the CuO_2 layers. Along the c-direction, i.e. in perpendicular direcefficiently suppressed in regions between superconducting layers. Apart from that whether screenestimated that strong anisotropic Bi and Tl high-T_c compounds and artificial high-T_c multilayers are the stacks of magnetic coupling superconducting planes in wide region temperature below T_c. Transverse coherence length ξ_c in these materials is less than interlayer distance s, so Josephson coupling between superconducting layers is vanishing. Current-voltage characteristics of extremely anisotropic high$-T_c$ superconductors [1,2] evidence the superconductivity in separate CuO_2 layers exists in *a* $2D$ sense.

$2D$ magnetic vortices in such systems have been described earlier in numerous papers (see, for example, [3-6]). In all these works there were used simplifying assumption that superconducting layers are just isotropic in basal plane. However, it is well known existence of an anisotropy in CuO_2 plane of $YBa_2Cu_3O_x$ crystals [7,8]. Effective electron masses along a- and b- crystal axes, μ_a and μ_b respectively, differs each from other so that $\mu_b \cong 1.4\,\mu_a$. The precision of μ_b/μ_a evaluation in Bi high$-T_c$ crystals is not enough to confirm the equality of the masses μ_a and μ_b [8,9].

2. Let us consider a stack of thin superconducting layers thickness d separated each from other by distance s. The Cartesian coordinate system we tie to the crystal axes: x_3-axis is oriented along c-axis and along the normal $\mathbf{n}$ to the layers, x_1- and x_2-axes are parallel to the symmetry axes in superconducting plane, a and b respectively.

First of all we consider an isolated $2D$-vortex placing in coordinate center of superconducting plane having number $k=0$. The distributions of vector potential $\mathbf{A}(\mathbf{x})$ throughout all space and screening currents $\mathbf{I}_k(\mathbf{x})$ in superconducting kth layers we will describe within the Pearl limit of very thin layers [4], such that current $\mathbf{I}_k(\mathbf{x})$ flows in ab-plane only. The full equation system is

$$\mathbf{A}(\mathbf{x}) = \frac{4\pi}{c}\sum_k \mathbf{I}_k(\mathbf{x}) = \frac{2}{\Lambda}\hat{\mu}^{-1}\sum_k\left(\frac{\phi_o}{2\pi}\vec{\nabla}\theta - \mathbf{A}\right)\delta(x_3 - ks), \tag{1.1}$$

$$\mathrm{div}\mathbf{I}_k = 0, \tag{1.2}$$

$$\mathbf{n}\mathbf{I}_k = 0. \tag{1.3}$$

Here ϕ_o is flux quantum, $\Lambda = 2\lambda^2/d$. Note that length parameter is equal to $\Lambda = 2\lambda^2/s$ in Lawrence–Doniach model for Josephson - decoupled layers. We take normalizing second rank tensor $\hat{\mu}$ to be $1 = \mu_1\mu_2$. A vector potential gauge we define by condition div $\vec{\nabla}\theta = 0$, θ is a phase of the order parameter. This condition doesn't depend on anisotropy parameters $\hat{\mu}$, so the phase gradient of $2D$-vortex has the same space distribution as in a case of isotropic layers.

To solve the Eqs.(1) we have defined the Fourier transforms:

$$\mathrm{A}_k(\mathrm{q}) = \int d^3\mathrm{q} e^{-i\mathrm{qx}}\mathrm{A}(\mathrm{x})\delta(x_3 - ks),$$

$$\mathrm{S}(\mathrm{q}) = i\phi_o\frac{\mathrm{q}\times\mathrm{n}}{q^2}\delta_{k0},$$

with $\mathrm{q} = (q_1, q_2, 0)$.

From Eqs.(1.2) and (1.3) we obtain the orientation dependence of current distribution,

$$\mathrm{I}_k(\mathrm{q}) = (\mathrm{q}\times\mathrm{n})i_k(\mathrm{q}), \tag{2}$$

where $i_k(\mathrm{q})$ is scalar function of vector q.

From this expression and Eq.(1.1) we obtain anisotropic form of vector potential,

$$\mathrm{A}_k(\mathrm{q}) = \mathrm{S}(\mathrm{q}) + \hat{\mu}(\mathrm{q}\times\mathrm{n})\frac{i\phi_o\Lambda}{qK(\mathrm{q})}[W_{k0} - \delta_{k0}]. \tag{3}$$

Here

$$W_{km} = \frac{\sinh(qs)[G - (G^2-1)^{1/2}]k - m}{K(\mathrm{q})(G^2-1)^{1/2}},$$

$$G = \cosh(qs) + K^{-1}(\mathrm{q})\sinh(qs), \quad K(\mathrm{q}) = \frac{\Lambda}{q}(\mathrm{q}\times\mathrm{n})\hat{\mu}(\mathrm{q}\times\mathrm{n}).$$

Exact solution (3) generalizes Clem's results [4] on the case of anisotropic superconducting layers.

In the limit of "large" distance, $qs < 1$, the well approximation of W_{km} is

$$W_{km} = \frac{\exp\left[-|k-m|sq\left(1 + \frac{2}{sqK(\mathrm{q})}\right)^{1/2}\right]}{K(\mathrm{q})\left(1 + \frac{2}{sqK(\mathrm{q})}\right)^{1/2}}.$$

This expression describes the screening of anisitropic vortex fields and currents by the stack of superconducting layers [4].

Eq.(3) tends to Fisher's solution [5] in Lawrence-Doniach model for Josephson-decoupled layers by anisotropy vanishing.

The distribution of $2D$-vortex screening current is given by

$$\mathrm{I}_k(\mathrm{q}) = \frac{ic\phi_o}{2\pi}\frac{\mathrm{q}\times\mathrm{n}}{qK(\mathrm{q})}[\delta_{k0} - W_{k0}]. \tag{4}$$

Note that, by description of current in central layer consisting $2D$-vortex, W_{k0} can be neglected with respect to unit by natural condition $\Lambda >> s$. In result, we obtain the strongly anisotropic current distributin central plane,

$$\mathrm{I}_0(\mathrm{x}) = \frac{c\phi_o}{4\pi^2\Lambda} \frac{\mathrm{n} \times \mathrm{x}}{\mathrm{x}\hat{\mu}\mathrm{x}}. \tag{5}$$

3. The free energy of arbitrary $2D$-vortex configuration is equal to

$$F = \frac{\phi_o^2}{16\pi^3} \sum_k \sum_m \int \frac{d^2\mathrm{q}}{\varsigma K(\mathrm{q})} [\delta_{km} - W_{km}] S_k(\mathrm{q}) S_m^*(\mathrm{q}). \tag{6}$$

Here $S_k(\mathrm{q}) = \sum_l \exp(i\mathrm{q}\mathrm{x}_{kl}^o)$ is structure factor of $2D$-vortices of kth layer, x_{kl}^o is coordinate of lth vortex center.

The self energy of a pair of $2D$-vortex and antivortex,

$$F_o(\mathrm{R}) = \left(\frac{\phi_o}{4\pi}\right)^2 \frac{2}{\pi\Lambda} \int \frac{d^2\mathrm{q}}{(\mathrm{q} \times \mathrm{n})\hat{\mu}(\mathrm{q} \times \mathrm{n})} (1 - e^{i\mathrm{q}\mathrm{R}}),$$

can be reduced to isotropic shape by the scale transformations

$$(\mathrm{q} \times \mathrm{n})\hat{\mu}(\mathrm{q} \times \mathrm{n}) = (\mathrm{q}')^2, \quad \mathrm{R}\hat{\mu}\mathrm{R} = (\mathrm{R}')^2, \tag{7}$$

R is intervortex distance. Note, that the self energy of vortex pair has just isotropic form in prime coordinate system. This is possible due to the anisotropy of coherence length $\vec{\xi}$, determining by relation $\vec{\xi} \cdot \hat{\mu} \cdot \vec{\xi} = (\vec{\xi}')^2$, where length $\vec{\xi}'$ doesn't depend on vector R' orientation.

The scale transformation opposite to (7) leads to

$$F_o(\mathrm{R}) = \left(\frac{\phi_o}{4\pi}\right)^2 \frac{1}{\Lambda} \ln\left(\frac{\mathrm{R}\hat{\mu}\mathrm{R}}{\xi\hat{\mu}\xi}\right) = \left(\frac{\phi_o}{4\pi}\right)^2 \frac{2}{\Lambda} \ln\left(\frac{R'}{\xi'}\right). \tag{8}$$

This means that $2D$-vortex-antivortex interaction in a stack of anisotropic superconducting films coincides with the well-known expression in isotropic case [5].

Now we consider the free energy of stacks of $2D$-vortices whose centers lie along c-axis and mutual position of these stacks are arbitrary. After summation over k in W_{km} of Eq.(6), we get the value of free energy per one layer,

$$F = \left(\frac{\phi_o}{4\pi}\right)^2 \frac{s}{2\pi} \int \frac{d^2\mathrm{q}|S(\mathrm{q})|^2}{\lambda_\parallel^2(\mathrm{q} \times \mathrm{n})\hat{\mu}(\mathrm{q} \times \mathrm{n}) + \frac{qs}{2}\coth(\frac{qs}{2})}. \tag{9}$$

Here $S(\mathrm{q})$ is structure factor of a system of $3D$-vortices. $\lambda_\parallel^2 = s\Lambda/2$ is penetration depth of $3D$-vortex magnetic field along the superconducting layer.

For $s \to \infty$ and $\coth(\frac{qs}{2})$ tends to unit, Eq.(9) describes the vortices in isolated anisotropic film [10].

For $s \to 0$ (in another words, for a "large" distance, $R >> s$, between vortices) the second term in denominator of Eq.(9) tends to unit and Eq.(9) describes $3D$-vortices in a bulk anisotropic superconductor [11]. In prime coordinate system (7), Eq.(9) has an isotropic form, so hexagonal vortex lattice [12] with isotropic deformation moduli [11] satisfy to its absolute minimum. The opposite to (7) scale transformation lets to easily obtain the all of stable vortex lattice structures and anisotropy of shear modulus [14].

4. In conclusion we discuss an influence of a superconducting layer anisotropy on general peculiarities of current-voltage ($I-V$) characteristics of Josephson-decoupled layered superconductors.

In Meissner phase the dissociation of $2D$-vortex-antivortex pairs take a place at the Kosterlitz-Thouless temperature [15]

$$T_{KT} = \frac{\phi_o^2}{16\pi^2\Lambda}. \tag{10}$$

In a stack of anisotropic layers the energy of vortex pair (8) has just isotropic form. An entropy contribution to the free energy doesn't depend on a choose of any coordinate system, so Kosterlitz- Thouless transition occurs at temperature T_{KT} in case anisotropy too.

At a small value of external current I, the energy of vortex pair (8) decreases, is given by

$$F = F_o(\mathbf{R}) - \frac{\phi_o}{c}(\mathbf{n}\times\mathbf{I})\mathbf{R} \tag{11}$$

and consists not an anisotropy parameter. Thus, the $I-V$ characteristics of anisotropic multilayers in Meissner phase must be the same as isotropic one, if a vortex mobility is isotropic too. Otherwise, a voltage scaling will give possibility to obtain the dependence of $2D$-vortex mobility on direction and anisotropy.

Now we consider the case of nonzero magnetic field and thermal disruption of $3D$-vortex. At the displacement of any $2D$-vortex on distance R from one stack axis, the free energy excess is given by

$$F_o(\mathbf{R}) = \frac{\phi_o^2}{8\pi^3\Lambda}\int\frac{d^2\mathbf{q}[1-\cos(\mathbf{q}\mathbf{R})]}{(\mathbf{q}\times n)\hat{\mu}(\mathbf{q}\times\mathbf{n})[1+\lambda_{\|}^2(\mathbf{q}\times\mathbf{n})\hat{\mu}(\mathbf{q}\times\mathbf{n})]}. \tag{12}$$

In primed coordinate system (7) this equation is isotropic and

$$F_o(\mathbf{R}') = \frac{\phi_o^2}{4\pi^2\Lambda}[\gamma + \ln(R'/2\lambda_{\|}) + K_o(R'/\lambda_{\|})], \tag{13}$$

where $\gamma = 0.5772\ldots$ is Euler's constant [4].

Following J.Clem [4] and using Eq.(13) we will be able to convince that thermal decoupling of the $3D$-vortex lattice doesn't depend on anisotropy parameters in primed coordinate system.

An influence of current I on disruption of $3D$-vortex is described by Eq.(11), where $F_o(R)$ is given by Eq.(12) or (13). To transform potential (11) into fully isotropic form, we supply the coordinate scaling (7) together with the current transformation

$$\mathbf{I}\hat{\mu}\mathbf{I} = (\mathbf{I}')^2. \tag{14}$$

From this expression the dependence of $I-V$ characteristics on current I direction follows immediately. For example, the critical current $\mathbf{I}_c$ of lattice melting is changed as $(\mathbf{I}\cdot\mu\cdot\mathbf{I})^{1/2}$ by current I rotation. The extreme values of $\mathbf{I}_c$ differ each from other in μ_a times. μ_a is roughly equal to 1.2 for artificial Y high$-T_c$ multilayers. The possible difference of critical currents in Bi or Tl high$-T_c$ compound can be an evidence of anisotropy of CuO_2-layers in this crystal.

Thus the $I - V$ characteristics of anisotropic multilayers depend on current direction and anisotropy parameters at presence of nonzero magnetic field only.

I believe that experimentally noticeable difference of values I_c at different current orientations can be obtained by method [2] and in artificial multilayers of polisulfur nitride $(CN)_x$, whose long molecules can be easily laying in fixed planes [16]. Thanks to this, a great anisotropy in basal plane can be reached and obtaining result would be easily examined.

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1. A.K.Asadov, Yu.A.Genenko, G.G.Levchenko et al., Physica C**206** 119 (1993).
2. Y.M.Wan, S.E.Hebboul, D.S.Harris and J.C.Harland, preprint, Submitted to Phys. Rev. Lett. (1993).
3. K.B.Efetov, Zh. Eksp. Teor. Fiz. **76** 1781 (1979).
4. J.R.Clem, Phys. Rev. B**43** 7837 (1991).
5. K.H.Fisher, Physica C**178** 161 (1991).
6. D.Feinberg, Physica C**194** 126 (1992).
7. L.Ya.Vinnikov, L.A.Gurevich, G.A.Emel'chenko, and Yu.A.Osip'yan, Pis'ma Zh. Eksp. Teor. Fiz. **47** 109 (1988).
8. G.J.Dolan, F.Holtzberg, C.Feild, and T.R.Dinger, Phys. Rev. Lett. **62** 2184 (1989).
9. D.J.Bishop, P.L.Gammel, D.A.Huse, and C.A.Murray, Science **255** 165 (1992).
10. A.Yu.Martynovich, Submitted to Zh. Eksp. Teor. Fiz. (1993).
11. L.J.Gampbell, M.M.Doria, and V.G.Kogan, Phys.Rev.B **38** 2439 (1988).
12. J.Matricon, Phys. Lett. **9** 289 (1964).
13. E.H.Brandt, J.Low Temp. Phys. **26** 709 (1977).
14. A.M.Grishin, A.Yu.Martynovich, and S.V.Yampolskii, Zh. Eksp. Teor. Fiz. **101** 649 (1992).
15. V.L.Berezinskii, Zh. Eksp. Teor. Fiz.**61** 1144 (1971); J.M.Kosterlitz and D.J.Thouless, J. Phys. C. 6 1181 (1973).
16. Y.Oda, H.Takenaka, H.Nagano and I.Nakada, Sol. St. Comm. **32** 659 (1979).