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SU(2)-EFFECTIVE ACTION WITH THE NONANALYTIC TERM

O.K.Kalashnikov

Department of Theoretical Physics P.N.Lebedev Physical Institute
117924 Moscow, Russia[1)]

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The nonanalytic g^3-term is calculated for $SU(2)$-effective action at finite temperature and the status of a gauge fields condensation and the problem of degeneracy are briefly discussed.

The gauge models with embeded external fields are a very important object of modern theoretical physics because their essence is rather realistic and adequate to many physical phenomena. The gauge fields condensation which can be simulated through a very simple external field is a typical phenomenon for many unified gauge models and its properties are intensively studied today by using mainly the effective action technique. However two-loop effective action calculated with a nonzero external field (at first in [1] and then for an arbitrary gauge parameter ξ in [2] for $SU(2)$-group and in [3,4] for $SU(3)$-one) displays that all its minima are physically equivalent although no evident reasons exist to consider this degeneracy being proved. This fact makes the phenomenon (where the gauge fields condensate arises spontaneously [5]) unreliable and it is very important to establish the real meaning of this scenario. Moreover the special attention should be paid to a gauge-invariance of the results found [4,6], especially a situation is not clear when the high-orders corrections being taken into account. The search of these corrections (at least the g^4-order ones) is a very actual task since only they can determine the status of this phenomenon and a role of degeneracy in building the nontrivial vacuum. Of course, the found degeneracy is partially determined by a symmetry of the breaking operator which destroys the initial gauge group but as concerns a trivial vacuum this degeneracy is false and results from the imperfection of the calculational scheme based on the lowest orders perturbative graphs.

The quantum SU(2)-lagrangian in the background gauge has the standard form

$$\mathcal{L} = -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{2\xi}[(\bar{\mathcal{D}}_\mu V_\mu)^a]^2 + \bar{C}\bar{\mathcal{D}}_\mu \mathcal{D}_\mu C, \tag{1}$$

where the gauge fields V^a_μ are decomposed in the quantum part Q^a_μ and the classical constant one $\bar{A}^a_\mu$ (here $V^a_\mu = Q^a_\mu + \bar{A}^a_\mu$). The gauge fields strength tensor $G^a_{\mu\nu}$ is determined through the new covariant derivative $\bar{\mathcal{D}}^{ab}_\mu = \partial_\mu \delta^{ab} + g f^{acb} \bar{A}^c_\mu$ but a term with the ghost fields $\bar{C}$ (and C) in (1) contains (besides $\bar{\mathcal{D}}(\bar{A})$) the usual derivative $\mathcal{D}^{ab}_\mu$ which depends on the total V^a_μ field. The parameter ξ fixes the internal gauge and the classical field has the form

$$\bar{A}^a_\mu = \delta_{\mu 4}\delta^{a3} A^{ext} = \delta_{\mu 4}\delta^{a3}\frac{\pi T}{g}x, \tag{2}$$

[1)]E-mail address: tdparticle@glas.apc.org

where x is a new variable. Here T is temperature and g is the standard coupling constant.

The effective action for this model (including the two-loop graphs) has been calculated by many authors [1,2] and has a rather simple form

$$
\begin{aligned}
W(x)/T^4 &= W^{(1)}(x)/T^4 + W^{(2)}(x)/T^4, \\
W^{(1)}(x)/T^4 &= \frac{2}{3}\pi^2[B_4(0) + 2B_4(\frac{x}{2})], \\
W^{(2)}(x)/T^4 &= \frac{g^2}{2}[B_2^2(\frac{x}{2}) + 2B_2(\frac{x}{2})B_2(0)] + \frac{2}{3}g^2(1-\xi)B_3(\frac{x}{2})B_1(\frac{x}{2}),
\end{aligned}
\tag{3}
$$

where $B_n(z)$ are the modified Bernoulli polynomials

$$
\begin{aligned}
B_1(z) &= z - \epsilon(z)/2 \ , \qquad B_3(z) = z^3 - 3\epsilon(z)z^2/2 + z/2, \\
B_2(z) &= z^2 - |z| + 1/6, \quad B_4(z) = z^4 - 2|z|^3 + z^2 - 1/30
\end{aligned}
\tag{4}
$$

with $\epsilon(z) = z/|z|$. Here we should consider that $\epsilon(0) = 0$ as it results from the direct calculations to make (3) be correct.

The action (3) has three extremum points

$$
\bar{x} = 0, \qquad \bar{x} = 1, \qquad \bar{x} = 2, \tag{5}
$$

where two of them ($\bar{x} = 0$ and $\bar{x} = 2$) are minima of the presented action. The effective action being put on these extremum points is a gauge independent quantity [4] but, unfortunately, the thermodynamical potential found within this approximation for the trivial vacuum ($\bar{x} = 0$) and for the nontrivial one ($\bar{x} = 2$) has the same value

$$
\Omega/T^4 = 2\pi^2 B_4(0) + \frac{3g^2}{2}B_2^2(0) = -\frac{\pi^2}{15} + \frac{g^2}{24}. \tag{6}
$$

This fact indicates that a degeneracy (which is probably a signal of the real one) takes place within this scenario and the multi-loop corrections are very essential for clearing the situation. However the direct calculation of a three-loop effective action (the g^4-order) is a hopeless task and therefore a nonperturbative scheme should be built to define the status of this phenomenon. Below the simplest summation is used to calculate the leading nonanalytic term in the nonperturbative expansion of $W(x)$ and we discuss its gauge dependence. This term is the g^3-order and for many physical phenomena plays a more essential role then the g^4-terms to come.

It is well known (see e.g.[7,8]) that for any non-Abelian gauge theory (despite of its more complicated structure) the leading nonanalytic term can be reproduced through the standard formula

$$
\frac{\partial W^{(cor)}(x)}{\partial g} = \frac{1}{\beta g}\sum_{k_4}\int \frac{d^3k}{(2\pi)^3}\mathrm{Tr}[\mathcal{D}(k)\Pi(k)] \tag{7}
$$

where the polarization tensor $\Pi(\bar{k}, k_4)$ should be calculated in the lowest order. For this calculation only $\Pi_{44}(|\bar{k}| \to 0, 0)$ is used and the final result has the form

$$
\Delta W^{(cor)} = -\frac{\Pi_{44}^{3/2}(0)}{12\pi\beta}\mathrm{Tr}(I) \tag{8}
$$

Here I is the unit matrix in the adjoint representation of the chosen gauge group (for SU(N) one has $\mathrm{Tr}(I) = N^2 - 1$).

Polarization tensor for the broken $SU(2)$-group (when $x \neq 0$) has two components $\Pi^{||}(\bar{k}, k_4)$ and $\Pi^{\perp}(\bar{k}, k_4)$ which are completely independent within the g^2-approximation. Their calculations are standard and exploit the usual temperature Green functions technique in the imaginary time space. To simplify what follows all details used are omitted and below the infrared limits of the $\Pi_{44}(\bar{k}, k_4)$-components are presented only by their leading terms.

The order g^2 infrared limit of $\Pi^{||}_{44}(\bar{k}, k_4)$ has the simple form

$$\Pi^{||}_{44}(|\bar{k}| \to 0, k_4 = 0) = 4g^2T^2B_2(\frac{x}{2}), \tag{9}$$

and for its calculating the standard prescription is used (here $k_4 = 0$ and then $|\bar{k}| \to 0$). It is very important to notice that only the expression (9) is generated by the effective action (3) through the usual formula

$$m^2_{||} = \frac{g^2}{\pi^2T^2}\frac{1}{4}[\partial^2/\partial(\frac{x}{2})^2]W(\frac{x}{2}) \tag{10}$$

and there is a possibility to improve (9) up to the order g^4 terms

$$\begin{aligned} m^2_{||} &= 4g^2T^2B_2(\frac{x}{2}) + \\ &+ \frac{g^4T^2}{\pi^2}\left\{B_1^2(\frac{x}{2}) + \frac{1}{2}[B_2(\frac{x}{2}) + B_2(0)] + (1-\xi)[B_1^2(\frac{x}{2}) + B_2(\frac{x}{2})]\right\}. \end{aligned} \tag{11}$$

The infrared limit of $\Pi^{\perp}_{44}(\bar{k}, k_4)$ cannot be found within (10) and it is calculated directly through the Green functions technique. Moreover there are some peculiarities which complicate a search of this limit when $x \neq 0$ since the initial gauge symmetry is broken. In the transversal sector all gauge bosons acquare a mass (a nonzero damping at the tree level) and the infrared limit of $\Pi^{\perp}_{44}(\bar{k}, k_4)$ should be determined near a new mass shell $\hat{k}_4 = 0$ (where $\hat{k}_4 = k_4 + \pi T x$). The calculations are standard and the order g^2 infrared limit of $\Pi^{\perp}_{44}(\bar{k}, k_4)$ has the form

$$\Pi^{\perp}_{44}(|\bar{k}| \to 0, \hat{k}_4 = 0) = 2g^2T^2\left(B_2(\frac{x}{2}) + B_2(0)\right), \tag{12}$$

which is a gauge-invariant quantity for any $x \neq 0$. This is not the case when all other possible infrared limits of $\Pi^{\perp}_{44}(\bar{k}, k_4)$ are studied and we consider that only the expression (12) should be used within formula (8).

Now gathering all expressions found for the infrared limits of $\Pi_{44}(\bar{k}, k_4)$ and using formula (8) we obtain the nonanalytic corrections as follows

$$\Delta W^{(cor)}/T^4 = -\frac{2g^3}{3\pi}\left\{B_2^{3/2}(\frac{x}{2}) + 2[\frac{1}{2}(B_2(\frac{x}{2}) + B_2(0))]^{3/2}\right\}, \tag{13}$$

which are gauge-invariant themselves and for the case $x = 0$ they coincide with the known results (see e.g. [7,8] for $SU(2)$-group).

$$\Delta\Omega^{(cor)}/T^4 = -\frac{g^3}{4\pi}\sqrt{(\frac{2}{3})^3}. \tag{14}$$

Unfortunately further only the result (14) has a physical meaning since all positions of the extremum points (at least for a small g) are to be the same as in (5).

The corrected points (which should be used for treating the order g^4 effective action) are also known [4] and for a small g these corrections are proportional to the g^2-terms

$$\bar{x}_{1,3} = 1 \pm \left[1 - \frac{g^2}{4\pi^2}(1 + \frac{1-\xi}{2})\right] \tag{15}$$

Being substituted to the lowest orders effective action (3) these points generate the gauge-dependent g^4-corrections

$$\frac{\Omega}{T^4} = \frac{\Omega^{(1)}}{T^4} + \frac{\Omega^{(2)}}{T^4} = 2\pi^2 B_4(0) + \frac{3g^2}{2}B_2^2(0) +$$

$$+\frac{g^4}{48\pi^2}(1 + \frac{1-\xi}{2})^2 - \frac{g^4}{24\pi^2}|1 + \frac{1-\xi}{2}|(1 + \frac{1-\xi}{2}) \tag{16}$$

and other ones which, however, are beyond the calculational accuracy. So the nonanalytic g^3-terms found above are the leading ones in the nonperturbative expansion of $W(x)$ and the both expressions (3) and (13) used jointly are the closed result till the g^4-terms are absent.

However all g^4-terms (or at least some part of them) should be calculated exactly to solve the problem of degeneracy as well as to check with the aid of (15) a gauge-invariance of the order g^4 thermodynamical potential. In particular, analysing (16) we consider that there is a possibility to calculate exactly within the three-loop graphs all g^4-terms which are proportional to $(1-\xi)$-multiplyer and then to combine these terms with the analogous ones in (16). Although these terms being put on the extremum points should be equal zero they are very important to check the necessary condition of a gauge-invariance for the order g^4 thermodynamical potential. Of course it is necessary to find all other terms which are proportional to the high powers of $(1-\xi)$-multiplyer but this task seems to be more complicated and it can be investigated in the second rate.

The problem of degeneracy should be also solved within g^4-terms. It is doubtless that a trivial vacuum will be splitted but some degeneracy seems to be kept because this possibility is embeded at once by a symmetry of the breaking operator to build in accordance with a structure of the chosen external field. However there are no reasons to consider this symmetry being proved for the order g^4 thermodynamical potential and a signal about breaking it will be received if one finds at least a part of terms which are not proportional to the truncated Bernoulli polynomials $\tilde{B}_2(z)$ and $\tilde{B}_4(z)$. Here $\tilde{B}_{2n} = B_{2n}(z) - B_{2n}(0)$ and these functions are periodic under substitution $|z| \to 1-|z|$. Unfortunately these g^4-terms should be calculated directly since any nonperturbative summations with using the g^2-terms keep the found degeneracy at least for a small gq. The g^3-terms obtained here display this fact although they are very important themselves when any applications of the found effective action are investigated.

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