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A SIMPLE WAY TO ESTIMATE THE VALUE OF $\bar{\alpha} \equiv \alpha(m_Z^2)$

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To obtain the value of electromagnetic coupling constant at $q^2 = m_Z^2$, $\bar{\alpha}$, which plays a key role in electroweak physics one has to integrate the cross-section of e^+e^--annihilation into hadrons divided by $(s - m_Z^2)$ over s from threshold to infinity. By combining, for each flavor channel, the contribution of lowest resonance with the perturbative QCD continuum, we obtain $1/\bar{\alpha} = 128.89 \pm 0.06$ a result which is close to known result obtained with purely experimental inputs, i.e. $1/\bar{\alpha} = 128.87 \pm 0.12$.

The detailed analysis of electroweak observables starts from three input parameters: G_μ, the Fermi coupling constant (extracted from muon decay), m_Z, the Z-boson mass (measured at LEP) and $\bar{\alpha}$, the electromagnetic coupling constant at $q^2 = m_Z^2$, obtained from dispersion relations. In fact, a Born aproximation to the minimal standard model which starts with $\bar{\alpha}$ (rather than $\alpha \equiv \alpha(0) = 1/137.0359895(61)$) reproduces the precise experimental values of the Z-decay parameters (obtained at LEP) and of the W mass (obtained at hadron colliders) with unexpectedly high accuracy [1,2]. For example for the ratio of vector and axial coupling constants of the Z-boson to charged leptons one obtains in this $\bar{\alpha}$ Born approximation [2]:

$$[g_V/g_A]_{\bar{\alpha}} = 0.0753(12) \ , \tag{1}$$

while the latest experimental numbers are [3]:

$$[g_V/g_A]_{LEP} = 0.0711(20) \ , \tag{2}$$

$$[g_V/g_A]_{LEP+SLD} = 0.0737(18) \ . \tag{3}$$

If instead of $\bar{\alpha}$ one uses $\alpha(0)$, then one gets:

$$[g_V/g_A]_{\alpha} = 0.152 \ ,$$

which is about $40\sigma's$ away from experiment as was stressed in [2]. The value of $\bar{\alpha}$ is of fundamental importance, and its error determines the uncertainty in the theoretical prediction (1).

$\bar{\alpha}$ is defined through the following formulas:

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha} \ , \tag{4}$$

$$\delta\alpha = \Sigma'_\gamma(0) - \frac{\Sigma_\gamma(m_Z^2)}{m_Z^2} \ , \tag{5}$$

[1)]Permanent address: ITEP, Moscow 117259, Russia where in (5) charged leptons and five quark flavor contributions in photon polarization operator should be taken into account. Contributions of $(t\bar{t})$ and $(W\bar{W})$ loops may be omitted in (5); they are numerically small and usually are attributed to proper electroweak radiative corrections [4]. The following integral representation for $\delta\alpha$ is valid:

$$\delta\alpha = \frac{m_Z^2}{4\pi^2\alpha}\int\frac{\sigma_{e^+e^-\to\text{all}}(s)}{m_Z^2 - s}ds \ , \tag{6}$$

where integral goes from threshold to infinity and its principal value at $s = m_Z^2$ should be taken. The lepton contribution of e, μ, and τ to (6) are readily calculated and one gets:

$$(\delta\alpha)_l = \frac{\alpha}{3\pi}[\Sigma\ln\frac{m_Z^2}{m_l^2} - \frac{5}{3}] = \frac{\alpha}{3\pi}[22.5 + 11.8 + 6.2] = 0.0314 \ . \tag{7}$$

For the hadronic contribution in [5] the following number was obtained (see also [6]):

$$(\delta\alpha)_h = 0.0282(9) \ . \tag{8}$$

To obtain this number the experimental cross-section for e^+e^--annihilation into hadrons below $s_0 = (40\text{GeV})^2$ and parton model result above s_0 was used in [5] and [6].

The difficulty in the theoretical determination of $(\delta\alpha)_h$ comes from its logarithmic dependence on the infrared cutoff. As it was mentioned in [7], the result of the dispersion calculation of $(\delta\alpha)_h$ can be reproduced by using perturbative QCD with the following effective "quark masses":

$$m_u = 53\text{MeV} \ , \ m_d = 71\text{MeV} \ , \ m_s = 174\text{MeV} \ ,$$
$$m_c = 1.5\text{GeV} \ , \ m_b = 4.5\text{GeV} \ . \tag{9}$$

Unfortunately one can not attribute any physical meaning to these values of m_u and m_d.

Our aim here is to present simplest sensible model for $\sigma_{e^+e^-\to\text{hadrons}}$ which can simulate the result given in (8). To do this we use one physical resonance $(\rho, \omega, \varphi, J/\psi$ and $\Upsilon)$ at the beginning of spectrum and then starting from $E_i = m_i + \frac{\Gamma_i}{2}$ the QCD improved parton model continuum in each quark channel.

For resonance contribution we use Breit-Wigner formula:

$$\sigma_{ee} = \frac{3\pi\Gamma_{ee}\Gamma}{E^2[(E-m)^2 + \Gamma^2/4]} \ . \tag{10}$$

Substituting in (6), neglecting terms of the order of $(m/m_Z)^2$ and integrating from $-\infty$ to $m + \frac{\Gamma}{2}$ we obtain:

$$(\delta\alpha)_{\text{resonance}} = \frac{3}{\alpha}\frac{\Gamma_{ee}}{m}\frac{3}{4} \ . \tag{11}$$

Thus vector meson contributions into $\delta\alpha$ are:

	ρ	ω	φ	J/ψ	Υ	
$\delta\alpha$	0.00274(13)	0.00024	0.00042	0.00053	0.000045 ,	(12)

where we take into account experimental uncertainty for ρ-meson contribution as the only noticeable.

For continuum contribution we use the following formulas:

$$\sigma_{I=1} = 2\pi\frac{\alpha^2}{s}(1+\frac{\alpha_s(s)}{\pi}) \ , \tag{13}$$

$$\sigma_{I=0} = \frac{2\pi}{9}\frac{\alpha^2}{s}(1+\frac{\alpha_s(s)}{\pi}) \ , \tag{14}$$

$$\sigma_{s\bar{s}} = \frac{4\pi}{9}\frac{\alpha^2}{s}(1+\frac{\alpha_s(s)}{\pi}) \ , \tag{15}$$

$$\sigma_{c\bar{c}} = \frac{16\pi}{9}\frac{\alpha^2}{s}\sqrt{1-\frac{4m_c^2}{s}}(1+\frac{2m_c^2}{s})(1+\frac{\alpha_s(s)}{\pi}) \ , \tag{16}$$

$$\sigma_{b\bar{b}} = \frac{4\pi}{9}\frac{\alpha^2}{s}\sqrt{1-\frac{4m_b^2}{s}}(1+\frac{2m_b^2}{s})(1+\frac{\alpha_s(s)}{\pi}) \ , \tag{17}$$

where we use for $\alpha_s(s)$ the following formula:

$$\alpha_s(s) = \frac{12\pi}{(33-2n_f)\ln s/\Lambda^{(n_f)2}} \tag{18}$$

with $\alpha_s(m_Z) = 0.129(5)$ as an input (this one loop value corresponds to 0.125(5) at three loops, which is extracted from latest LEP data [2]). We take $n_f = 5$ for $s > m_\Upsilon^2$, $n_f = 4$ for $m_\Upsilon^2 > s > m_{J/\psi}^2$, $n_f = 3$ for $m_{J/\psi}^2 > s > m_\varphi^2$ and $n_f = 2$ for $m_\varphi^2 > s > (m_\rho + \Gamma_\rho/2)^2$. This corresponds to $\Lambda^{(5)} = 160\,\mathrm{MeV}, \Lambda^{(4)} = 220\,\mathrm{MeV}, \Lambda^{(3)} = 270\,\mathrm{MeV}$ and $\Lambda^{(2)} = 300\,\mathrm{MeV}$.

Substituting (13) – (18) into (6) with $m_c = m_b = 0$ we get:

	$I=1$	$I=0$	$s\bar{s}$	$c\bar{c}$	$b\bar{b}$	
$\delta\alpha$	0.01174	0.00133	0.00249	0.00741	0.00123.	(19)

Summing up contributions of (12) and (19) we get:

$$(\delta\alpha)_h = 0.0282 \ , \ \bar{\alpha} = (128.87)^{-1}. \tag{20}$$

Comparing with obtained by integrating experimental data results (8) $(\delta\alpha)_h = 0.0282$ and $\bar{\alpha} = [128.87(12)]^{-1}$ we see that agreement is astonishing. The contribution of α_s correction in (19) is rather small, $0.00087 + 0.00010 + 0.00018 + 0.00042 + 0.00006 = 0.00163$, so even if light gluino octet slow down α_s running in order to accomodate α_s values measured at quarkonium decays [8] $(\delta\alpha)_h$ will decrease by 0.0002 only.

We have to make a few comments:

(1) taking contributions $\sim \alpha_s^2$ in continuum cross-section into account and using next-to-leading order formula for $\alpha_s(s)$ we increase $(\delta\alpha)_h$ by 0.00045; negative contribution of $\sim \alpha_s^3$ term appeares to be approximately two times larger. In beauty and charm channels third loop gives much smaller contribution than second (numerically both are negligible) so we can trust our continuum calculation. In strange channel third loop contribution equals that of second, while in $I = 1$ and $I = 0$ channels it is two times larger. So below, say, 1.5GeV perturbative continuum can not be approved. Allowing physical continuum variation at the level of ±15% around tree plus one loop perturbative continuum value in the domain 1 - 2GeV we get ±0.0004 variation in $(\delta\alpha)_h$;

(2) experimental uncertainty in Γ_{ll} of vector resonances lead to $(\delta\alpha)_h$ variation of the order of 0.0002, while that in $\alpha_s(m_Z)$ – to 0.0001 variation. Both are small compared with the uncertainty 0.0009 in (8);

(3)subtracting from the ρ contribution the integral over Breit-Wigner formula from $-\infty$ to two pion threshold we diminish it by:

$$\delta\alpha_{sub} = \frac{3\Gamma_{ee}}{2\pi\alpha m_\rho} 2\arctan\Gamma_\rho/(2(m_\rho - 2m_\pi)) = 0.00017; \tag{21}$$

(4) taking into account heavy quark masses $m_c = 1.6$ GeV, $m_b = 4.7$ GeV, we decrease $(\delta\alpha)_h$ correspondingly by:

$$(\delta\alpha_h)_m = 0.00031 + 0.00008 = 0.00039; \tag{22}$$

(5) finally, at energies $E = m_i + \frac{\Gamma_i}{2}$ our model curve for $\sigma_{e^+e^-\to\text{hadrons}}$ is discontinuous. To understand $(\delta\alpha)_h$ sensitivity for the details of the model we change it in the following way: we continue ρ, ω, φ and J/ψ resonance curves up to their intersection with quarks continuum. In this way $(\delta\alpha)_h$ increases :

$$\delta(\delta\alpha)_h = 0.00051. \tag{23}$$

Subtracting from (23) sum of (22) and (21) and taking uncertainty from point (1) above for total shift we get:

$$(\delta\alpha)_h = 0.0281(4), \quad \bar{\alpha} = (128.89(6))^{-1}. \tag{24}$$

So it is evident that the value of $(\delta\alpha)_h$ is rather insensitive to the details of the model of $\sigma_{e^+e^-\to\text{hadrons}}$. More refined model which takes all known resonances in each flavour channel into account gives $(\delta\alpha)_h = 0.0275(2)$ [9].

For real progress in diminishing error in (8) systematic error in cross section of e^+e^--annihilation into hadrons in background region below 3 GeV should be improved [5, 10].

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