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THREE-PARTICLE ANYON EXCITONS

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We propose a model of an anyon exciton (AE) consisting of a hole and several anyons [1], and apply it to the spectroscopy of an incompressible quantum liquid (IQL). Fractionalization of the electron charge makes properties of such entities quite different from those of usual magnetoexcitons. The model describes a number of properties established by few-particle simulations, including an abrupt change in emission spectra *vs* electron-hole asymmetry of the system.

Experimental findings [2] in the optical spectroscopy of an IQL [3], underlying the Fractional Quantum Hall Effect (FQHE) [4], stimulated recent theoretical activity in this field [5-8]. It is one of the most remarkable properties of IQL's that their elementary excitations carry fractional charge [3], and are anyons [9,10], i.e., obey fractional statistics [11,12]. The theory based on simulations [5,8] predicts that fractional charges should manifest themselves by abrupt changes in the electronic density distribution for an exciton existing against the background of an IQL. These changes set in when the charge asymmetry of electron-electron and electron-hole interactions is changed, and are accompanied with dramatic changes in the position and the intensity of the emission band *vs* the charge asymmetry parameter [5c,8]. The ratio h/l, where h is a distance between electron and hole confinement planes, and $l=(c\hbar/eH)^{1/2}$ is the magnetic length, may be chosen as such a parameter. Numerical simulations for few-electron systems are accessible only for small values of $h/l<2$. For the opposite limit case of strongly asymmetric systems, $h/l \gg 1$, anyon concept seems to be most promising. An exciton, appearing against a background of an electron IQL, is a neutral entity consisting of a valence hole and several anyons. E.g., if the filling factor $\nu=1/3$, the charge of anyons $e^*=-e/3$, their statistical charge $\alpha=-1/3$ (for comparison, $\alpha=0$ for bosons, and $\alpha=1$ for fermions) [9], and the number of anyons $N=3$. If $h \gg l$, the mean separation between anyons in an exciton is about h, which is larger than the anyon size, l. Therefore, anyons are well defined particles, anyon-anyon and anyon-hole interactions follow a Coulomb law, and the Coulomb energy is small as compared to the IQL gap width. When $h<l$, only qualitative results may be expected from the AE model. Nevertheless, we show that they are quite encouraging. Recently there has been a significant activity in the hierarchy theory of the FQHE [13,14], and some experimental data have been discussed in these terms [15]. While the hierarchies provide a level classification for free anyons, AEs may also give insight into the effect of an external (Coulomb) field and the treatment of optical data.

We consider a model of an AE consisting of a hole and two semions, anyons with $e^* = -e/2$, $\alpha = -1/2$. If the hard-core constraint [12,13] is imposed, one should chose $\alpha = 3/2$. For spin polarized IQLs $e^* = -e/q$, where q is odd. Our two anyon model is the simplest one giving insight into the properties of the more realistic models with $q \geq 3$. We assume that the magnetic field is strong enough, i.e., the Coulomb energy $\varepsilon_C = e^2/\epsilon l \ll \hbar\omega_c$, ω_c is the cyclotron frequency, and use dimensionalless variables scaled in units of ε_C and l. Such an AE, consisting of three particles, is described by three quantum numbers. Since it is a neutral entity, a two component momentum **K** may be ascribed to it. Therefore, the internal motion in it is characterized by a single quantum number, and the charge fractionalization (CF) should result in a single-parameter array of energy bands, instead of the single band of an usual magnetoexciton.

The preexponential factor in the wave function of a positively (negatively) charged particle is a polynomial in complex coordinate z $(\bar{z})$, $z = x + iy$. We introduce for anyons the Jacobi coordinates $z = z_1 - z_2$ and $z_0 = (z_1 + z_2)/2$, and relate z_0 to a hole coordinate, z_3, by the usual exciton transformation. The new coordinates are z, $\zeta = z_0 - z_3$, and $Z = (z_0 + z_3)/2$, and Halperin pseudo wave functions [9] of the three non-interacting particles with a momentum **K** are:

$$\Psi_{\mathbf{K}n}(z_1, z_2, z_3|\alpha) = B_n(\alpha)\exp\{i\mathbf{KR} + i(\rho_x Y - \rho_y X)/2\}\times$$

$$\times \exp\{-(1/4)(\vec{\rho} - \vec{\kappa})^2\}|z|^\alpha \bar{z}^n \exp(-|z|^2/16).$$

Vectors **R**, $\vec{\rho}$ and **r** correspond to complex coordinates Z, ζ and z; $\vec{\kappa} = \hat{z} \times \mathbf{K}$, $\hat{z}$ is a unit vector in the direction perpendicular to the confinement plane, $-\vec{\kappa}$ is the dipole moment of the exciton, and $B_n(\alpha)$ is a normalization factor. The quantum number $n \geq 0$ is the relative angular momentum of anyons. The Coulomb interaction may be written as $V = V_{aa} + V_{ah}$. The first term is diagonal in n:

$$< n_1|V_{aa}|n_2 >= \delta_{n_1 n_2}\frac{\Gamma(n_1 + \alpha + 1/2)}{8\sqrt{2}\Gamma(n_1 + \alpha + 1)}. \qquad (1)$$

When $m = |n_1 - n_2| =$ odd, all $< n_1|V_{ah}|n_2 >= 0$, which ensures the correct interchange symmetry; only even n have a physical meaning. For even m:

$$< n_1|V_{ah}|n_2 >=$$

$$= -\{\Gamma(\frac{n_1 + n_2 + m}{2} + \alpha + 1)/2^{m/2}m![\Gamma(n_1 + \alpha + 1)\Gamma(n_2 + \alpha + 1)]^{1/2}\}\times$$

$$\times \int_0^\infty q^m \exp(-q^2/2 - qh)J_m(Kq)\times$$

$$\times\Phi(\frac{n_1 + n_2 + m}{2} + \alpha + 1, m + 1; -q^2/2)dq, \qquad (2)$$

where Φ is the confluent hypergeometric function. For $\alpha = -1/2$ the integral $< n|V_{aa}|n >$ diverges logarithmically for $n = 0$. This is the price for using the oversimplified model, $N = 2$. We use a cut-off $V_{aa} = 1/4(r^2 + a^2)^{1/2}$. For the hard core model, $\alpha = 3/2$, all integrals are regular.

For $K = 0$ only diagonal matrix elements survive, and eqs.(1) and (2) give the exact solution of the three particle problem.

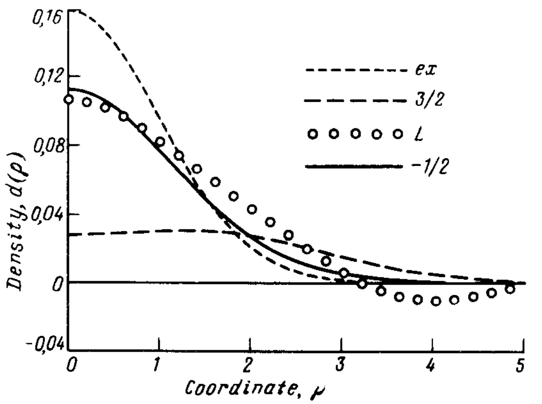

Fig.1. Electron density, $d(\rho)$, for the ground state of a free magnetoexciton (ex), an exciton in the presence of the IQL with $\nu = 1/3$ (L), and for anyon excitons with statistical charges of anyons $\alpha = -1/2$, and 3/2; $K = 0, h = 0$. The last curve also describes the first excited state for $\alpha = -1/2$

Fig.1 shows the effect of statistics on the distribution of electron density, $d(\rho)$, around a hole in an AE for $K = n = 0$. The functions $d(\rho)$ for a free exciton, $d_{ex}(\rho) = \exp(-r^2/2)/2\pi$, and an exciton in the presence of an IQL ($\nu = 1/3, h = 0$), are also shown. In the last case the excess density, $d_L(\rho)$, is plotted [8]. The most striking property of d_L is a considerable increase in the spread of the density as compared to d_{ex}, which is caused by the Pauli exclusion principle. This property is reproduced by the anyon model, primary because of the increase in the magnetic length, $l^* > l$. The curve $d_{-1/2}(\rho)$ is even in reasonable quantitative agreement with d_L; a realistic comparison may be done only for odd q. The first excited state of an $\alpha = -1/2$ exciton coincides with the ground state of an $\alpha = 3/2$ exciton. The latter curve, having a flat minimum at $\rho = 0$, highly resembles the distribution of the electronic density for an exciton against the background of an IQL, $\nu = 1/3$ (Fig.2, curve 3 in Ref. [8]). This similarity in $d(\rho)$ for the two lowest states found both by simulations and for AE strongly suggests that when h is small, the hard-core constraint for quasiparticles [12,13] is violated in an exciton by the attractive field of a hole.

There are several distinctive features of AEs caused by CF. If $K = 0$, and h increases, Ψ_{Kn} remain exact eigenfunctions, but the level arrangement is changed; the larger is h, the higher is the value of n for the ground level, and the wider is the density distribution, $d(\rho)$, for it. The $n = 0$ and $n = 2$ levels interchange at $h_{cr} \approx 1.66$ (for $a = 1$), at this point $d(\rho)$ for the ground state changes abruptly from $\alpha = -1/2$ for $\alpha = 3/2$ curve, Fig.1. Since the K-dependence of diagonal matrix elements, $< n|V|n >$, is the stronger the less is n, energy levels draw together at different values of K. At $K = 0$ non-diagonal elements of V vanish, and the level rearrangement shows the patterns of the level crossing, while at $K \neq 0$ of the level anticrossing. This behaviour is seen in Fig.2 where dispersion laws, $\varepsilon(K)$, and densities, $d(0)$, at $\rho = 0$, are shown. While $\varepsilon(K)$ are always monotonic for the ground state, and changes smoothly for excited states, $d(0)$ show dramatic changed near level anticrossings. The humps seen in Fig.2c reflect a transfer of $n = 0$ component between wave functions of adjacent states. Dispersion is strongly suppressed as compared to an usual exciton, Fig.2a, since in the $K \to \infty$ limit only one anyon moves away from the hole, while another remains in a bound state

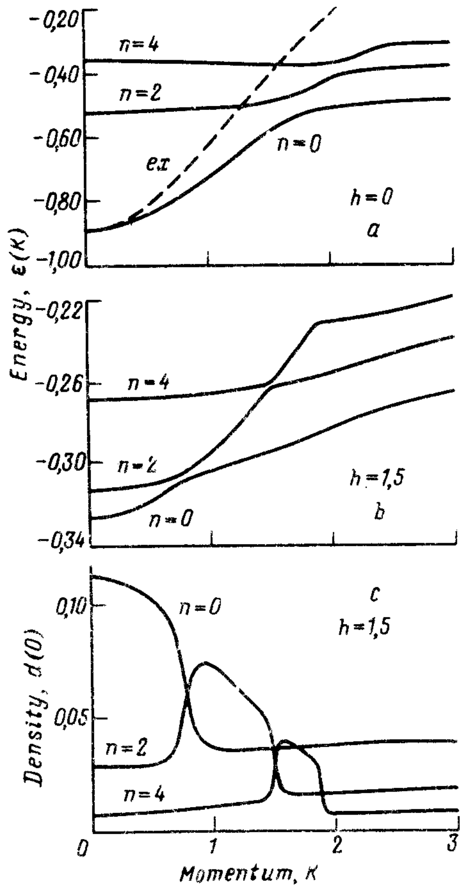

Рис.2

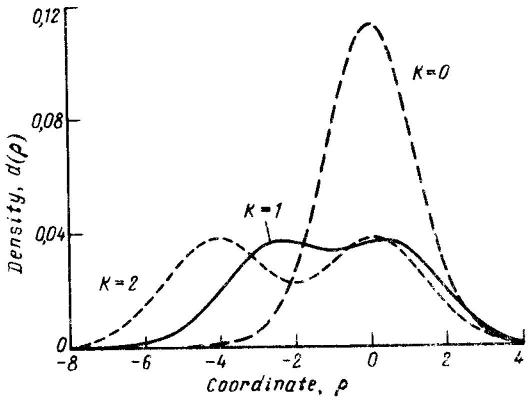

Рис.3

Fig.2. Dispersion law, $\epsilon(K)$, for the three lowest spectrum branches; $\alpha = -1/2$. When h increases, anticrossings become narrower, and $\epsilon(K)$ for the ground state flatter. The curve for a free exciton (ex) is shifted to facilitate a comparison with $n = 0$ curve

Fig.3. Distribution of the electron density in the ground state of an anyon exciton along the direction $\vec{\rho} \perp \mathbf{K}$; $h = 1.5$

and makes with the hole an ion (the existence of anyon ions was established in Ref. [5c] by supported by more recent calculations [16]). If $h > h_{cr}$, dispersion in the ground state is even more suppressed. Both the abrupt change in the ground state with increasing h, and the suppression of the dispersion are in conformity with the patterns found by simulations [5c,8].

The level intersections at $K = 0$ have important implications for optical transitions. The matrix elements for them are:

$$M_n(\alpha) \propto \int \Psi_{0n}(z_1, z_2, z_3|\alpha)\delta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)d\mathbf{r}_1 d\mathbf{r}_2,$$

here $\delta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ is a δ-shape function of r_{13} and r_{23} having a width about 1, in units of l. After the angular integration over $\mathbf{r}$, only the $M_n(\alpha)$ with $n = 0$ survive. Therefore, exciton transitions are allowed in the emission at $T = 0$, i.e., from the ground state, only if $h < h_{cr}$. This result is in agreement with numerical data [5c,8] which show that only weak transitions assisted by magnetorotons (MR)

[17] are allowed at $h > h_{cr}$.

The effect of the CF becomes even more spectacular when the ground state density, $d(\vec{\rho})$, is plotted for $K \neq 0$. In Fig.3 $d(\vec{\rho})$ is shown along the symmetry line, $\vec{\rho} \parallel \vec{\kappa}$; it is symmetric with respect to $\rho = -K$ for all K. When K increases, a single humped distribution changes into a camel-back type. The right hand part, centered near $\rho = 0$, corresponds to the ground state of the ion, and the left hand part to a free anyon. When $K \to \infty$, the separation between maxima approaches $K/|e^*| = 2K$, and the electron density distribution in both wings approaches $d(\rho) = \exp(-\rho^2/4)/8\pi$, which describes the shape of both a free anyon and ion. In the same limit the energy equals $\varepsilon(K) \approx -\sqrt{\pi}/4 - 1/8K$, the second term is the same as for MR's.

We have concentrated on the small h region, the least favorable for the AE model, since the comparison with numerical data is available only for it. All the more, the success of the model is impressive. However, there are two peculiarities of the exciton ground state found by simulations [5,8], which the simple AE model does not describe: i) at $h = 0$ the function $d(\vec{\rho})$ is of a single-hump type and only feebly depends on K, and ii) at $h = h_{cr}$ the minimum of $\varepsilon(K)$ shifts from $K = 0$ to $K_{min} \neq 0$, where K_{min} is close to the roton minimum [17]. These facts unambiguously signal the AE−MR coupling should be invoked. The importance of it is implied by Fig.3. The separation between an anyon and ion increases with K, and an AE produces a strong Coulomb field acting on the IQL, which is known to become unstable when an external charge about e^* approaches it [18]. A simple idea that the AE creates a virtual MR and makes a bound state with it describes the behaviour at $h \approx 0$ very well [8]. When h increases, the exciton dispersion curves flatten and become closer, Fig.2*b*. As a result, a pseudo-Jahn-Teller effect sets in, and a new ground state with a broken symmetry and the momentum K_{min} close to the MR minimum appears. In this state the shape of $d(\vec{\rho})$ found by simulations, Ref. [5c], is reminiscent of the curve $K = 1$, Fig.3. AE−MR coupling manifests itself also in the oscillatory behaviour of $d_L(\rho)$, Fig.1 (similarly to the screening of charged impurities [17,18]).

In conclusion, we have proposed a model of anyon excitons for the description of optical properties of IQL's. We show that the model reflects distinctive features of excitons in the presence of an IQL, and establishes properties of excitons, for which their coupling to magnetorotons is of a crucial importance.

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