

Self-compression and catastrophic collapse of photon bullets in vacuum

*M. Marklund*¹⁾, *B. Eliasson*⁺, *P. K. Shukla*^{+*}

Department of Electromagnetics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

+ Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

** Department of Physics, Umeå University, SE-901 87 Umeå, Sweden*

Submitted 5 January 2004

Photon–photon scattering, due to photons interacting with virtual electron–positron pairs, is an intriguing deviation from classical electromagnetism predicted by quantum electrodynamics (QED). Apart from being of fundamental interest in itself, collisions between photons are believed to be of importance in the vicinity of magnetars, in the present generation intense lasers, and in intense laser-plasma/matter interactions; the latter recreating astrophysical conditions in the laboratory. We show that an intense photon pulse propagating through a radiation gas can self-focus, and under certain circumstances collapse. This is due to the response of the radiation background, creating a potential well in which the pulse gets trapped, giving rise to photonic solitary structures. When the radiation gas intensity has reached its peak values, the gas releases part of its energy into ‘photon wedges’, similar to Cherenkov radiation. The results should be of importance for the present generation of intense lasers and for the understanding of localized gamma ray bursts in astrophysical environments. They could furthermore test the predictions of QED, and give means to create ultra-intense photonic pulses.

PACS: 12.20.Ds, 95.30.Cq

In classical electrodynamics, as described by the Maxwell equations, photons do not interact as long as there is no material medium present. However, due to the interaction of photons with virtual electron–positron pairs, quantum electrodynamics (QED) predicts photon–photon scattering in vacuum [1]. This is commonly modeled by the Heisenberg–Euler (H-E) Lagrangian, which neglects dispersive effects. The H-E Lagrangian gives rise to cubic nonlinear corrections to Maxwell’s vacuum equations, similar to the self-interaction terms encountered in optics of Kerr media [2]. The H-E corrections give rise to both single particle effects, such as photon splitting [3, 4], lensing effects in strong magnetic fields [5], like the ones in magnetar environments [6], and to coherent field effects such as harmonic generation [7] and self-focusing of photon beams [8]. Efforts to detect these collisions are being made by using state-of-the-art super-conducting microwave facilities [9]. Recently, it has been shown theoretically that QED effects can give rise to two-dimensional collapsing photonic structures in a radiation gas [10], which could be of importance for photon propagation in stellar atmospheres and in the early Universe. Studies of photon pulses in a radiation background and of optical pulses in nonlinear media reveal that they share common features

which can be described mathematically by a Schrödinger equation with a nonlinear potential [8, 9].

Dispersive effects can play an important role for short optical pulses when the spatial gradients and time variations become large. One of the most important generic effects of the dispersion is to permit pulse splitting along the direction of propagation [11, 12], which has also been experimentally verified [13]. The pulse splitting is of great interest in applications to normal dispersive media, where the collapse of light pulses can be arrested (which is not possible in anomalous dispersive media [14]), giving rise to a train of high-intensity pulses; these pulses may then work as a source of white light generation [15]. The quantum vacuum also possesses such dispersive effects [8], which can be of importance in the present generation intense lasers [16] where ultra-short high-intensity photon pulses will be produced. Rapidly varying fields are also of interest for the frequency up-shift of photons in photon acceleration [17–19], which is an important ingredient in studies of plasma-based charged particle accelerators and laser-plasma/matter interactions [20] where ultra-strong fields [21–23] are expected to reach from petto zeta-watt powers [24]. In this Letter, we present for the first time results dealing with the nonlinear propagation of *three-dimensional intense photon pulses* in vacuum where QED effects play a major role. Specifically, we report on new features of nonlinear propagation

¹⁾e-mail: marklund@elmagn.chalmers.se

of a linearly polarized intense photon pulse on a radiation gas background, assuming that there is no pair creation and that the field strength E is below the critical Schwinger field, i.e. $\omega \ll m_e c^2/\hbar \simeq 8 \cdot 10^{20} \cdot \text{rad s}^{-1}$ and $|E| \ll m_e c^2/e\lambda_c \simeq 10^{18} \text{ V} \cdot \text{m}^{-1}$, respectively, where ω is the photon frequency, m_e the electron mass, e the magnitude of the electron charge, and λ_c the Compton wavelength. The derivative corrections to the H-E Lagrangian gives rise to a nonlinear dependence of the photon frequency on the wavenumber, which is shown to permit three-dimensional self-focusing. For moderate intensities of the photon pulse, the self-focusing will be followed by pulse splitting and the formation of stable photonic solitary-pulses. However, for high initial powers of the photon pulse, the latter undergoes catastrophic collapse, giving rise to field amplitudes exceeding the Schwinger limit $10^{29} \text{ W} \cdot \text{cm}^{-2}$. At these intensities, our theory breaks down and one must consider higher-order nonlinear effects and pair productions. It is argued that higher-order nonlinear effects become important before the Schwinger limit is reached, arresting the collapse and giving rise to ultra-high intensity three-dimensional solitary photonic pulses. Thus, the results presented here gives, apart from its immediate fundamental interest and astrophysical applications, a mechanism for creating ultra-high intensity photon pulses.

The evolution of an intense short photon pulse and the radiation background, is governed by the Karpman-like system of equations [10, 8, 25, 26]

$$i \left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) E + \frac{v_g}{2k_0} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial E}{\partial r} \right) - \beta_z \frac{\partial^2 E}{\partial z^2} \right] + \kappa \mathcal{E} E = 0, \quad (1a)$$

and

$$\left(\frac{\partial^2}{\partial t^2} - \frac{c^2}{3} \nabla^2 \right) \mathcal{E} + \mu \epsilon_0 \left(\frac{\partial^2}{\partial t^2} + c^2 \nabla^2 \right) |E|^2 = 0, \quad (1b)$$

where \mathcal{E}_0 and \mathcal{E} is the background and perturbation of the radiation energy density, respectively, $v_g = c(1 - \mu - \mu^2\delta)$ is the group velocity, $\beta_z \simeq 2\mu^2\delta$ the vacuum dispersion coefficient, and $\kappa \simeq ck_0(1 + \mu\delta)/\mathcal{E}_c$ the vacuum nonlinear refraction parameter [10]. Here, $\delta \equiv k_0^2/k_c^2$ and $\mu \equiv \mathcal{E}_0/\mathcal{E}_c$, and the critical parameters $k_c^{-1} \sim 10^{-13} \text{ m}$, the Compton wavelength divided by 2π , and $\mathcal{E}_c \sim 10^{27} \text{ Jm}^{-3}$ are defined by the QED properties of the vacuum.

If the time response of the radiation background is slow, then Eq. (1b) may be integrated to yield $\mathcal{E} \simeq 3\mu\epsilon_0|E|^2$ and from Eq. (1a) we obtain the standard equation for analyzing ultra-short intense pulses in normal dispersive media, see Refs. [2, 11, 12] and references therein. It is well-known that the evolution of a pulse

within this equation is modulationally unstable [2], and displays first self-focusing, then pulse splitting along the direction of the pulse propagation [11, 12].

We have analyzed the system (1) numerically and analytically; in Figure, we display the intensity of the electric field E in the left panels and the perturbation \mathcal{E} of the background radiation in the right panels. As an initial condition, we use a Gaussian pulse for the electric field envelope, $E = E_0 \exp\{-[r^2 + (z + z_0)^2]/a_0^2\}$, while the radiation perturbation is initially set to zero, and $v_g \simeq c$ together with $\beta_z = 10^{-7}$ (relevant to astrophysical applications); see the upper panels of Figure. The z derivatives in Eqs. (1) are calculated numerically with a pseudo-spectral method, the r derivatives with a second-order difference scheme, and the system is advanced in time with a fourth-order Runge–Kutta scheme.

Displayed in the middle panels of Figure is the initial pulse which has been moving in the z direction with a speed close to the speed of light. The pulse has self-focused, and exhibits a structure stretched along the z direction. The self-focusing can be understood in the framework of the two-dimensional nonlinear Schrödinger equation; during the initial phase of the pulse propagation, the dispersive term plays a minor dynamical role. If the t and z variations are neglected in Eq. (1b), we obtain $\mathcal{E} \simeq 3\mu\epsilon_0|E|^2$. Thus, Eq. (1b) becomes the standard, nonlinear Schrödinger equation in which the cubic nonlinear potential has the same sign as the diffraction term, supporting two-dimensional collapse of the pulse [2]. The time for *complete* two-dimensional collapse of a pulse can be estimated by means of Rayleigh–Ritz optimization. Using normalized units (see Figure), two-dimensional collapse will occur when the intensity and pulse width satisfies the inequality $|E_0|^2 \gtrsim 1.3a_0^{-2}$. The resulting collapse time $t_c \simeq 4.3a_0^2(E_0^2a_0^2 - 1.3)^{-1/2}$. In Figure, pulse splitting occurs after $\sim t_c/3$, thus well before field strengths reaches critical levels.

However, here the collapse of the pulse is arrested due to the backscattering from the background; the core of the pulse slows down because of the nonlinear interaction with the background, while the flanks of the pulse continue to propagate with the speed of light, creating a fan-like structure in front of the slower moving pulse; see the middle left panel of Figure. The pulse then splits into several parts with local maxima; one wider pulse which can be seen at $k_0z = 35$ in the lower left panel followed by two smaller, narrower pulses at $k_0z \approx 27$. We have carried out simulations with different values of β_z , which reveal that the flattening of the leading edge pulse is less pronounced for smaller values of β_z . The wider pulse is correlated with a slight depletion of the radiation energy density. Letting the pulse propagate

Место для тонового рисунка

The collapse of the pulse and radiation gas disturbance displayed using normalized electric field intensity $|E|^2$ (left panels) and energy densities \mathcal{E} (right panels). We have scaled the pulse electric field E by $2(3\sqrt{\mu\epsilon_0/\mathcal{E}_c})^{-1}$ and the radiation background \mathcal{E} by $2\mathcal{E}_c/3$, while the time scale is normalized by $(ck_0)^{-1}$ and the length scale by k_0^{-1} . The initially Gaussian pulse has the amplitude $E_0 = 0.64(3\sqrt{\mu\epsilon_0/\mathcal{E}_c})^{-1}$ and width $a_0^2 = 50/k_0^2$. The upper panels show the initial configuration, while the middle panels give the system after 18 time units and the lower panels after 36 time units. The initial photonic pulse in the upper left panel self-focuses and then splits into two pulses, as can be seen in the left middle and lower panels. The energy background emanates Cherenkov-like radiation, clearly seen in the lower right panel as two yellow bands created behind the pulse

further, the transverse variation for the wide pulse will become much smaller than the longitudinal variation, leading to an almost one-dimensional structure (depending weakly on the transverse coordinate r), thus making the nonzero dispersive term essential. This pulse moves with a supersonic speed ($> c/\sqrt{3}$). For these broad supersonic pulses, we may solve Eqs. (1) for the stationary state. The acoustic equation (1b) gives $\mathcal{E} = -3\mu(v_g^2 + c^2)/(3v_g^2 - c^2)\epsilon_0|E|^2$. We note that the pulse

will experience resonance phenomena as the group velocity approaches the sound speed. With the solution to Eq. (1b), one can describe the localized stationary solitary solutions for a pulse moving close to the speed of light according to [2] $|E| \simeq E_0 \operatorname{sech}[(3\epsilon/\delta)^{1/2}k_0(z - v_g t)]$, where E_0 is the constant amplitude of the pulse, and $\epsilon = \epsilon_0 E_0^2/\mathcal{E}_0$ is the relative energy density of the pulse. We point out that these pulses are not necessarily of extremely high amplitudes. The remnants of radially

collapsing pulse asymptotically form a train of low-amplitude axially modulated solitary pulses. Moreover, the pulses show a parabolic self-compression, which will compensate for the small, but non-zero, diffraction of the pulse, making the one-dimensional approximation valid over a longer period of time. In the final stage, we can see the formation of a ‘photon wedge’ (seen in the lower right panel), which is due to the pulse propagation with a supersonic speed, so that part of the energy of the pulse is released into Cherenkov-like radiation behind the pulse, analogous to the sonic shocks created behind supersonic flights in air. Thus, the dynamics of the photonic pulse is surprisingly complex, exhibiting a multitude of nonlinear phenomena.

Simulations with higher initial amplitudes of the pulse show that the pulse again experience two-dimensional collapse in which the intensity may grow above the Schwinger limit before photon pulse splitting occurs. Because of the nonlinear dominance, the speed of the pulse decreases below the sound speed of the background, and the Cherenkov energy loss through the photon wedges will therefore be small, reinforcing the photon pulse collapse. This mechanism leads to amplitudes where our model breaks down; thus, the photon collapse becomes catastrophic close to the Schwinger limit [15]. However, before the Schwinger limit is reached, higher order nonlinear effects are likely to arrest the collapse [27]. This effect is similar to ultra-short intense laser pulses in air, where the formation of a plasma due to self-focusing gives rise to filamentation and halted collapse [28]. The plasma formation gives rise to a significantly longer propagation range for laser pulses in air, a behavior which the QED pulse propagation presented here is expected to share, but with the plasma formation replaced by higher order QED effects and the pair creation.

Astrophysical environments can be of very extreme nature, exhibiting the largest energy levels known to man. In the case of regular neutron stars the surface magnetic field strengths reach $10^{10} - 10^{13}$ G, while in magnetars they can reach $10^{14} - 10^{15}$ G, the latter being close to the Schwinger limit. An interesting possibility arises in the context of neutron star and magnetar quakes, in which magnetic fields build up tensions in the star crust over long periods of time, and sudden bursts of energy are released from the star during the quakes. There, it is expected that large quantities of low-frequency photons would be ejected, forming an almost incoherent spectrum of waves [29]. This photon gas could reach energy densities $\mathcal{E}_0 \sim 10^{17} - 10^{26} \text{ J m}^{-3}$, corresponding to $\mu \sim 10^{-10} - 10^{-1}$. A short high intensity electromagnetic pulse, with wave-

lengths from the UV to gamma range (corresponding to $\delta \sim 10^{-10} - 10^{-2}$), with its evolution modeled by the nonlinear Schrödinger eq. (1a), passing through this low-frequency photon gas dynamically governed by the acoustic-like wave eq. (1b), could be a source for gamma-ray bursts. The latter are short (\lesssim tens of second) emissions of photons in the gamma range [30]. Apart from the pulse collapse described by the system (1), there will also be a significant blue-shift of the photon pulse, due to the formation of steep intensity gradients in the system [15], which we can see in the numerical results of Figure. We note that the timescales for these events could be extremely short depending on the frequency of the photons. One can therefore expect the formation of intense photon pulses with frequencies up to the gamma regime, within the incoherent photon gas created by magnetar quakes, possibly giving insight into the dynamics of gamma-ray bursts, since we here have a mechanism both for blue shifting, pulse compression and high intensity field generation. However, we note that, in accordance with the standard relativistic fireball model, it would be more appropriate to model incoherent photons by a wave kinetic equation, instead of a Schrödinger equation.

To summarize, we have considered the implications of the dispersive properties of the quantum vacuum for the case of intense photon pulses propagating on a radiation background. In the slowly varying acoustic wave limit, the pulse evolves similar to an ultra-short high-intensity pulse in nonlinear, normal dispersive media, with pulse collapse and splitting as a result. The analysis of the full system of equations shows that the slowly varying acoustic limit is far from generic, and that the response of the radiation gas can have the same timescale as the pulse evolution. It is due to the self-generation of potential wells, giving an attractive force between the photonic pulse peak and the acoustic disturbance. This can give rise to three-dimensional catastrophic photonic pulse collapse, where the pulse and radiation gas power increases towards the Schwinger limit. Moreover, given suitable initial conditions, the photonic pulses can evolve into a stable localized structure with high field strengths. The application of our work to astrophysical settings has been discussed. Specifically, the present investigation sheds light on gamma-ray burst dynamics, and gives a means for obtaining pulse intensities surpassing the ones achievable by known mechanisms.

This work was partially supported by the European Commission (Brussels, Belgium) through contract No. HPRN-CT-2000-00314 for carrying out the task of the Human Potential Research Training Networks “Turbulent Boundary Layers in Geospace Plasmas”, as well as by the Deutsche Forschungsgemeinschaft (Bonn, Ger-

many) through the Sonderforschungsbereich 591 entitled “Universelles Verhalten Gleichgewichtsferner Plasmen: Heizung, Transport und Strukturbildung”.

1. J. Schwinger, Phys. Rev. **82**, 664 (1951).
2. Y.S. Kivshar and G. P. Agrawal, *Optical Solitons*, Academic Press, San Diego, 2003.
3. S. L. Adler, J. N. Bahcall, C. G. Callan, and M. N. Rosenbluth, Phys. Rev. Lett. **25**, 1061 (1970).
4. M. G. Baring and A. K. Harding, Astroph. J. **507**, L55 (1998).
5. A. K. Harding, Science **251**, 1033 (1991).
6. C. Kouveliotou et al., Nature **393**, 235 (1998).
7. Y. J. Ding and A. E. Kaplan, Phys. Rev. Lett. **63**, 2725 (1989).
8. N. N. Rozanov, JETP **86**, 284 (1998).
9. G. Brodin, M. Marklund, and L. Stenflo, Phys. Rev. Lett. **87**, 171801 (2001).
10. M. Marklund, G. Brodin, and L. Stenflo, Phys. Rev. Lett. **91**, 163601 (2003).
11. P. Chernev and V. Petrov, Opt. Lett. **17**, 172 (1992).
12. J. E. Rothenberg, Opt. Lett. **17**, 583 (1992).
13. J. K. Ranka, R. W. Schirmer, and A. L. Gaeta, Phys. Rev. Lett. **77**, 3783 (1996).
14. Y. Silberberg, Opt. Lett. **15**, 1282 (1990).
15. A. L. Gaeta, Phys. Rev. Lett. **84**, 3582 (2000).
16. G. A. Mourou, C. P. J. Barty, and M. D. Perry, Phys. Today **51**, 22 (1998).
17. S. C. Wilks, J. M. Dawson, W. B. Mori et al., Phys. Rev. Lett. **62**, 2600 (1989).
18. N. L. Tsintsadze, J. T. Mendonça, and L. O. Silva, Phys. Rev. **E58**, 4890 (1998).
19. J. T. Mendonça, *Photon acceleration*, IOP Publishing, Bristol, 2001.
20. A. Pukhov, Rep. Prog. Phys. **66**, 47 (2003).
21. V. Malka et al., Science **298**, 1596 (2002).
22. A. Pukhov, S. Gordienko, and T. Baeva, Phys. Rev. Lett. **91**, 173002 (2003).
23. R. Bingham, Nature **424**, 258 (2003).
24. S. V. Bulanov, T. Esirkepov, and T. Tajima, Phys. Rev. Lett. **91**, 085001 (2003).
25. V. I. Karpman, Plasma Phys. **13**, 477 (1971).
26. V. I. Karpman, *Nonlinear Waves in Dispersive Media*, Pergamon Press, Oxford, 1975.
27. D. D. Tskhakaia, Phys. Rev. Lett. **48**, 484 (1982).
28. S. Tzortzakis et al., Phys. Rev. Lett. **87**, 213902 (2001).
29. V. N. Kondratyev, Phys. Rev. Lett. **88**, 221101 (2002).
30. T. Piran, Phys. Rep. **314**, 575 (1999).