The constraints on the non-singlet polarised parton densities from the infrared-renormalon model

A. L. Kataev[1)]

Institute for Nuclear Research RAS, 117312 Moscow, Russia

Submitted 7 April 2003

Using the infrared-renormalon approach, we obtain the constraints on the next-to-leading order non-singlet polarised parton densities. The advocated feature follows from the consideration of the effect revealed in the process of the next-to-leading order fits to the data for the assymetry of polarised lepton-nucleon scattering which result in the approximate nullification of the $1/Q^2$-correction to $A_1^N(x,Q^2)$.

PACS: 11.15.Tx, 12.88.+e, 13.85.Hd

The study of the QCD predictions for the photon-nucleon asymmetry $A_1^N = (\sigma_{1/2} - \sigma_{3/2})/(\sigma_{1/2} + \sigma_{3/2})$, where subscripts denote the total angular momentum of the photon-nucleon pair along the incoming lepton's direction, plays the essential role in the analysis of polarised deep inelastic scattering (DIS) (see e.g. Ref. [1]). It is related to the well-known structure function g_1^N of polarised DIS by the following way

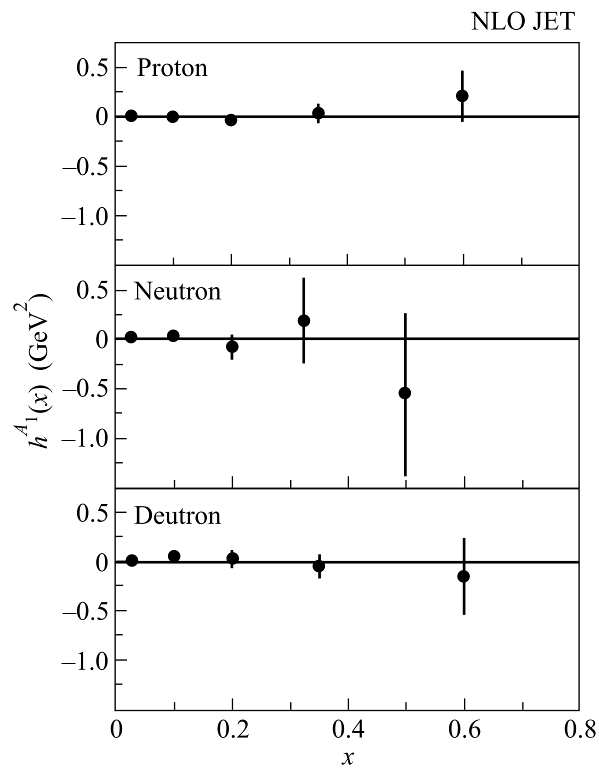
$$A_1^N(x,Q^2) = (1+\gamma^2)\frac{g_1^N(x,Q^2)}{F_1^N(x,Q^2)}, \qquad (1)$$

where the kinematic factor γ is defined as $\gamma = 4M_N^2x^2/Q^2$ and $g_1^N(x,Q^2)$ is the structure function (SF) of polarised DIS, while $F_1^N(x,Q^2)$ SF enters into the cross-section of unpolarised charged lepton-hadron DIS (see e.g. Ref. [2]). Quite recently several procedures of the study of the Q^2-behaviour of A_1^N were discussed in the literature (see Refs. [3 – 6]). Moreover, in Refs. [5, 6, 2] by fitting existing data for polarized DIS obtained at accelrators of scientific centers CERN, DESY and SLAC in the kinematical region $0.005 \leq x \leq$ ≤ 0.75 and 1 GeV$^2 \leq Q^2 \leq 58$ GeV^2 the $1/Q^2$ dynamical power correction to A_1^N was extracted. In general it gives additional contribution to the perturbation theory part of $(A_1^N)_{PT}$ and can be parameterised as

$$A_1^N(x,Q^2) = (A_1^N(x,Q^2))_{PT} + h^{A_1}(x)/Q^2. \qquad (2)$$

It is interesting, that in the process of the fits of Refs. [5, 6, 2] it was found that the x-shape of $h^{A_1}(x)$ is consistent with zero (see e.g. Figure from Ref. [2]).

In this note we are describing the possible consequences of this effect in the non-singlet (NS) approximation, which is valid for the x-cut $x \gtrsim 0.25$. Our consideration will be based on the infrared-renormalon



The results of extraction of $h^{A_1}(x)$ from the next-to-leading order fits of Ref. [2] in the JET scheme [7]

(IRR) approach, developed in QCD in Ref. [8] and reviewed in detail in Ref. [9]. This approach was used in Ref. [10] to study the behaviour of the $1/Q^2$ corrections to the NS contributions to F_2 and F_1 SFs of unpolarised DIS of charged leptons on nucleons and the pure NS xF_3 SF of νN DIS using $\overline{MS}$-scheme calculations[2)].

[1)]e-mail: kataev@ms2.inr.ac.ru

[2)]Note that we avoid considerations of the IRR renormalon free expansions in QCD coupling constants with the "freezing-type" behaviour at small Q^2 (see Refs. [11]).

It is interesting that the predicted in Ref. [10] x-shape of the IRR induced power corrections to xF_3 was supported in Refs. [12, 13] by the leading order (LO) and next-to-leading order (NLO) fits to CCFR'97 data (the detailed description and refinements of the fits of Ref. [12] is given in Refs. [14, 15]). Therefore, it is worth to consider the consequences of calculations of the IRR contributions to the NS part of g_1^N SF of polarised deep-inelastic scattering, which was also performed in Ref. [10].

Let us rewrite Eqs.(1), (2) in the following way

$$A_1^N = (1+\gamma^2)\frac{g_1^N(x,Q^2)(1+\dfrac{h^{g_1}(x)}{Q^2\, g_1^N(x,Q^2)})}{F_1^N(x,Q^2)(1+\dfrac{h^{F_1}(x)}{Q^2\, F_1^N(x,Q^2)})} \qquad (3)$$

where $h^{g_1}(x)/Q^2$ and $h^{F_1}(x)/Q^2$ are the model-independent parameterisations for the twist-4 contributions to g_1^N and F_1^N SFs, which in general are non-zero. Using the above mentioned effect of approximate nullification of the twist-4 correction to A_1^N we get

$$\frac{h^{g_1}(x)}{Q^2\, g_1^N(x,Q^2)} \approx \frac{h^{F_1}(x)}{Q^2\, F_1^N(x,Q^2)}. \qquad (4)$$

At the next step we will use the existing inequality for $g_1^N(x,Q^2)$ SF, namely

$$|g_1^N(x,Q^2)| \leq F_1^N(x,Q^2). \qquad (5)$$

Combining it with Eq. (4) we arrive to to the following bound

$$|h^{g_1}(x)| \leq |h^{F_1}(x)|. \qquad (6)$$

It should be stressed that the calculations of Ref. [10] predict that in the NS approximation (or in the valence-quarks approximation) the contributions of the $1/Q^2$ corrections to F_1 and xF_3 SFs are the same. Indeed, the corresponding results of Ref. [10] can be re-written in the following way:

$$h^{F_1}(x,\mu^2) = h^{F_3}(x,\mu^2) = A_2^{'} \int_x^1 \frac{dz}{z} C_1(z) q^{NS}(x/z,\mu^2), \qquad (7)$$

where

$$C_1(z) = -\frac{4}{(1-x)_+} + 2(2+x+2x^2) - \\ - 5\delta(1-x) - \delta^{'}(1-x), \qquad (8)$$

the '+' – prescription, for any test function, is defined as

$$\int_0^1 F(x)_+ f(x)dx = \int_0^1 F(x)[f(x)-f(1)]dx, \qquad (9)$$

and

$$q^{NS}(x,\mu^2) = \\ = \sum_{i=1}^{n_f}\left(e_i^2 - \frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2\right)\left(q_i(x,\mu^2) + \overline{q}_i(x,\mu^2)\right) \qquad (10)$$

are the NS parton densities, μ^2 is the normalisation point of order 1 GeV^2 and A_2' is the IRR model parameter, to be extracted from the fits to the concrete data. Its value was extracted from the low-energy xF_3 data, collected by the IHEP-JINR Neutrino detector at the IHEP 70 GeV proton synchrotron [16]. (see also Ref. [17] for a review). The result of Ref. [16] $A_2^{'}=-0.10\pm0.09$ (experimental) GeV^2 is in agreement with the value extracted from the NLO analysis of the CCFR'97 xF_3 data [18], namely with $A_2^{'}=-0.125\pm0.053$ (statistical) GeV^2 [15]. It should be noted, that the identity of Eq. (7) does not contradict point of view, expressed in Refs. [3, 4], that to study the Q^2 behaviour of $A_1(Q^2)$ in the NS approximation it might be convenient to use the concrete xF_3 data instead of theoretical expression for F_1^N.

Consider now the case of g_1^N SF of polarised DIS. In general, the IRR contributions to g_1^N were studied in Ref. [19]. In the NS approximation the IRR contributions to g_1^N were calculated in Ref. [10], where the following result was obtained

$$h^{g_1}(x,\mu^2) = A_2 \int_x^1 \frac{dz}{z} C_1(z) \Delta^{NS}(x/z,\mu^2). \qquad (11)$$

Here $\Delta^{NS}(x,\mu^2)$ are the NS polarised parton densities, namely

$$\Delta^{NS}(x,\mu^2) = \\ = \sum_{i=1}^{n_f}\left(e_i^2 - \frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2\right)\left(\Delta q_i(x,\mu^2) + \Delta\overline{q}(x,\mu^2)\right) \qquad (12)$$

and the IRR model coefficient function $C_1(z)$ is the same, as in the case of the IRR model contributions to the $1/Q^2$ corrections for F_1 and xF_3 SFs of unpolarised deep-inelastic scattering (see Eq. (8)). As to the IRR model parameter A_2, in general one should not expect that it has the same value as the parameter $A_2^{'}$ in Eq. (7). In principle, it should be extracted from the separate fits to g_1 data in the NS approximation. However, it is worth to stress, that in the NS approximation the IRR contributions to g_1^N and F_1^N are closely related (the similar feature was revealed while comparing IRR model contributions to the Bjorken sum rule for g_1^N SF [20] and still unmeasured Bjorken sum rule for $F_1^{\nu N}$ SF [21]).

Using now Eqs.(6), (7) and Eq.(11), we get the following constraint

$$|A_2\Delta^{NS}(x,\mu^2)| \leq |A_2^{'} q^{NS}(x,\mu^2)| \qquad (13)$$

which is valid both at the LO and NLO. This constraint is the main result of our note. The consequences for its Q^2-dependence can be further studied using the machinery of the DGLAP equations [22].

It is rather impressive that the NLO constraint of Eq. (13) is similar to the well-known LO bound of Ref. [1], namely

$$|\Delta(x,Q^2)| \leq q(x,Q^2). \qquad (14)$$

Moreover, Eq. (13) can be also transformed to the LO relation between the IRR model parameters of Eq. (3) and Eq. (7), namely

$$|A_2| \sim |A_2^{'}|. \qquad (15)$$

We hope that it will be possible to check the relation of Eq. (15) using the fits of the concrete data for g_1 SF.

The final version of this work was presented at COMPASS meeting at Dubna (5 March 2003). I am grateful to M. G. Sapozhnikov for invitation. It is the pleasure to thank A. V. Sidorov and D.B. Stamenov for discussions of the preliminary version of this note and to G. Altarelli and B. Webber for the useful questions and comments.

This work is done within the program of the RFBR Grants # 02-01-00601 and # 03-02-17177.

1. G. Altarelli, S. Forte, and G. Ridolfi, Nucl. Phys. **B534**, 277 (1998) [hep-ph/9806345].
2. E. Leader, A. V. Sidorov, and D. B. Stamenov, hep-ph/0212085, to be published in Phys. Rev. **D**.
3. A. V. Kotikov and D. V. Peshekhonov, JETP Lett. **65**, 7 (1997) [hep-ph/9612319].
4. A. V. Kotikov and D. V. Peshekhonov, Eur. Phys. J. **C9**, 55 (1999) [hep-ph/9810224].
5. E. Leader, A. V. Sidorov, and D. B. Stamenov, in *Proc. of 9th Lomonosov Conference on Elementary Particle Physics: Particle Physics on the Boundary of the Millenium*, Moscow, Russia, 20-26 Sep. 1999, Ed. A. I. Studenikin, World Scientific, 2001, p. 78.
6. E. Leader, A. V. Sidorov, and D. B. Stamenov, Eur. Phys. J. **C23**, 479 (2002) [hep-ph/0111267].
7. R. D. Carlitz, J. C. Collins, and A. H. Mueller, Phys. Lett. **B214**, 229 (1988); M. Anselmino, A. Efremov, and E. Leader, Phys. Rept. **261**, 1 (1995) [Erratum-ibid. **281**, 399 (1997)] [hep-ph/9501369]; D. Muller and O. V. Teryaev, Phys. Rev. **D56**, 2607 (1997) [hep-ph/9701413].
8. V. I. Zakharov, Nucl. Phys. **B385**, 452 (1992).
9. M. Beneke, Phys. Rept. **317**, 1 (1999) [hep-ph/9807443].
10. M. Dasgupta and B. R. Webber, Phys. Lett. **B382**, 273 (1996) [hep-ph/9604388].
11. N. V. Krasnikov and A. A. Pivovarov, Mod. Phys. Lett. **A11**, 835 (1996) [hep-ph/9602272]; D. V. Shirkov and I. L. Solovtsov, Phys. Rev. Lett. **79**, 1209 (1997) [hep-ph/9704333]; Y. A. Simonov, Phys. Atom. Nucl. **65**, 135 (2002) [Yad. Fiz. **65**, 140 (2002)] [hep-ph/0109081].
12. A. L. Kataev, A. V. Kotikov, G. Parente, and A. V. Sidorov, Phys. Lett. **B417**, 374 (1998) [hep-ph/9706534].
13. S. I. Alekhin and A. L. Kataev, Phys. Lett. **B452**, 402 (1999) [hep-ph/9812348].
14. A.L. Kataev, G. Parente, and A. V. Sidorov, Nucl. Phys. **B573**, 405 (2000) [hep-ph/9905310].
15. A. L. Kataev, G. Parente, and A. V. Sidorov, Fiz. Elem. Chast. Atom. Yadra **34**, 43 (2003) [Phys. Part. Nucl. **34**, 20 (2003)] [hep-ph/0106221].
16. S. I. Alekhin, V. B. Anykeyev, A. A. Borisov et al., Phys. Lett. **B512**, 25 (2001) [hep-ex/0104013].
17. A. L. Kataev, hep-ph/0107247, in *Proc. of 15th Les Recontre de Physique de la Vallee d'Aosta: Results and Perspectives in Particle Physics*, 2001, Ed. M. Greco, Frascati Physics Series, v. XXII, 2001, p. 205.
18. W. G. Seligman, C. G. Arroyo, L. de Barbaro et al., Phys. Rev. Lett. **79**, 1213 (1997).
19. E. Stein, M. Maul, L. Mankiewicz, and A. Schafer, Nucl. Phys. **B536**, 318 (1998) [hep-ph/9803342].
20. D. J. Broadhurst and A. L. Kataev, Phys. Lett. **B315**, 179 (1993) [hep-ph/9308274].
21. D. J. Broadhurst and A. L. Kataev, Phys. Lett. **B544**, 154 (2002) [hep-ph/0207261].
22. V. N. Gribov and L. N. Lipatov, Yad. Fiz. **15**, 781 (1972) [Sov. J. Nucl. Phys. **15**, 438 (1972)]; G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977); Y. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977) [Zh. Eksp. Teor. Fiz. **73**, 1216 (1977)].