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HALL TUNNELING OF VORTICES IN HIGH TEMPERATURE SUPERCONDUCTORS

*M.V.Feigel'man, V.B.Geshkenbein, A.I.Larkin, S.Levit**

Landau Institute for Theoretical Physics, Moscow, 117940, Russia
Weizmann Institute of Science, Rehovot, 76100, Israel

* *Weizmann Institute of Science, Rehovot, 76100, Israel*

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Quantum tunnelling of vortices in very clean type-II superconductors is shown to be governed by the Hall term in the equation-of-motion. An effective action determining the tunnelling probability is calculated. It is argued that high-temperature superconductors may well belong to a class of very clean materials and posseses the Hall tunnelling at low temperatures.

At very low temperatures, large, temperature independent magnetic relaxation have been observed in a chevrel phase [1], heavy fermion [2], organic [3] and high temperature [4,3] superconductors, suggesting the existence of vortex motion by quantum tunneling. The tunneling rates for single vortices and for vortex bundles have been determined within the framework of weak collective pinning theory [5,6]. This theory is in a reasonable agreement with experimental data [3] and can explain why quantum creep was observed in these "exotic" compounds whereas "more conventional" superconductors do not show such behaviour. The important difference between quantum tunneling and classical thermal activation is the time component of the motion. For thermal activation the time is irrelevant. The probability of the process with exponential accuracy is given by the height of the barrier between the metastable states and the dynamical properties of the vortices influence only the preexponential factor. On the contrary, during tunneling, the vortex moves under the barrier, the process is virtual and the longer the time which the vortex has to spend under the barrier, the less chances are for this process to happen. That's why it is important to understand what are the dynamical equations describing the vortex motion. The simplest possible dynamics is the massive dynamics for which the question of quantum decay of a metastable state is well studied. That is probably the reason why the problem of the vortex mass was raised from the very beginning of quantum creep study [1,5-7]. The known expressions for the vortex mass were obtained either from different kinds of timedependent Ginzburg-Landau theories [8] (which definitely are not applicable at low temperature where quantum tunneling was observed) or from various qualitative estimates [5-7]. There is no microscopic derivation of the vortex mass. From microscopic theory of superconductivity it is possible to deduce that the vortex motion should be dissipative [9] for not very clean materials. In the quantum collective creep theory the estimates for the vortex mass were mainly given in order to show that the massive term is not important and the vortex tunneling is dissipative.

In this case the Euclidean action S_E^{eff} which determines the relaxation of the magnetization M: $\partial \ln M / \partial \ln t \approx -\hbar / S_E^{eff}$ is given by

$$\frac{S_E^{eff}}{\hbar} \simeq \frac{\hbar}{e^2} \frac{L_c}{\rho_n}. \tag{1}$$

Here ρ_n is the normal state resistivity extrapolated to zero temperature and L_c the collective pinning length which can be expressed through the coherence length ξ, depairing and critical current densities j_0 and j_c, $L_c \simeq \xi (j_0/j_c)^{1/2}$.

However there should be no dissipation for very clean superconductors at very low temperatures. In this limit the flux motion should be similiar to the motion of the vortices in superfluid Helium, where vortices are dragged by the superfluid flow. The criteria distinguishing dissipative versus nondissipative vortex motion was given by Kopnin and Kravtsov [10]. Starting from microscopic theory of superconductivity they derived the following equation of motion in flux flow regime:

$$\eta \mathbf{v}_L + \alpha [\mathbf{v}_L \times \mathbf{n}] = \frac{\Phi_0}{c} [\mathbf{j} \times \mathbf{n}]. \tag{2}$$

Here j is the transport current density, $\mathbf{v}_L$ is the velocity of the vortex line, $\mathbf{n}$ is the unit vector along the vortex, η and α are viscous and Hall drag coefficients respectively. This coefficients are determined by the interaction of normal excitation existing in the bound states [11] at the vortex core with impurities and depend strongly on the parameter $\omega_0 \tau$, where τ is transport time and ω_0 is the spacing between the low lying levels in the vortex core, $\omega_0 \simeq \Delta^2/\varepsilon_F$. Viscous flux flow ($\alpha \ll \eta$) with Bardeen–Stephen's expression for $\eta \simeq \frac{\Phi_0 H_{c_2}}{\rho_n c^2} \simeq \pi \hbar n \omega_0 \tau$ corresponds to $\omega_0 \tau \ll 1$, whereas the opposite limiting case $\omega_0 \tau \gg 1$ corresponds to nondissipative flow like in Helium II with $\alpha = \pi \hbar n_s$, where n and n_s are electron density and density of superconducting electrons respectively, $\mathbf{j}_s = e n_s \mathbf{v}_s$.

The condition for the nondissipative flow $\omega_0 \tau \gg 1$, expressed through the mean free path $l \gg \xi \varepsilon_F / \Delta$ is much stronger than the usual condition for clean limit $l \gg \xi$ and is practically never realized for usual superconductors, where $\varepsilon_F/\Delta \sim 10^3$. However high T_c superconductors have much larger value of Δ with $\varepsilon_F/\Delta \sim 10$. An estimate for ω_0 gives $\omega_0 \sim 10$ K and $\xi \varepsilon_F / \Delta \sim 10 - 20\,nm$. Extrapolation of the normal state resistivity to zero temperature gives mean free path $l \sim 70\,nm$. So it is very likely that HTSC are in a "superclean" limit $l \gg \xi \varepsilon_F / \Delta$.

Note that this simple estimates result in a very strong conclusion that Hall angle in the flux flow regime in HTSC $\Theta_H = \arctan(\alpha/\eta) \approx \pi/2$ which was not confirmed experimentally yet. However the measurements of the Hall resistivity which we are aware of are performed at high temperature. Due to flux pinning the Hall angle goes to zero by lowering temperature [12]. So it is very important to perform the measurements of the Hall effect in the flux flow regime at very low temperature. A usefull tool for that can be AC measurements at very high frequencies, where pinning effects are unimportant.

Let us consider quantum tunneling in nondissipative case. For the sake of simplicity we start with 2D case where vortices are point like objects. The generalization to 3D case is trivial and will be given later. The classical equation of motion is now

$$\alpha [\mathbf{v} \times \mathbf{n}] = -\nabla U(\mathbf{r}), \tag{3}$$

where $\alpha = \pi \hbar n_s^{(2)}$ (with $n_s^{(2)}$ being a superfluid density per one superconductive layer) and $U(\mathbf{r})$ is a pinning potential. This is the same equation which describes the motion of charge with zero mass in magnetic field B (for this case α would be equal to eB/c). Writing (3) in components

$$\alpha\frac{dx}{dt}=\frac{\partial U(x,y)}{\partial y}\quad;\quad\alpha\frac{dy}{dt}=-\frac{\partial U(x,y)}{\partial x},\tag{4}$$

we immediately see that the system (4) is of Hamiltonian form. Rescaling $\sqrt{\alpha}x=q$, $\sqrt{\alpha}y=p$ our potential energy U transforms to the Hamiltonian $U(x,y)=H(q,p)$. Action which produces Eqs.(4) has a form

$$S=\int(\alpha\dot{x}y-U(x,y))dt.\tag{5}$$

The first term is the same as $\frac{e}{c}\mathbf{v}\mathbf{A}$ term for a particle in magnetic field with gauge $A_x=By$. Now coordinates x and y are canonical variables with commutator

$$[x,y]=\frac{i\hbar}{\alpha}=\frac{i}{\pi n_s^{(2)}}.\tag{6}$$

The same approach was used in Ref.[13] for calculation of the probability of quantum creation of vortices in a superfluid Helium, and in Ref. [14] for studying of tunneling in a high magnetic field. From incertainty relation one can see that quantum fluctuations of the vortex position are of the order of the average distance between the elecrons $\langle\Delta r^2\rangle_Q\sim 1/n_s^{(2)}$.

Now the problem is reduced to one-dimensional quantum mechanics with Hamiltonian $U(x,y)$. Note, however, that our Hamiltonian $U(x,y)$ is in general symmetric and random function of canonical "coordinate" x and "momentum" y. It gives some beauty to the problem, but doesn't help to solve it. It is then worthwhile to consider some model potential for which the problem can be solved exactly:

$$U(x,y)=U_0(\frac{y^2}{r_p^2}+\frac{x^2}{r_p^2}-\frac{x^3}{r_p^3}).\tag{7}$$

This is not an artificial potential. If current $j\parallel y$ is applied (so the Lorentz force is directed along x), then near criticality the generic form of the energy potential is given by Eq.(7). After the transformation to the canonical variables ($y\rightarrow p$, $x\rightarrow q$) the Hamiltonian (7) describes the one-dimentional motion of a particle with mass $m=\frac{r_p^2}{2U_0}$ in the potential $U^{(1)}(x)=U(x,y=0)$. In the quasiclassical approximation (i.e. when $r_p\gg 1/\sqrt{n_s}$) the tunnelling probability $W\propto\exp(-S^{eff}/\hbar)$ can be calculated straightforwardly:

$$\frac{S^{eff}}{\hbar}=\frac{8}{15}\frac{r_p^2\alpha}{\hbar}=\frac{8}{15}r_p^2n_s^{(2)}.\tag{8}$$

One can notice that the result for the effective action (8) is independent on the strength of the pinning potential U_0. In 1D quantum mechanics the action determining tunneling probability is equal to $2\int p\cdot dq$ taken along the classically forbidden path. In our problem it corresponds to

$$S^{eff}=\alpha\oint ydx$$

(cf. Eq.(5)) which is nothing but α times the area spanned by the vortex trajectory (note that, contrary to the usual massive or viscous tunneling, vortex trajectory in our case has no turning point - it is not "bounce" but the orbit with non-zero area). Thus, just due to the "geometrical" quantization (canonical variables are just coordinates) the effective action depends upon the length scale of the potential and not upon its magnitude.

Generalization of this result for the 3-dimentional case is simple: in this case

$$\frac{S^{eff}}{\hbar} = \frac{\alpha}{\hbar} V = \pi n_s V, \tag{9}$$

where V is the volume spanned by the trajectory of the vortex line or vortex bundle. In the collective pinning picture r_p is of the order of the vortex core size ξ, whereas the length of the tunnelling vortex segment is L_c (in the single vortex pinning limit). Hence the result for the tunnelling exponent is

$$\frac{S^{eff}}{\hbar} \sim n_s^{(2)} \xi^2 \quad (\text{at} \quad D = 2); \quad \sim n_s \xi^2 L_c \quad (\text{at} \quad D = 3). \tag{10}$$

For usual BCS-type superconductors at low temperatures in clean limit n_s coincides with the carrier density n. Results (10) give the order of magnitude estimate for the tunnelling exponent $\sim 10^2$ in agreement with the experimental data [3,4]. In the 2D case tunnelling exponent is independent on the strength of pinning potential, in agreement with the results of Ref. [15], where quantum creep in $Bi_2Sr_2CaCu_2O_8$ with columnar defects was studied, and quantum relaxation rate was shown to be only weakly affected by columnar defect whereas the critical current substantially increased. The reason for such a behaviour is that columnar defect in Bi-compound (where pinning, especially at low T, is of 2D nature) has radius close to ξ, so the space scale of the potential was not modified with respect to unirradiated sample.

This experiment, however, cannot be considered as an evidence against dissipative vortex tunnelling [5]: for a 2D dissipative tunnelling we can use Eq.(1) with L_c replaced by d and the result will again be independent on pinning strength. Actually the results (10) can be obtained qualitatively with the same kind of dimensional estimates as it was done in [5], just by substitution of the viscosity η by the Hall coefficient α in formulas for the dissipative tunnelling. However such a simple transformation works for one-parameter estimates like (10) only, whereas the dependences of the tunnelling rate on temperature, external current, *etc* are quite different in dissipative and Hall cases. In particular, low-temperature corrections to the result (10) are exponentially small ($\propto \exp(-T_0/T)$), whereas in the dissipative case they are $\propto T^2$. Also when we study tunneling near the critical force then the length scale along x and along y in Eq.(7) are different. Due to the smearing of potential near criticality there will be additional factor $(\frac{j_c-j}{j_c})^{1/2}$ before x^2/r_p^2 term in (7) and the hopping distance $x_{hop} \sim r_p(\frac{j_c-j}{j_c})^{1/2}$ goes to zero. In this case for tunneling with dissipation $S_d^{eff} \propto (j_c - j)$ whereas for Hall tunneling result will be similiar to the massive one and $S_H^{eff} \propto (j_c - j)^{5/4}$ (see [6]).

It means that dissipation becomes more important near criticality. The approach developed above can be generalized for the case when both dissipation and Hall motion are important. The Euclidean action can be written in the form

$$S_E = \int dt \left[\int dt' \frac{\eta}{4\pi} \frac{(x(t)-x(t'))^2 + (y(t)-y(t'))^2}{(t-t')^2} + i\alpha \dot{x} y + U(x,y) \right] \qquad (11)$$

The first term in the above expression is the Caldeira–Leggett term [16] accounting for dissipation; in the case when the potential $U(x,y) = U_0 y^2/r_p^2 + U^{(1)}(x)$ the change of variable $y \to -iy$ transforms two last terms of Eq.(11) to the form corresponding to the classical motion of the particle in a magnetic field and in the "inverted" potential $-U^1(x)$; however, such a simple analogy is not valid for a general form of the potential $U(x,y)$.

Going into the Fourier representation in (11) and performing Gaussian integration over y variable we obtain

$$S_E = r_p^2 \int \frac{d\omega}{2\pi} \left\{ \left[\frac{\eta \mid \omega \mid}{2} + \frac{\alpha^2 \omega^2}{2 + \eta\omega} \right] x_\omega^2 + x_\omega^2 - (x^3)_\omega \right\}. \qquad (12)$$

Here we rescaled $x, y \to x r_p, y r_p$. Now the problem is reduced to a one-dimensional motion in a cubic potential with a dispersive kinetic term. At $\alpha \gg \eta$ we are back with the usual one-dimentional tunnelling problem, whereas in the opposite limit dissipative tunnelling takes place. Characteristic scale of the displacement in y direction is given by $(1+\eta/\alpha)^{-1}$. Minimization of the action (12) is complicated by the presence of the non-local in ω cubic term, and the resulting integral equation can be solved only numerically. However one can see that the general evolution of the optimal trajectory is from circular orbit at $\eta = 0$ via its narrowing in the y direction with η increasing until its collapse to a straight line along x axis in the limit $\alpha \ll \eta$. At small η relative correction to a purely non-dissipative result are of the order η/α, whereas in the opposite limit "Hall" corrections to a dissipative result are $\propto (\alpha/\eta)^2$. This last result is due to the fact that the tunnelling rate should be independent on the α sign and analitic at small α. A very interesting approach in dealing with Hall effect in superconductors was proposed by Ao and Thouless [17] (see also [18] where similiar approach was used for vortices in helium films). They have shown that the Magnus force acting on a vortex line can be obtained by calculating of the Berry phase for an adiabatic motion of the vortex along the closed loop. In this approach the formula (9) can be interpreted as the Berry phase acquired by the vortex during tunneling and equal to π times the number of superconductive electrons inside the volume spanned by the trajectory of the vortex line. Note however that the microscopic derivation of the Hall coefficient α is still a subject of controversy. As it was found in [10] (see also [19]) the Magnus force is almost completely canceled at relatively weak disorder (i.e. in a usual "clean" though not "superclean" regime). The problem of the topological Berry phase dependence on the strength of disorder is very important and clearly needs further investigation. In this paper we tried to touch more phenomenological level of the problem, starting from a given value of the Hall constant α.

In conclusion, the "vortex mass" problem is not relevant for quantum tunnelling of vortices; depending on the disorder strength the tunnelling can be of the dissipative or Hall nature. It is likely that HTSC belong to the class of very clean materials and Hall tunnelling does realized at low temperatures. In this case the estimates for the tunnelling rate are given by Eqs.(9), (10) in a reasonable

Note added: when this work was completed we became aware that the problem consi dered was also studied recently by D.Thouless with coworkers [20]. We are grateful to D.Thouless for the discussion followed.

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