ADIABATIC TRANSITION OF THE PUMP INTO SECOND OPTICAL HARMONIC

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Submitted 17 May 1993

Mode-type solutions with z-independent ratio $E_{2\omega}/E_\omega^2$ are found for a medium suppoting $\hbar\omega \leftrightarrow 2\hbar\omega$ interaction for arbitrary wave vector mismatch $\Delta k = k_2 - 2k_1$. Complete adiabatic energy transition from E_ω to $E_{2\omega}$ is proposed for a medium with gradually z-dependent mismatch $\Delta k(z)$.

The process of second harmonic generation is described by coupled wave equations for slowly varying envelopes:

$$\frac{\partial E_2}{\partial z} = i\Delta k \cdot E_2 + i\mu E_1^2 \frac{\partial E_1}{\partial z} = i\mu E_1^* E_2, \tag{1}$$

where $\mu = 2\pi\chi^{(2)}\omega/cn > 0$ is nonlinear coupling coefficient. It is usually assumed that most efficient energy transfer $E_\omega \to E_{2\omega}$ is achieved for zero value of wave vector mismatch $\Delta k = k_2 - 2k_1$. In particular, for $\Delta k = 0$ there is a well-known solution of (1):

$$E_1 = E_0/\cosh(\mu|E_0|z), \qquad E_2 = i\frac{E_0^2}{|E_0|}\tanh(\mu|E_0|z), \tag{2}$$

which corresponds to zero amplitude of second harmonic in the input, $E_{2\omega}(z=0) = 0$. For $z \to \infty$ that solution describes asymptotically complete energy transfer $E_\omega \to E_{2\omega}$. However, the resulting second harmonic field $E_{2\omega}$ in a medium with $\Delta k = 0$ is unstable relative to the parametric decay down to the waves E_ω.

The suggestion of this paper is to use a medium with gradually changing $\Delta k(z)$ in such a way that almost pure second harmonic $E_{2\omega}$ appears only in the region where large $|\Delta k|$ ($|\Delta k| \gg 2\mu|E_0|$) makes the wave $E_{2\omega}$ to be stable.

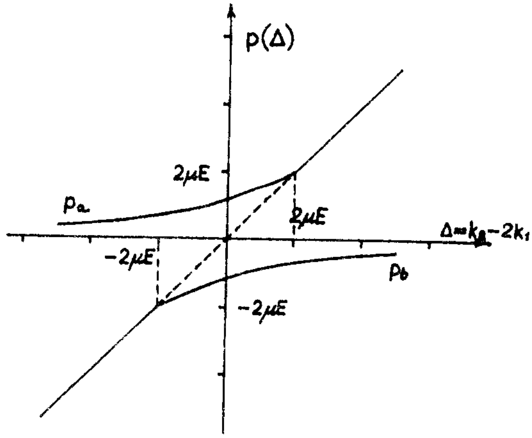
With this aim in the mind, we consider first the mode-type solutions of Eqs.(1) for a medium with constant value of Δk. (Mode-type solutions for even more general case of 3-wave interaction $\hbar\omega_1 + \hbar\omega_2 = \hbar\omega_3$, but for zero phase mismatch $\Delta k = 0$, were considered recently by A. E. Kaplan [1]). By the term "mode" we mean the solution with a dynamic balance of energy transfer $E_\omega \to E_{2\omega}$ and back $E_{2\omega} \to E_\omega$, i. e. a solution of the type

$$E_1(z) = E_0 e^{i\frac{p}{2}z}\cos\psi, \qquad E_2(z) = \frac{E_0^2}{|E_0|}e^{ipz}\sin\psi, \tag{3}$$

where $|E_0|^2$ is the conserved value of total energy flux.

There is one "trivial" mode of Eqs.(1),

$$E_1 \equiv 0, \qquad E_2 = |E_0|\, e^{2i\alpha + ipz}, \qquad p \equiv \Delta k, \qquad \alpha = \text{const}, \tag{4}$$



Dependence of eigenvalues p on the wave vector mismatch $\Delta = k_2 - 2k_1$ for different modes: "a", "b" and trivial mode $p = \Delta$. Dashed line notes the instability region of the trivial mode

corresponding to pure second harmonic light. That mode is unstable relative to the parametric decay in the interval of wave vector mismatch $|\Delta k| < 2\mu |E_0|$ and is stable outside this interval, see Fig.

Beside that, for $-\infty < \Delta k < 2\mu |E_0|$ there is a mode "a" with

$$\cos\psi_a = \sqrt{1 - \frac{p_a^2}{(2\mu |E_0|)^2}}, \qquad \sin\psi_a = \frac{p_a}{2\mu |E_0|}$$

$$p_a = \frac{\Delta k}{3} + \sqrt{\left(\frac{\Delta k}{3}\right)^2 + \frac{4}{3}(\mu |E_0|)^2} > 0 \tag{5}$$

In this mode E_ω and $E_{2\omega}$ are mixed with such phases, that time-averaged value of the cube of real optical field is positive, $\langle E^3_{real}(z,t)\rangle = \frac{3}{4} |E_0|^3 \cos^2\psi_a \sin\psi_a > 0$, see [2]. For $\Delta k \to -\infty$ the "a"-mode consists predominantly of the pump E_ω, with small admixture of second harmonic, $E_{2\omega} \approx \mu E_\omega^2/\Delta k$. For $\Delta k \to 2\mu |E_0| - 0$ the "a"-mode passes into almost pure second harmonic $E_{2\omega}$, with small admixture of fundamental frequency wave E_ω. So, we may consider trivial $E_{2\omega}$-mode for $\Delta k > 2\mu |E_0|$ as the direct continuation of "a"-mode.

In an analogous way, for $-2\mu |E_0| < \Delta k < +\infty$ there is a "b"-mode with

$$\cos\psi_b = \sqrt{1 - \frac{p_b^2}{(2\mu |E_0|)^2}} \qquad \sin\psi_b = \frac{p_b}{2\mu |E_0|},$$

$$p_b = \frac{\Delta k}{3} - \sqrt{\left(\frac{\Delta k}{3}\right)^2 + \frac{4}{3}(\mu |E_0|)^2} < 0, \tag{6}$$

and for this mode $\langle E^3_{real}(z,t)\rangle = \frac{3}{4} |E_0|^3 \cos^2\psi_b \sin\psi_b < 0$. There is a continuous transition of this mode into the almost pure pump for $\Delta k \to +\infty$ and into the "trivial" $E_{2\omega}$-mode for $\Delta k < -2\mu |E_0|$. For both modes ("a" and "b") the absence of energy exchange between fundamental frequency wave and second harmonic is quite in coincidence with zero or 180° phase shift between $E_{2\omega}$ and E_ω^2.

The main idea of this letter is to take the medium with slowly varying $\Delta k(z)$. If the pump E_ω enters the medium at the region with $\Delta k(z=0) \ll -2\mu \mid E_0 \mid$, then almost pure "a"-mode is excited. The hypothesis (which we hope to confirm by numerical simulation) is that slow change of $\Delta k(z)$ from $\Delta k \ll -2\mu \mid E_0 \mid$ up to $\Delta k \gg 2\mu \mid E_0 \mid$ adiabatically keeps the system in "a"-mode and hence results in almost 100% transfer of energy of the pump E_ω to second harmonic $E_{2\omega}$. Usual adiabaticity condition takes the form:

$$|d\Delta k/dz| \ll (\mu |E_0|)^2. \tag{7}$$

This process is somewhat analogous to adiabatic following of polarization in smoothly inhomogeneous nematic liquid crystals, see e.g. [3-5], adiabatic passage in magnetic and optical resonance [6], Landau–Zener picture of predissociation of two-atomic molecules [7], etc. However, in our case the equations are essentially nonlinear, as opposed to above mentioned problems. The modes here are not orthogonal in any reasonable sense and the principle of superposition is not valid. The number of modes (three) is even larger than the number of degrees of freedom (two). Therefore the results obtained for the above mentioned adiabatic proccesses can not be applied to our problem, and numerical simulation is a necessity. However, the existence of large splitting of eigenvalues,

$$p_a - p_b = 2\sqrt{\frac{(\Delta k)^2}{9} + \frac{4}{3}(\mu |E_0|)^2} \tag{8}$$

makes the hypothesis of adiabatic passage in condition (7) to look reasonable.

There are several possibilities of physical realization of a medium with $d\Delta k/dz$. One of them is the gradient of chemical compound or temperature along the crystal. Another one is connected with the artifitial phase-matching schemes using periodic domain structures, so that $\Delta k(z) = k_2 - 2k_1 - q(z)$, where $q = 2\pi/\Lambda$, and Λ is the period of domain's grating. In the latter case one should take the grating with slowly varying period $\Lambda(z)$.

Author is grateful to B. Ya. Zel'dovich, E. Van Stryland, G. Stegeman for the discussions and to A. E. Kaplan for the information about his work [1] prior to the publication.

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