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On projection (in)dependence of dual superconductor mechanism of confinement

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We study the temperature dependence of the monopole condensate in different Abelian projections of the $SU(2)$ gauge theory on the lattice. Using the Fröhlich-Marchetti monopole creation operator we show numerically that the monopole condensate depends on the choice of the Abelian projection.

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The confinement of color in QCD is one of the most interesting issues in the modern quantum field theory. Numerical simulations of non–Abelian gauge theories on the lattice show [1] that the confinement of quarks happens due to a formation of the chromoelectric strings spanned between quarks and anti-quarks. Although an analytical derivation of the color confinement is not available, the physical reason of the emergence of the string seems to be known. According to the dual superconductor model [2], the vacuum of a non–Abelian gauge model may be regarded media of condensed Abelian monopoles. The monopole condensate squeezes the chromoelectric flux (coming from the quarks) into a flux tube due to the dual Meissner effect. This flux tube is an analogue of the Abrikosov vortex in an ordinary superconductor.

The basic element of the dual superconductor is the Abelian monopole. This object does not exist on the classical level in QCD, but is can be identified with a particular configuration of the gluon fields with the help of the so–called Abelian projection [3]. The Abelian projection uses a partial gauge fixing of the SU(N) gauge symmetry up to an Abelian subgroup. In the Abelian projection the Abelian monopoles appear naturally due to compactness of the residual Abelian group.

Many numerical simulations show that the Abelian degrees of freedom in an Abelian projection are likely to be responsible for the confinement of quarks (for a review, see, e.g., Ref. [4]). For example [5], the Abelian gauge fields provide a dominant contribution to the tension of the fundamental chromoelectric string ("Abelian dominance"). Moreover, the internal structure of the string energy, such as energy profile and the field distribution are very well described by the dual superconductor model [1].

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Since qualitative features of the confinement mechanism in the real QCD with the $SU(3)$ gauge group and in the $SU(2)$ gauge theory are the same, in this Letter we restrict ourselves to the simplest case of the $SU(2)$ gluodynamics. The most convincing results supporting the dual superconductor scenario were obtained in the so called Maximal Abelian (MA) projection [6]. This gauge is defined by the maximization of the lattice functional (σ_i are the Pauli matrices),

$$\max_\Omega R_{MA}[U^\Omega],\qquad R_{MA}[U] = \sum_{s,\hat\mu} \mathrm{Tr}\Big(\sigma_3 U(s,\mu)\sigma_3 U^\dagger(s,\mu)\Big), \tag{1}$$

with respect to the gauge transformations, $U(s,\mu) \to \to U^\Omega(s,\mu) = \Omega(s)U(s,\mu)\Omega^\dagger(s+\hat\mu)$. In the continuum limit the condition (1) corresponds to the minimization of the functional $\int d^4x\,((A^1_\mu)^2 + (A^2_\mu)^2)$. A local condition of the MA gauge can be written in the form of the differential equation, $(\partial_\mu + igA^3_\mu)(A^1_\mu - iA^2_\mu) = 0$, which is invariant under the residual U(1) gauge transformations, $\Omega^{\mathrm{Abel}}(\omega) = \mathrm{diag}(e^{i\omega}, e^{-i\omega})$, where ω is an arbitrary scalar function.

The MA gauge is a good candidate for a realization of the dual superconductor scenario because the MA gauge makes the off–diagonal gluon fields as small as possible reducing their role. Thus the Abelian dominance [5] is a natural effect in the MA gauge. According to numerical simulations [7 – 9] the monopole condensate in the MA gauge is formed in the low temperature (confinement) phase and it disappears in the high temperature (deconfinement) phase in a perfect agreement with the dual superconductor scenario.

Besides the MA gauge there are also various gauges which are defined by a diagonalization of certain SU(2) functionals $X[U]$ with respect to the gauge transformations,

$$X_x[U] \to X_x[U^{(\Omega)}] \equiv \Omega^\dagger X_x[U] \Omega = \mathrm{diag}(\lambda_1, \lambda_2)\,. \quad (2)$$

To define an Abelian gauge the functional $X[U]$ must transform in an adjoint representation of the $SU(2)$ gauge group [3]. After the Abelian projection is fixed, the matrix $X[U]$ becomes diagonal and the theory possesses the (residual) $U(1)$ gauge symmetry. The most popular examples of the diagonalization gauges are the Abelian Polyakov (AP) gauge and the Abelian field strength gauge (F_{12} gauge). The Polyakov gauge corresponds to the diagonalization of the Polyakov line,

$$X_x[U] \equiv U_{\mathbf{x},x_4,0} U_{\mathbf{x},x_4+1,0} \cdots U_{\mathbf{x},x_4-1,0},$$

where $x = \{\mathbf{x}, x_4\}$. The F_{12} gauge is defined by the diagonalization of the 12-plaquette, $X_x[U] \equiv U_{x,12}$, where $U_{x,\mu\nu} = U_{x,\mu} U_{x+\hat\mu,\nu} U^\dagger_{x+\hat\nu,\mu} U^\dagger_{x,\nu}$.

There are conflicting reports on the gauge independence of the dual superconductor mechanism of the color confinement (a review of the current literature on this subject can be found in Ref. [10]). Needless to say that this question is important for understanding of the properties of the QCD vacuum. It is natural to think that the confinement – as a gauge–invariant phenomenon – can not be described by a gauge–dependent model. On the other hand the Abelian projection by itself can be considered as just a gauge–dependent tool to associate the confining gluon configurations with the Abelian monopoles. This tool may work well in one gauge and may not work in another gauge.

Analytical considerations of Ref. [10] show that in the AP gauge (contrary to the MA gauge) the dual superconductor mechanism can not be realized. It was concluded that the Abelian projection mechanism is projection dependent. However, the projection (in)dependence of the monopole condensate could not be proven within analytical approach. To check this issue we study below the condensate of the Abelian monopoles in the finite temperature $SU(2)$ gauge theory in the AP and the F_{12} projections and compare it with the condensate in the MA gauge.

We study numerically the $SU(2)$ gauge theory with the standard Wilson action, $S[U] = -\beta \sum_P \mathrm{Tr}\, U_P$, where the sum goes over the plaquettes P and β is the lattice bare gauge coupling related to the continuum gauge coupling g, $\beta = 1/4g^2$. The $SU(2)$ link field is parameterized in the standard way:

$$U_{x\mu} = \begin{pmatrix} \cos\phi_{x\mu}\, e^{i\theta_{x\mu}} & \sin\phi_{x\mu}\, e^{i\chi_{x\mu}} \\ -\sin\phi_{x\mu}\, e^{-i\chi_{x\mu}} & \cos\phi_{x\mu}\, e^{-i\theta_{x\mu}} \end{pmatrix}$$

In an Abelian projection the allowed gauge transformations have a diagonal form. Under these transformations the diagonal field θ transforms as an Abelian gauge field, the off-diagonal field χ changes as a double charged matter field, while the field ϕ remains intact:

$$\theta_{x\mu} \to \theta_{x\mu} + \omega_x - \omega_{x+\hat\mu}\,,$$
$$\chi_{x\mu} \to \chi_{x\mu} + \omega_x + \omega_{x+\hat\mu},$$
$$\phi_{x\mu} \to \phi_{x\mu}.$$

The $SU(2)$ plaquette action contains [11] various interactions between these fields as well as the action for the Abelian gauge field θ:

$$S[U] = -\sum_P \beta_P[\phi] \cos\theta_P + \ldots \quad (3)$$

Here $\theta_P = \theta_1 + \theta_2 - \theta_3 - \theta_4$ is a lattice analogue of the Abelian field strength tensor and $\beta_P[\phi]$ is an effective coupling constant dependent of the fields ϕ, Ref. [11].

Following Ref. [7] we apply the monopole creation operator of Fröhlich-Marchetti [12] to the Abelian part of the non–Abelian action (3). Effectively, this operator shifts the Abelian plaquette variable θ_P as follows:

$$\Phi_{mon}(x) =$$
$$= \exp\left\{\sum\nolimits_P \beta_P[\phi] \left[\cos(\theta_P + W_P(x)) - \cos(\theta_P)\right]\right\}, \quad (4)$$

where the tensor W_P can be written in a compact way with the help of the differential form formalism on the lattice [4]. In this formalism, δ– (d–) operator is the backward (forward) derivative on the lattice which decreases (increases) by one the rank of the form on which it is acting. The rank of the form is determined by a dimensionality of the lattice cell on which this form is defined. For example, a scalar function is a 0-form, the vector function is a 1-form *etc.* Suppose that A is a lattice vector, then δA is a scalar (a lattice analogue of the divergence, $\partial_\mu A_\mu$) while dA is an antisymmetric tensor (a lattice analogue of the curl, $\partial_{[\mu,} A_{\nu]}$). The lattice Laplacian is $\Delta = \delta d + d\delta$, and Δ^{-1} denotes the inverse Laplacian. The lattice Kronecker symbol is denoted as δ_x: it is a scalar function which is equal to unity at the site x and zero otherwise. The *-operator relates the forms on the dual and original lattice. For example, if B is a scalar function (0-form) on the original lattice then $^*B$ is a 4-form on the dual lattice.

The plaquette function W in Eq. (4) is defined as follows[2)]: $W = 2\pi\delta\Delta^{-1}(D_x - \omega_x)$. Here the lattice 1-form $^*\omega_x$ represents the Dirac string attached to the monopole on the dual lattice (thus ω_x is a 3-form on the original lattice). The form $^*\omega_x$ is zero outside the string position and it is equal to plus or minus unity (depending

2) We omit a lengthy derivation of the tensor W and refer an interested reader to Ref. [12].

on the orientation) on the string. The three dimensional form D_x is called "the Dirac cloud" because it represents a lattice analogue of the radially symmetric (Coulomb) magnetic field of a monopole. This form satisfies the equation $\delta^* D_x = {}^*\delta_x$, which is a lattice analogue of the Maxwell equation in continuum, $\mathrm{div}\mathbf{H}(x) = \rho(x)$. Here $\rho(x)$ is the density of the monopoles. In our case we introduce one monopole at the origin of the lattice therefore the lattice monopole density is just a Kronecker symbol, $\rho = \delta_x$.

A partition function of any compact U(1) model can be rewritten as a sum over closed monopole trajectories [4, 7]. The quantum average of the operator $\Phi_{mon}(x)$ can be represented as a sum over all closed monopole trajectories plus one open trajectory which begins at point x. Thus this operator does create a monopole at the point x. The operator is invariant under local gauge transformations. Another property of the operator (4) is that it is defined up to a complex phase: the operator $e^{i\alpha}\Phi_{mon}(x)$ also creates a monopole at the point x. For the sake of definiteness we study the positively defined operator (4) with $\mathrm{Im}\,\Phi_{mon} = 0$ and $Re\,\Phi_{mon} \geqslant 0$.

The object of our interest in this paper is the effective potential on the monopole field. According to Ref. [7] this potential is defined as follows:

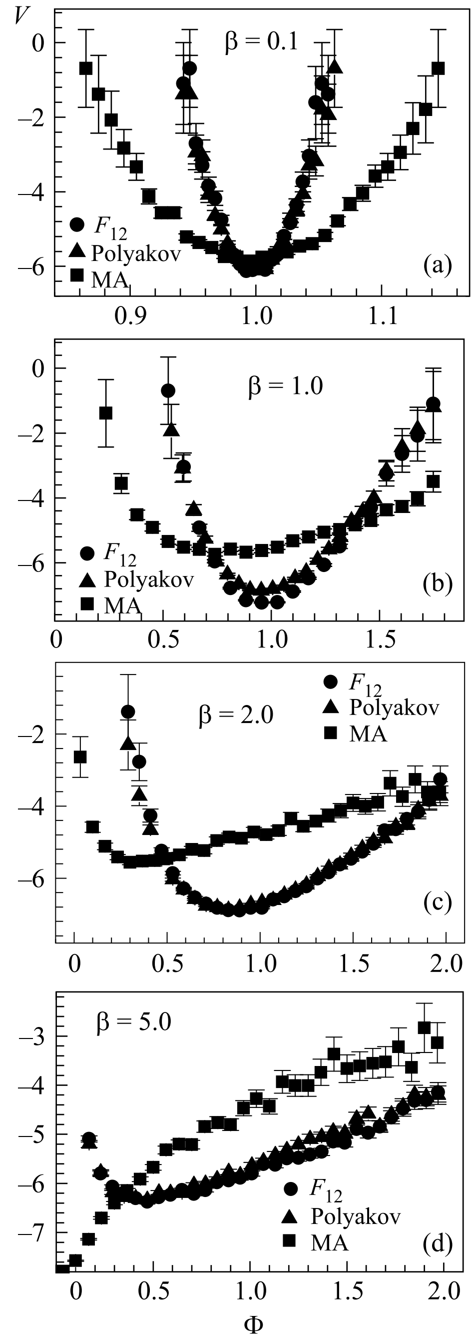
$$V(\Phi) = -\ln\langle\delta(\Phi - \Phi_{mon}(x))\rangle. \quad (5)$$

The minimum of $V(\Phi)$ corresponds to the monopole condensate.

We simulate the $SU(2)$ on the lattice $16^3 \times 4$ with C–periodic boundary conditions in space directions [13]. The C–periodicity corresponds to the anti-periodicity for the Abelian gauge fields which is required by the Gauss law [7]. In the case of $SU(2)$ gauge group the C–periodic boundary conditions are almost trivial: on the boundary we have $U_{x,\mu} \to \Omega^+ U_{x,\mu}\Omega$, $\Omega = i\sigma_2$.

To get the effective potential we use 400 configurations of the $SU(2)$ gauge fields. On each configuration the distribution of the monopole creation operator is evaluated in 20 points. The logarithm of the distribution provides us with the effective potential (5). To evaluate the errors of the potential we use the bootstrap method.

In Figures we show the effective potential $V(\Phi)$, Eq. (5), at various values of the gauge coupling β in the MA, AP and F_{12} gauges. The potential is shown for positive real values of the field Φ: $\mathrm{Re}\,\Phi \geqslant 0$, $\mathrm{Im}\,\Phi = 0$. The minimum of the effective potential correspond to the value of the monopole condensate (in lattice units). The results for the MA gauge (quoted below) are taken from Ref. [7]. The critical gauge coupling corresponding to the temperature phase transition at chosen lattice geometry is $\beta_c \approx 2.3$. Thus Figure a,b,c correspond to the confinement phase while Figure d is plotted for the deconfinement phase.



The potential on the monopole field in the MA, AP and F_{12} gauges at various values of the gauge coupling β. The data for the MA gauge are taken from [7]

First of all we note that for all considered values of β (i) the minima of the potentials in the AP and F_{12} gauges coincide with each other within numerical errors; (ii) the potential in the MA gauge is different from AP and F_{12} potentials. According to Figure a in the strong coupling limit ($\beta = 0.1$) the minima of the monopole potential in all three gauges are located at the same point, $\Phi_{\rm min} \approx 1$. However, as one can see from other Figures this coincidence is lifted as we increase β. As we increase the value of β the difference in the monopole condensates in the MA gauge and in the AP and F_{12} gauges appears evidently, Figure b,c. Moreover, in the deep deconfinement phase, Figure d the monopole condensate vanishes in the MA gauge while in the AP and F_{12} gauges the condensate is non–zero.

Summarizing, we have presented an evidence that the monopole condensate in different Abelian projections coincide with each other only in the (unphysical) strong coupling region. Generally, the condensate depends on the choice of the Abelian projection. Our results are in contradiction with conclusions of Ref. [14] where condensate was found to be projection–independent. Since our results are based on the well–justified theory of the Fröhlich–Marchetti operator we suggest that the difference between the results of our paper and Ref. [14] originates in the improper choice of the monopole creation operator in Ref. [14].

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1. V. Singh, D. A. Browne, and R. W. Haymaker, Phys. Lett. **B306**, 115 (1993); G. S. Bali, K. Schilling, and C. Schlichter, Phys. Rev. **D51**, 5165 (1995); G. S. Bali, C. Schlichter, and K. Schilling, Prog. Theor. Phys. Suppl. **131**, 645 (1998); F. V. Gubarev, E. M. Ilgenfritz, M. I. Polikarpov, and T. Suzuki, Phys. Lett. **B468**, 134 (1999); Y. Koma, M. Koma, E. M. Ilgenfritz, and T. Suzuki, hep-lat/0308008.
2. G. 't Hooft, in *High Energy Physics*, Ed. A. Zichichi, EPS International Conference, Palermo, 1975; S. Mandelstam, Phys. Rept. **23**, 245 (1976).
3. G. 't Hooft, Nucl. Phys. **B190**, 455 (1981).
4. M. N. Chernodub and M. I. Polikarpov, in *Confinement, duality, and nonperturbative aspects of QCD*, Ed. P. van Baal, Plenum Press, p. 387, hep-th/9710205.
5. T. Suzuki and I. Yotsuyanagi, Phys. Rev. **D42**, 4257 (1990); H. Shiba and T. Suzuki, Phys. Lett. **B333**, 461 (1994); G. S. Bali, V. Bornyakov, M. Müller-Preussker, and K. Schilling, Phys. Rev. **D54**, 2863 (1996).
6. A. S. Kronfeld, M. L. Laursen, G. Schierholz, and U. J. Wiese, Phys. Lett. **B198**, 516 (1987); A. S. Kronfeld, G. Schierholz, and U. J. Wiese, Nucl. Phys. **B293**, 461 (1987).
7. M. N. Chernodub, M. I. Polikarpov, and A. I. Veselov, Phys. Lett. **B399**, 267 (1997); Nucl. Phys. Proc. Suppl. **49**, 307 (1996).
8. V. A. Belavin, M. N. Chernodub, and M. I. Polikarpov, JETP Lett. **75**, 217 (2002) [Pisma ZhETF **75**, 263 (2002)]; Phys. Lett. **B554**, 146 (2003).
9. A. Di Giacomo and G. Paffuti, Phys. Rev. **D56**, 6816 (1997).
10. M. N. Chernodub, hep-lat/0308031.
11. M. N. Chernodub, M. I. Polikarpov, and A. I. Veselov, Phys. Lett. **B342**, 303 (1995).
12. J. Fröhlich and P. A. Marchetti, Commun. Math. Phys. **112** (1987) 343.
13. U. J. Wiese, Nucl. Phys. **B375**, 45 (1992).
14. A. Di Giacomo, B. Lucini, L. Montesi, and G. Paffuti, Phys. Rev. **D61**, 034503, 034504 (2000).