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A UNIFIED DESCRIPTION OF CONFINEMENT AND SUPERCONDUCTIVITY IN TERMS OF VACUUM CORRELATORS

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We argue that both confinement and superconductivity may be described in framework of Vacuum Correlator method. The closed similarity of this phenomena is stressed. The fundamental vacuum correlator functions D and D_1 are expressed through current correlators (ordinary and monopolelike).

It is widely accepted point of view, that confinement in SU(N) gluodynamics and QCD is a dual Meissner effect [1]-[3]. There is a strong support for this opinion from recent Monte-Carlo studies in the framework of the so-called abelian projection method [4]-[8]. Even the profile of the QCD string is similar to that of the Abrikosov vortex line [9]-[13].

At the same time there are strong objections against the full similarity of the underlying dynamics. Namely, in superconductivity one can work out all dynamical equations for fields as classical equations, following from e.g. Landau–Ginzburg Lagrangian, i.e. there are classical configurations behind the Abrikosov line and superconducting vacuum. In contrast to that in QCD (or SU(N) gluodynamics) it is unlikely that field configurations are classical, since stable classical solutions are topological and the net topological charge of vacuum is zero. Also in lattice calculations the nonperturbative physics is ensured by a rather large set of configurations, and this can be checked in the so-called cooling method [15, 14].

Therefore there is a necessity to formulate both superconductivity and confinement using one and most general language, which does not depend on the classical equations of motion. We suggest in this letter to use vacuum field correlators (current correlators) to describe both phenomena and demonstrate explicitly which correlators are necessary for that and what duality means in this language. It is remarkable that the same correlators are responsible for confinement both in abelian and nonabelian theory. As an outcome we have a possibility of purely quantum superconductivity, described by quantum correlators and not by classical equations.

1. Wilson loop & correlation functions

We consider abelian theory, like QED with possible admission of magnetic monopoles, since all final equations hold true for nonabelian case too with obvious insertion of parallel transporters, traces etc. Confinement is usually characterized by the area law of the Wilson loop along the trajectory of charges $e, -e$

$$\langle W(C)\rangle = \langle \exp\ ie \int_C A_\mu dx_\mu\rangle = \langle \exp\ ie \int F_{\mu\nu} d\sigma_{\mu\nu}\rangle = \exp(-\sigma S), \qquad (1)$$

where S is the area of the contour C in 14 plane, and string tension σ is expressed through field strength correlators (FSC) $\langle F_{\mu\nu}(x)F_{\rho\lambda}(y)\rangle$, [16, 17]

$$\sigma = \frac{1}{2}e^2 \int d^2x \langle F_{14}(x)F_{14}(y)\rangle + \cdots, \tag{2}$$

where dots imply contribution of higher correlators $\langle FFFF\rangle$ etc, which are unimportant for our purpose here.

On general grounds of Lorentz invariance FSC can be expressed in terms of two basic scalar functions $D(x)$ and $D_1(x)$ [17] and we shall write separately FSC for electric and magnetic fields

$$\langle E_i(x)E_j(y)\rangle = \delta_{ij}(D^E + D_1^E + h_4^2\frac{\partial D_1^E}{\partial h^2}) + h_ih_j\frac{\partial D_1^E}{\partial h^2}, \tag{3}$$

$$\langle H_i(x)H_j(y)\rangle = \delta_{ij}(D^H + D_1^H + h^2\frac{\partial D_1^H}{\partial h^2}) - h_ih_j\frac{\partial D_1^H}{\partial h^2}, \tag{4}$$

where $h = (h_\mu h^\mu)^{1/2}$, $h_\mu = x_\mu - y_\mu$.

For Lorentz-invariant vacuum, like that of QED or QCD, one has $D^E = D^H = D$, $D_1^E = D_1^H = D_1$. However, for the same theories but at nonzero temperature T, electric and magnetic correlators do not coincide. For superconducting material Lorentz invariance is violated too, and again $D^E \neq D^H, D_1^E \neq D_1^H$.

For a contour C in the 14 plane one has from (3)

$$\sigma = \frac{e^2}{2}\int d^2x D(x) + \cdots. \tag{5}$$

Let us now consider a magnetic charge g in the superconductor of second kind and similarly to (2) introduce the Wilson loop operator

$$\langle W^*(C)\rangle = \langle \exp\ ig \int F^*_{\mu\nu}d\sigma_{\mu\nu}\rangle = \exp(-\sigma^* S). \tag{6}$$

Here σ^* is expressed through the dual fields F^*_{14} as in (2), but since $F^*_{14} = \varepsilon_{1234}F_{23} = H_1$, one has due to (4)

$$\sigma^* = \frac{g^2}{2}\int d^2x D_1^H(x). \tag{7}$$

Comparison of (5) and (7) gives an exact meaning of the notion of dual Meissner effect without reference to the underlying equations of motion. To proceed one needs first to define $D(x)$ and $D_1(x)$ more explicitly together with integrals (5) and (7) which may diverge. As can be seen in (4) $D^E(x)$ can exist only due to magnetic monopoles [17] , indeed applying ∂_i to both sides of (4), one has

$$\langle \mathrm{div}\boldsymbol{H}(x), \mathrm{div}\boldsymbol{H}(y)\rangle = -\partial^2 D^H(x-y). \tag{8}$$

Hence $D^H(x)$ does not contain purely perturbative contributions (at least in lowest orders) . In contrast to that $D_1(x)$ may contain perturbative contributions, which should be subtracted from it in (7) , to lowest order one has

$$\bar{D}_1(x) = D_1(x) - \frac{4e^2}{x^4} \tag{9}$$

and one should replace in (7) D_1 by $\bar{D}_1$. Hence the phenomenon of Abrikosov string depends on the nonperturbative contents of $\bar{D}_1$, i.e. on the possibility to create a mass parameter, characterizing the size of the string.

To see the mechanism of this mass creation, one can use the Ginzburg-Landau equations to derive [1)]

$$D_1^{LG}(x-y) = \left(e^2|\phi|^2 - \partial^2\right)^{-1}_{xy}, \tag{10}$$

which for the region outside of the string core, $r \gg \xi$ (ξ is the coherence length [18]) , when $|\phi| = \phi_0 = \text{const}$ yields exponential decay at large distances with the mass parameter

$$D_1(x) \sim e^{-mx}, \qquad m = e|\phi_0| = 1/\delta, \tag{11}$$

where δ is the London (Landau) penetration length.

It is interesting to compare this behaviour with that of $D(x)$, measured recently in SU(3) gluodynamics [19],

$$D(x) \sim e^{-\mu|x|}, \qquad \mu \sim 1 \text{ Gev}. \tag{12}$$

One can visualize in (11), (12) that the notion of duality of QCD string and Abrikosov string has a more detailed correspondence. To see more of this correspondence one can compare profiles (density distributions off the string) for QCD string and Abrikosov string.

2. Profiles of the string

In the first case one can probe field inside the QCD string using so-called connected ρ^c and disconnected ρ^{disc} plaquette averages around the Wilson loop [11- 13]

$$\rho^c = \frac{\langle W(C_\sigma)\rangle - \langle W(C)\rangle\langle W(\sigma)\rangle}{\langle W(C)\rangle}, \tag{13}$$

where C_σ is the contour formed by connecting $\Delta\sigma$ and the Wilson loop C, while $W(\sigma)$ is the Wilson loops for $\Delta\sigma$ contour. In terms of vacuum correlators method (VCM) one can obtain the following expression for the Wilson loop

$$W(C_\sigma) \simeq \langle W(C)\rangle(1 + e^2 \int d\sigma_{14}(y)\Delta\sigma_{\mu\nu}\langle E_1(y)\Phi F_{\mu\nu}(x)\Phi^+\rangle) + \cdots \tag{14}$$

where dots stand for higher cumulants and $O((\Delta\sigma)^2)$ terms. Here Φ denotes the parallel transporter $\Phi(x,y) = P\exp ie\int_y^x A_\mu dz_\mu$.

As one can see the ρ^c quantity measures the spatial distribution of the components of the field strength tensor $F_{\mu\nu}$ in presence of charges. The MC simulations shows that electric field in confinement phase is squeezed in flux tubes with an exponential [13] behaviour of the flux tube profile inside the string. In

[1)] In more general form it can be shown that twopoint correlation function (which does describe a response of vacuum on the source presence) coincides with the second derivative of effective Ginsburg-Landau Hamiltonian

$$\Delta^{-1} = \left[\frac{\delta^2 H_{eff}}{\delta H^2}\bigg|_{H=H_0}\right]^{-1}.$$

case of the very long string one obtaines a simple analytic result for ρ_{14}^c. The transverse shape measured at the middle is given by

$$\rho_{14}^c = \frac{2\pi a^2}{\mu^2}[D(0)(1+\mu x) - D_1(0)\frac{1}{2}(\mu x)^2]e^{-\mu x}, \tag{15}$$

where the obtained parameters are

$$\mu \approx 0.190382\text{fm}, \ \ a^2 D(0) \approx 3.91468 \times 10^7, \ \ D_1(0) = D(0)/3,$$

$$m \text{ from (11) is given by } m \approx \mu, \quad \text{with a} \quad \chi/d.o.f = 0.17.$$

One can see that such behavior of ρ_{14}^c is in good agreement with dual Meissner effect picture, when the asymptotic of the field distribution of the vortex line is exponential

$$H(r) = \frac{\phi_0}{(8\pi\, r\, \delta^3)^{1/2}} \exp(-r/\delta), \tag{16}$$

where δ =const in $\xi \to 0$ limit, and $\delta = \delta_{eff} = \int_0^\infty H(r)\, dr/H(\infty)$, when δ goes to zero.

3. Twopoint FSC in terms of currents

Let us consider $U(1)$ electrodynamics with monopoles (there may be, for example, Dirac monopoles, or topological defects in compact $U(1)$ theory)

$$\partial_\mu F_{\mu\nu} = j_\nu, \qquad \partial_\mu \overset{*}{F}_{\mu\nu} = \overset{*}{j}_\nu,$$

where variables j_ν, $\overset{*}{j}_\nu$, describe normal and monopolelike current correspondingly. In line with [20] we can express the observed field strength tensors $F_{\mu\nu}$ in terms of currents by redoubling field strength tensors $H_{\mu\nu}$, $G_{\mu\nu}$ as follows

$$F_{\mu\nu} = H_{\mu\nu} + \overset{*}{G}_{\mu\nu}, \ \ \overset{*}{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta},$$

$H_{\mu\nu}$, $G_{\mu\nu}$ satisfy Maxwell equations:

$$\partial_\mu H_{\mu\nu} = j_\nu, \ \ \partial_\mu G_{\mu\nu} = \overset{*}{j}_\nu,$$
$$\partial_\mu \overset{*}{H}_{\mu\nu} = 0, \ \ \partial_\mu \overset{*}{G}_{\mu\nu} = 0.$$

As a consequence one can define the dual pair of potentials A_ν, $\overset{*}{A}_\nu$ in usual form

$$H_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$
$$G_{\mu\nu} = \partial_\mu \overset{*}{A}_\nu - \partial_\nu \overset{*}{A}_\mu .$$

Then in the Lorentz gauge for the Fourier components one has

$$A_\nu(k) = \frac{1}{k^2} j_\nu(k), \ \ \overset{*}{A}_\nu(k) = \frac{1}{k^2} \overset{*}{j}_\nu(k).$$

For the field correlator one easily obtains

$$\begin{aligned} \langle F_{\mu\sigma}(-k)F_{\nu\rho}(k)\rangle = \frac{1}{k^2}[\langle \overset{*}{j}{}^2\rangle(k)\{\delta_{\mu\rho}\delta_{\sigma\nu} - \delta_{\mu\nu}\delta_{\rho\sigma}\} + \\ + \{\langle j^2 + \overset{*}{j}{}^2\rangle(k)\}\{\frac{k_\mu k_\nu}{k^2}\delta_{\rho\sigma} - \frac{k_\nu k_\sigma}{k^2}\delta_{\mu\rho} + (\mu\nu \leftrightarrow \sigma\rho)\}] \end{aligned} \tag{17}$$

while for the dual cumulant, which is responsible for the confinement of magnetic charge, one has the same expression with the replacement $j \leftrightarrow \overset{*}{j}$. From (17) we deduce expressions for Fourier components of $\tilde{D}$ and $\tilde{D}_1$ function

$$\frac{\partial\ \tilde{D}_1(k)}{\partial k^2} = \frac{1}{k^2}\frac{\langle j^2 + \overset{*}{j}{}^2\rangle(k)}{k^2},\quad \tilde{D}(k) + 2\tilde{D}_1(k) = \frac{\langle \overset{*}{j}{}^2\rangle(k)}{k^2},$$

or, in formal way, for the space forms

$$D_1(h) = 2\,\Box^{-1}\Box^{-1}\frac{\partial}{\partial h^2}\langle j^2 + \overset{*}{j}{}^2\rangle(h),$$

$$D(h) = -2\,D_1(h) - \Box^{-1}\langle \overset{*}{j}{}^2\rangle(h).$$

Conclusions

We have described both confinement and superconductivity using field correlators D, D_1. In the first case due to Lorentz invariance $D^E = D^H = D$ and this correlator ensures confinement. In the case of superconductivity and in absence of the condensate of magnetic monopoles only D_1^H is nonzero and responsible for confinement of magnetic charges and formation of Abrikosov fluxes. Correlators D, D_1 are expressed through correlators of charge and monopoles currents. We have shown that duality of confinement and superconductivety goes beyond symmetric expressions for string tensions (5), (7), and reveals itself also in the form of the field correlators (11), (12) and string profiles (15), (16).

All treatment above referred to the zero temperature case. It would be very interesting to extend this approach of field (current) correlators to nonzero temperatures and especially to the phase transition region.

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