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ASYMPTOTIC FREEDOM AT LARGE DISTANCES AND THE IR RENORMALON PROBLEM

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Perturbation theory is formulated in the confining background field B_μ. All diagrams are gauge invariant and at large N_c limit depend on B_μ only through the Wilson loops $W(C, B_\mu)$. Assuming the area law for the latter one obtains the charge renormalization at large distances which yields behaviour $\alpha_s(p) \sim 4\pi[\ln \frac{p^2+m^2}{\Lambda^2}]^{-1}$, where m is a process-dependent mass of the order of 1 GeV. Then for $m \gg \Lambda$ the asymptotic freedom persists for all Euclidean momenta and IR renormalons are shown to disappear.

1. Perturbation expansion (PE) in the free vacuum of QCD is diverging at large distances due to the ghost pole in the running coupling $\alpha_s(p)$. In addition sum of PE is not Borel summable due to the IR renormalons [1]. In this letter we instead consider PE in the framework of the background field formalism [2,3], splitting in the lagrangian L the total field A_μ into the nonperturbative (NP) part B_μ and perturbation a_μ:

$$A_\mu = B_\mu + a_\mu, \quad L(A) = L_0(B) + \sum_{i=1}^{4} L_i(B, a). \qquad (1)$$

Then the usual diagrammatic technic can be used for the expansion in powers of ga_μ, with propagators of the quantum field a_μ, $G_{\mu\nu}(x, y; B)$, depending on the background field B_μ. We also use as in [3] the gauge fixing (gf) term $\frac{1}{2\xi}(D_\mu(B)A_\mu)^2$ and the corresponding Faddeev – Popov ghosts. Prescribing gauge transformations for $a_\mu, B_\mu \to V^+(a_\mu, B_\mu - \frac{i}{g}\partial_\mu)V$ all physical amplitudes can be written for large N_c with the help of the Feynman-Schwinger representation (FSR) [4] through the average of a product of Wilson loops $W(C; B)$ of the field B_μ.

We are in particular interested in the large distance behaviour of the new PE, and assume for $< W(C, B) >$ the area law $\exp(-\sigma S_{min})$, which follows naturally from the cluster expansion [5] or lattice calculations [6]. Thus the large distances are described by only one input parameter – the string tension σ, which is used as the only characteristics of the NP fields $\{B_\mu\}$.

Our specific goal is to calculate the running coupling $\alpha_s(p)$ at all Euclidean momenta p including the IR region $p \to 0$. Two different NP definitions of the charge are used:

i) the energy of static charges at distance R

$$E(R) = E_{NP}(R;B) - \frac{4}{3}\frac{\alpha_s(R)}{R}, \tag{2}$$

where $E(R)$ is computed as

$$E(R) = -\lim_{T\to\infty}\{\frac{1}{T}\ln < W(C, B+a) >\} \tag{3}$$

and the contour C is a rectangular $R \times T$;

ii) the extended method of [3], where the two-point functions with external B lines have been calculated. In the lowest order for $B = 0$ the charge Z factor, Z_g, is given by a gluon a_μ loop and a ghost loop diagrams, yielding familar charge renormalization.

In the full treatment for $B \neq 0$ one considers a gauge-invariant generalization of the same loop diagrams, where gluon and ghost Green's functions are not free but contain B_μ to all orders.

In the first definition i) the case without background renormalization of the Wilson loop was considered in [7] and the result for $\alpha_s(R)$ can be obtained from [8] taking the limit of heavy quark masses:

$$\alpha_s(R) = \alpha_s(\mu)(1 + \alpha_s(\mu)f^{(0)}(R) + ...) \ . \tag{4}$$

The same diagrams as in the free vacuum case can be considered with the confining background which reveals itself as the area law for the Wilson loop. In this specific situation when the side length T of the Wilson loop tends to infinity and N_c is large, so that gluon lines are replaced by double fundamental lines, one can show that the internal fundamental loops have the asymptotics [9] at large $| x - y |$

$$< \Pi(x, y; B) >_B \sim exp(-m_1 \mid x - y \mid). \tag{5}$$

The mass m_1 is a mass of two charges connected by the string, like the mass of ρ meson and can be computed through σ [10]. Typically [9,10] $m_1 \approx (3-4)\sqrt{\sigma} \sim 1$ GeV. Using the asymptotics (5) in the computation of $f^{(0)}(R)$ one obtains instead of (4)

$$\frac{1}{R}f^{(B)}(R) \approx I(R) \equiv \int_\delta^\infty \frac{dr}{r}e^{-m_1 r}(\frac{1}{R}\Theta(R-r) + \frac{1}{r}\Theta(r-R)). \tag{6}$$

At small R, $I(R)$ has the familiar asymptotic freedom behaviour

$$I(R) \approx \frac{1}{R}\ln\frac{R}{\delta} \ , \quad m_1 R \ll 1. \tag{7}$$

This coincides with $f^{(0)}$ upon renormalization $\frac{1}{\delta} \to \mu$.

For large R, $m_1 R \gg 1$, the logarithmic growth of $f^{(B)}(R)$ is "screened" in (6):

$$f^{(B)}(R) \approx ln \frac{1}{m_1 \delta} \sim ln \frac{\mu}{m_1} + 0(\frac{1}{Rm_1}). \tag{8}$$

From the Fourier transform of (2) one easily obtains

$$\alpha_s(p) \approx \alpha_s(\mu)(1 + \alpha_s(\mu)\frac{b_0}{4\pi} \ln \frac{p^2 + m_1^2}{\mu^2} + ...) \tag{9}$$

Similar results one obtains for the definition ii), where the contribution to Z_g is calculated from the two-point function:

$$H_{\mu\nu}(x, y) = \frac{g^2}{(4\pi)^2} b_0(\partial_\mu \partial_\nu - \partial^2 \delta_{\mu\nu})\bar{\Pi}(x, y) \tag{10}$$

and the one-loop function $\bar{\Pi}$ contains sum of contributions of the gluon a_μ and ghost in the background B_μ, which can be computed using FSR [4,10] and the proper-time Hamiltonian technic [9,10].

For us it is only important the asymptotics behaviour which can be found by mentioned above methods:

$$\bar{\Pi}(x, y) \sim \exp(-m_2 \mid x - y \mid), \quad \mid x - y \mid \to \infty \tag{11}$$

Here the value of m_2 is close to the two-gluon glueball mass in the state 0^{++}. As was computed in [11] m_2 is around 2 GeV. Using (11) one can compute Z_g to one loop approximation and $\alpha_s(p)$ appears to be the same as in (9) with replacement $m \to m_2$. We note that m_i depends on the process, since the infrared asymptotics of the renormalization gluon loop depends on the surroundings in the diagram where it enters – the loop is attached by strings to the overall Wilson contour.

It is important that due to the gauge invariance of PE the renormalization of g and B_μ are connected [3] : $Z_g = Z_B^{-1/2}$ and the product gB_μ is the renorminvariant, implying the same property for σ and m_1, m_2. Thus it is not surprising that m^2 enters (9) on the same footing as the momentum p^2. Moreover, the dependence on the normalization mass (scale parameter) μ is the same as in the free case, which means that Gell-Mann-Low equations do not change (at least to one-loop order)

$$\frac{d \ln g}{d \ln \mu} = -\frac{b_0 g^2}{16\pi^2}. \tag{12}$$

Solving (12) with the initial condition (9) one obtains $\alpha_s(p)$ in the form with the $\bar{\Lambda}$ parameter

$$\alpha_s(p) = \frac{4\pi}{b_0 \ln \frac{m^2 + p^2}{\bar{\Lambda}^2}}. \tag{13}$$

Here $\bar{\Lambda}$ may differ from the usual QCD parameter, and coincides with it when $m \to 0$.

The form (13) goes over into the familiar one when $p^2 \gg m_i^2$ and is universial in this limit. However, for $p^2 < m_i^2$ the value of $\alpha_s(p)$ is not universal (i.e. depends on the process where it enters) which is reflected by the nonuniversality of m_i^2 in our two definitions.

In addition, we have systematically dropped in the process of derivation of (9), (13) constant terms and powers of $(\mu^2/(p^2+m^2))$, so that the result (9) is a leading term for large $\ln \frac{m^2+p^2}{\mu^2} \gg 1$.

The estimate of $\alpha_s(p=0)$ in (13) for $m \cong 1$ GeV and $\Lambda \simeq 0.2$ GeV, $b_0 = 9$ yields $\alpha_s(p=0) = 0.43$ and $\alpha_s(p \approx m_\rho) \approx 0.38$ in agreement with sum rule estimates in [12]. Thus (13) gives a phenomenologically reasonable behaviour whereby the asymptotic freedom persists up to small values of momentum.

Finally we turn to the renormalon problem [1]. Following [13] we write the generalization of the contribution of the set of IR renormalon diagrams to the Euclidean correlator $\Pi(Q^2)$ of e.m. currents as

$$\Delta\Pi = \frac{\alpha_s(Q^2)}{8\pi^3}\sum_n\left(\frac{b_0\alpha_s(Q^2)}{4\pi}\right)^n \int_0^{Q^2}\frac{k^2dk^2}{Q^4}\ln^n\left(\frac{Q^2+m^2}{k^2+m^2}\right) \tag{14}$$

where we have used (13) and have taken the normalization point at Q^2+m^2, $Q^2 \gg m^2$. When $m=0$ (14) goes over into the familiar expression (see e.g. Eq.(9) of ref. [13]). The integration in (14) yields

$$\Delta\Pi(Q^2) = \frac{1}{2\pi^2 b_0}\sum_{n\gg 1}^{\infty}\left(\frac{\alpha_s(Q^2)b_0}{8\pi}\right)^n q_n \tag{15}$$

where

$$q_n = \begin{cases} (n-1)!, & n < 2n_0 \\ \frac{(2n_0)^n}{n}, & n > 2n_0 \end{cases} \tag{16}$$

and $n_0 = ln\frac{Q^2+m^2}{m^2} \gg 1$.

In the limit $m \to 0$, $n_0 \to \infty$ one obtains in (15) a factorially diverging series, with the Borel transform $\Delta\tilde{\Pi}(t)$ having a pole in the Borel parameter t at $8\pi/b_0$ – the so called IR renormalon [1] precluding the Borel summation of the series (15). For finite values of m, however, the factorial growth of q_n stops at $n \approx 2n_0$ and the series (15) is summable by usual methods yielding asymptotically, $n_0 \gg 1$, the simple expression.

$$\Delta\Pi(Q^2) \cong -\frac{1}{2b_0\pi^2}\ln\frac{\alpha_s(Q^2)}{\alpha_s(0)}. \tag{17}$$

Thus the confining background drastically improves the properties of PE and gives hopes that the perturbation theory of QCD can be managed at all distances. For that NP background can be described by the lowest correlators $< FF >$, $< FFF >$... or just by the string tension (at large distances), while from the PE one can keep only few lowest order terms.

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