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WEAK LOCALIZATION IN QUANTUM WELLS WITH SPIN-ORBIT INTERACTION

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The quantum interference correction to the counductivity is calculated for $2d$ electrons in cubic A_3B_5 srystal with spin-orbit splitting by solving appropriate Cooperon equation. The spin dependent vector-potential due to spin orbit-interaction leads to considerable changes compare to Larkin – Hikami – Nagaoka expression.

Weak localization corrections to the conductivity of $2d$ structures in magnetic field were obtained by Hikami et al in their pioneering work [1]. Only the spin-orbit skew scattering mechanism (Elliot-Jaffet mechanism) of electron spin relaxation was considered as the origin of spin-orbit effect on conductivity.In subsequent works on the weak localization the spin-orbit effects for $2d$ conductors without inversion center were treated exclusively in terms Dyakonov – Perel spin relaxation time in close analogy with the skew scattering effect (see e.g.[2, 3]).Recently [4 – 6] it was demonstrated that the spin-orbit interaction effect on the weak localization and universal conductance fluctuations should be considered as an effect of spin-dependent vector potential and important terms connected with this vector potential were shown to exist in the Cooperon equation.Such an approach considerably changes the results [1, 2] on the spin-orbit interaction effects.In the present paper we study the anomalous magnetoresistance.Its experimental investigation is the most convenient tool for the examination of the weak localization effects [3,7,8],and the improvement of theoretical formulae is important for the determination of various relaxation times as well as the spin orbit splitting.For this purpose we solve here exactly the Cooperon equation which is obtained by direct using Green functions which explicitely include spin-orbit terms in the Hamiltonian.

We consider here quantum wells with the normal to $2d$ plane in (001) direction of A_3B_5 cubic crystal. In that case the Hamiltonian for $2d$ electrons has the form (we assume $\hbar = 1$).

$$H = \frac{k^2}{2m} + \vec{\sigma}\vec{\Omega} \qquad (1)$$

Where σ_i are Pauli matrices and $\Omega_x = -\Omega_1 \cos\varphi - \Omega_3 \cos 3\varphi$, $\Omega_y = \Omega_1 \sin\varphi - \Omega_3 \sin 3\varphi$, $\Omega_1 = \gamma k(k_z^2 - \frac{k^2}{4})$, $\Omega_3 = \gamma\frac{k^3}{4}$, $\tan\varphi = \frac{k_y}{k_x}$, $k^2 = k_x^2 + k_y^2$. Here $k_z^2 = \int |\frac{\partial\psi}{\partial z}|^2 dz$ is the mean square of electron momentum in the direction perpendicular to $2d$ plane, ψ is the electron wave-function, k_x, k_y are the components of inplane electron momentum, γ is the constant of spin-orbit interaction. We assume anisotropical elastic scattering and introduce the probability of the scattering $W(\vartheta)$ per angle ϑ per unit time.

The weak localization correction is expressed in terms of Cooperon amplitude $C(q, \vartheta, k_F)$, where q is small total Cooperon momentum, ϑ is the angle defining the position of electron momentum on the Fermi surface $k = k_F$. (See e.q. review

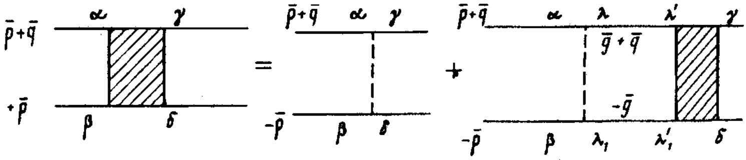

Fig.1. Graphical representation of Cooperon equation

[5]) which is defined by graphical equation on fig.1. Green functions (fat lines) entering Cooperon equation are expressed in terms of Hamiltonian (1) and elastic relaxation time τ_0

$$G^{+}(k,\omega) = \frac{I}{\omega - \frac{k^2}{2m} - \vec{\sigma}\vec{\Omega} \pm \frac{i}{2\tau_0}}, \tag{2}$$

where $G^{\pm}$ is 2×2 matrix in spin indices, I is unit matrix and

$$\frac{1}{\tau_0} = \int W(\vartheta)d\vartheta. \tag{3}$$

By usual in weak localization theory procedure the equation for Cooperon amplitude $C = C(q,\vartheta,k_F)$ can be solved by perturbation theory assuming $1/\tau_0$ large compare to spin orbit energy splitting and $v_F q$ (v_F is Fermi velocity). The Cooperon amplitude can be expanded in Fourrier harmonics of ϑ and it turns out that arising first and third harmonics are small and can be expressed in terms of zero harmonic C_0. The substitution of higher order harmonics in the zero harmonic equation gives the effective matrix equation for $C_0(k_F,q)$:

$$\hat{L}C_0 = \frac{1}{2\pi\nu_0\tau_0}, \tag{4}$$

where

$$\hat{L} = \tau_0\{\frac{1}{\tau_\varphi} + \frac{1}{2}v_F^2q^2\tau_1 + 2(\Omega_1^2\tau_1 + \Omega_3^2\tau_3)(1 + \sigma_+\rho_- + \sigma_-\rho_+) -$$

$$-\tau_1 v_F\Omega_1[(\sigma_+ + \rho_+)q_+ + (\sigma_- + \rho_-)q_-]\}. \tag{5}$$

Here we introduce the inverse phase relaxation time $1/\tau_\varphi$ as a qut off at small q, the transport times

$$\frac{1}{\tau_1} = \int(1-\cos\theta)W(\theta)d\theta, \qquad \frac{1}{\tau_3} = \int(1-\cos 3\theta)W(\theta)d\theta, \tag{6}$$

the density of states $\nu_0 = \frac{m}{2\pi}$, $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$, $\rho_\pm = \frac{1}{2}(\rho_x \pm i\rho_y)$ are the combinations of Pauli matrices acting on the upper and lower spin indices of fig.1 respectively, I is the product of the unit matrices in the same basis, $q_\pm = q_x \pm iq_y$.

The solution of the equation (4) can be represented in the form

$$C^{\alpha\gamma}_{\beta\delta}(q) = \frac{1}{2\pi\nu_0\tau_0}\sum_r \frac{1}{E_r}\psi_{r,q}(\alpha,\beta)\psi^*_{r,q}(\gamma,\delta), \tag{7}$$

where $\psi_{r,q}$, $E_r(q)$ are normalized eigenfunctions and eigenvalues of the operator $\hat{L}$:

$$\hat{L}\psi_r = E_r\psi_r. \tag{8}$$

These eigenfunctions can be classified by the value of the total spin momentum of two particles: antisimmetrical singlet $l = 0$ $(r = 0)$, and symmetrical $l = 1$ $(r = 1, 2, 3)$, in the latter case we use also the basis of functions with integer spin momentum projection on Z-axis $\Phi_{l=1,m}$ $(m = 1, 0, -1)$.

The quantum interference correction to the conductivity is proportional to (see e.g. [2]) the sum

$$S(q) = \sum_{\alpha,\beta} C^{\alpha\beta}_{\beta\alpha} = \frac{1}{2\pi\nu_0\tau_0}(-\frac{1}{E_0} + \sum_{r=1}^{3}\frac{1}{E_r}). \tag{9}$$

The singlet eigenvalue E_0 does not depend on spin orbit term

$$E_0 = (Dq^2 + \frac{1}{\tau_\phi})\tau_0, D = \frac{1}{2}v_F^2\tau_1.$$

The triplet eigenvalues can be also easily found by solving the equation (8) (for the total momentum $l = 1$) with

$$L = \tau_o[Dq^2 + \frac{1}{\tau_\phi} + 2(\Omega_1^2\tau_1 + \Omega_3^2\tau_3)(2 - J_z^2) - \tau_1 v_F\Omega_1\sqrt{2}(J_+q_+ + J_-q_-)], \tag{10}$$

where J is angular momentum operator: $\mathbf{J} = \frac{1}{2}(\vec{\sigma} + \vec{\rho})$ and $J_\pm = \frac{1}{\sqrt{2}}(\sigma_\pm + \rho_\pm)$. It is convenient to express the sum of inverse eigenvalues in (8) directly in terms of the coefficients of secular equation.Such an approach gives us a possibility to find the solution of the problem in the case of applied magnetic field. Using the expression (8) for S we get by standart procedure the weak localization correction to the conductivity without magnetic field retaining only logarithmic terms

$$\Delta\sigma(0) = -\frac{e^2}{\pi}\nu_0\tau_0 D\int S(q^2)\frac{d^2q}{(2\pi)^2} =$$

$$= -\frac{e^2}{2\pi^2\hbar}\{-\frac{1}{2}\ln\frac{\tau_1}{\tau_\varphi} + \ln(\frac{\tau_1}{\tau_\varphi} + \frac{\tau_1}{\tau_{Sxx}}) + \frac{1}{2}\ln(\frac{\tau_1}{\tau_\varphi} + \frac{\tau_1}{\tau_{Szz}})\} \tag{11}$$

Here $\frac{1}{\tau_{Sxx}} = \frac{1}{\tau_{Syy}} = \frac{1}{2\tau_{Szz}} = 2(\Omega_1^2\tau_1 + \Omega_3^2\tau_3)$, where τ_{Sii} are the spin relaxation times defined by equation $\frac{dS_i}{dt} = -\frac{S_i}{\tau_{Sii}}$ [7, 8]. From the expression (11) we see that with the logarithmic accuracy the spin-dependent vector potential terms in Cooperon equation are not essential for temperature dependent corrections to the conductivity [6].

In the presence of magnetic field B the quantities $q_\pm$, q^2 entering eq.(5,10) are defined by gauge invariance and become $q_+ = \sqrt{2}Sa$, $q_- = \sqrt{2}Sa^+$, where $S^2 = \frac{2eB}{c}$, a, a^+ are operators increasing and decreasing the number n of Landau level of the wave function F_n:
$(a^+a)F_n = (n + \frac{1}{2})F_n$, $aF_n = \sqrt{n}F_{n-1}$, $a^+F_n = \sqrt{n+1}F_{n+1}$.

The eigenvalue $E_0(n)$ does not depend on spin-orbit interaction and is given in the paper [1].According to eqn. (8), (10) the solution $\vec{\psi}(n)$ in the basis of the functions $\Phi_{l=1,m}$ with momentum projection m has the form

$$\vec{\psi}_1(n) = (f_{1,r}(n)F_{n-2}, f_{0,r}(n)F_{n-1}, f_{-1,r}(n)F_n). \tag{12}$$

The substitution of (12) in the equation (8), (10) gives the system of algebrac linear equations for the determination of f_{ir} and the determinant of the appropriate matrix gives the secular equation for the eigenvalues $E_r(n)$ $(r = 1,2,3)$. In the case $n = 0, F_{n-1}, F_{n-2}$ must be set to zero and there is only one eigenvalue $E_1(0)$. In the case $n = 1, F_{n-2}$ must be set to zero and there are two eigenvalues $E_1(1)$, $E_2(1)$ defined by appropriate quadratic equation. For all other $n \geq 2$ there are three eigenvalue $E_r(n)(r - 1,2,3)$ defined by cubic equation. As well as for the case without magnetic field the sum (9) can be expressed directly through the coefficients of secular equation without solving it. Being short of place we give the final expression for the conductivity correction:

$$\Delta\sigma(B) = -\frac{e^2}{4\pi^2\hbar}\left\{\frac{1}{a_0} + \frac{2a_0 + 1 + \frac{H_{SO}}{B}}{a_1(a_0 + \frac{H_{SO}}{B}) - 2\frac{H'_{SO}}{B}} - \right.$$

$$-\sum_{n=1}^{\infty}\left[\frac{3}{n} - \frac{3a_n^2 + 2a_n\frac{H_{SO}}{B} - 1 - 2(2n+1)\frac{H'_{SO}}{B}}{(a_n + \frac{H_{SO}}{B})a_{n-1}a_{n+1} - 2\frac{H'_{SO}}{B}[(2n+1)a_n - 1]}\right] +$$

$$\left. +2\ln\frac{H_{tr}}{B} + \psi(\frac{1}{2} + \frac{H_\varphi}{B}) + 3C\right\}, \quad (13)$$

where

$$a_n = n + \frac{1}{2} + \frac{H_\varphi}{B} + \frac{H_{SO}}{B}, \quad H_\varphi = \frac{c\hbar}{4eD\tau_\varphi}; \quad H_{SO} = \frac{c\hbar}{4eD}(2\Omega_1^2\tau_1 + 2\Omega_3^2\tau_3);$$

$$H'_{SO} = \frac{c\hbar}{4eD}2\Omega_1^2\tau_1; \quad H_{tr} = \frac{c\hbar}{4eD\tau_1}; \quad \psi(1+z) = -C + \sum_{n=1}^{\infty}\frac{z}{n(n+z)} \quad (14)$$

and C is Euler constant. If we omit terms containing $\frac{H'_{SO}}{B}$ we obtain Hikami–Larkin–Nagaoka expression from (13):

$$\Delta\sigma(B) - \Delta\sigma(0) = \frac{e^2}{2\pi^2\hbar}\left\{\psi(\frac{1}{2} + \frac{H_\varphi}{B} + \frac{H_{SO}}{B}) + \frac{1}{2}\psi(\frac{1}{2} + \frac{H_\varphi}{B} + \frac{2H_{SO}}{B}) - \right.$$

$$\left. -\frac{1}{2}\psi(\frac{1}{2} + \frac{H_\varphi}{B}) - \ln\frac{H_\varphi + H_{SO}}{B} - \frac{1}{2}\ln\frac{H_\varphi + 2H_{SO}}{B} + \frac{1}{2}\ln\frac{H_\varphi}{B}\right\} \quad (15)$$

but for magnetic field $B \sim H_{SO}$ formulae (13) and (15) are numerically different.Fig.2 gives the dependence $\Delta\sigma(B) - \Delta\sigma(0)$ calculated in accordance with eq.(13) for $H_{SO}/H_\phi = 4$, $H'_{SO}/H_{SO} = 1, 1/2, 1/4$ and $H'_{SO} = 0$ when (15) is applicable.Realistic values of $\frac{H'_{SO}}{H_{SO}}$ are close to 1 and we see that in this case the value of $\Delta\sigma(B) - \Delta\sigma(0)$ for B more large than H_φ and H_{SO} is essentially increased.It is explained by the fact that for nonvanishing H'_{SO} according to (13) $\Delta\sigma(0)$ contains beside the logarithmic term (11) a large additional contribution which also vanishes in strong magnetic field.

The other difference (besides the difference in τ_1, τ_3) compare to the papers [2, 3] is in the value of H_{SO} in (13), (15) which is twice as large in terms of the same Hamiltonian. The relaxation time introduced in [2, 3] $\frac{1}{\tau_{SOi}} = <\Omega_i^2> \tau$ (here $<\Omega_i^2>$ is the average over the Fermi surface) by definition does not coinside with spin relaxation times, which are [9, 10]

$$\frac{1}{\tau_{Sii}} = 2(<\Omega^2> - <\Omega_i^2>)\tau.$$

(For comparing results it is necessary to remind that in [9, 10] the spin orbit Hamiltonian is in form $H_{SO} = 1/2\vec{\sigma}\vec{\Omega}$, instead of (1)).Similiar remark concerns 3d case which was considered in [2].

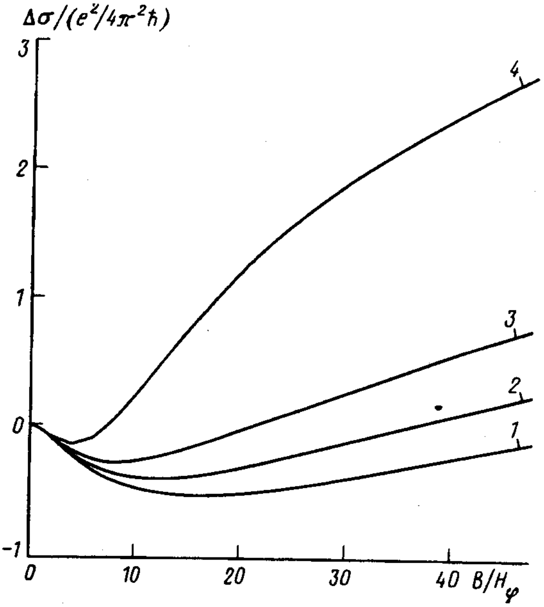

Fig.2. Magnetic field dependence of the conductivity corrections in the units $e^2/4\pi^2\hbar$ for $H_{SO}/H_\varphi = 4$, and $H'_{SO}/H_{SO} = 0$ *(1)*, 0,25 *(2)*, 0,5 *(3)*, 1 *(4)*

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1. S.Hikami, A.Larkin, and Y.Nagaoka, Progr. Theor. Phys. **63**, 707 (1980).
2. B.L.Altshuler, A.G.Aronov, A.I.Larkin, and D.E.Khmelnitzki., JETP **81**, 788 (1981).
3. P.Dresselhaus, C.Papavassillion, R.Wheeler, and R.Sacks, Phys. Rev. Lett. **68**, 106 (1992).
4. H.Mathur and A.Stone. Phys. Rev. Lett. **68**, 2864 (1992).
5. Yu.B.Lyanda-Geller and A.D.Mirlin. Phys. Rev. Lett. **72**, 1894 (1994).
6. A.G.Aronov and Yu.B.Lyanda-Geller, Spin-Orbit flux effect in disordered conductors, Preprint, University of Karlsrhue 1993, to be published.
7. G.Bergmann, Phys. Reports. **107**, 1 (1984).
8. W.Knap, C.Skierbiszewski, E.Litwin-Staszewska et al, Phys. Rev. **B**, in press.
9. M.Dyakonov and V.Perel, JETP **33**, 1053 (1971).
10. G.Pikus and A.Titkov, In "Optical Orientation" ed. F.Mayer, B.Zaharchenya. North Holland, Amsterdam (1984).