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ON THE PARTON INTERPRETATION OF QUARK FRAGMENTATION INTO HADRONS WITH DIFFERENT SPINS

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We calculate the spin density matrix of the hadron h created via quark fragmentation in the process $e^-e^+ \to q\bar{q} \to h + X$. In the case of $h = \Lambda$ the experimental data could possibly elucidate the problem of s-quark contribution to the spin of the Λ-hyperon, to be compared with the case of the proton ("spin crisis"). Generally we find that for hadrons with spin 1/2 the parton description, using only probabilities, works much better than for vector particles, since in the former case the non diagonal matrix elements of the hadron spin density matrix are suppressed by a small factor $p_T/(z\sqrt{s})$, where p_T is the hadron transverse momentum inside the jet.

1. Introduction

The spin content of baryons, in terms of their constituents, is far from being clearly understood. The last deep inelastic scattering experiments [1−3], probing the internal structure of polarized nucleons with polarized leptons, have upset our original simple spectroscopic picture of three valence quarks sharing the baryon spin (and other quantum numbers), while gluons and sea quarks only contribute to the baryon momentum. Orbital angular momentum, gluon and non perturbative contributions have also to be invoked and taken into account to explain the experimental data on polarized nucleon structure functions; the quarks alone do not seem to be able to account for the proton spin [4].

Any further experimental information on the internal spin structure of baryons is therefore of great importance and usefulness in our attempts of better understanding the subtleties of quark and gluon bound states. In this respect, as it has been known for some time [5] and recently reproposed [6,7], the production of Λ-particles in e^-e^+ annihilations seems to be particularly interesting. According to the $SU(6)$ quark model, the Λ-hyperon has a very simple spin-flavour wave function, in that all its spin is carried by the s-quark, while the remaining ud pair is in a $S = I = 0$ state. s-quarks in e^-e^+ annihilations ($e^-e^+ \to s\bar{s}$) at large energies are produced strongly polarized; in particular, at the Z_0 pole, the (negative) longitudinal polarization of s-quarks can be as high as $P \simeq -0.9$ and almost independent of the production angle. Fast Λ's produced in e^-e^+ annihilations may be thought as the direct result of s-quark fragmentations; moreover, their weak decays allow a precise measurement of their polarization; it is then natural to propose a study [6,7] of the correlation between the (observed) Λ polarization and the (computable) s-quark polarization to see to what extent the latter is transferred to the former.

If s-quark and Λ polarizations turned out to be uncorrelated, then a "Λ spin crisis" would add to the "proton spin crisis" [8], an intriguing result indeed. Actually, there already exist several experimental indications that it might be so in the puzzling polarization data of large p_T Λ's (and other hyperons) inclusively produced in several processes as fragments of unpolarized nucleons [9]; in all of them it appears that unpolarized quarks (inside the unpolarized nucleons) give rise to strongly polarized Λ's, what cannot be understood at the constituent level.

In this paper we consider the problem at a more general level by studying the production of a hadron h in e^-e^+ annihilations via the two-step process $e^-e^+ \to q\bar{q} \to h+X$; we relate the helicity density matrix of the h hadron, $\rho_{\lambda_h \lambda'_h}(h)$, to the helicity density matrix of the $q\bar{q}$ pair, $\rho_{\lambda_q \lambda_{\bar{q}};\lambda'_q \lambda'_{\bar{q}}}(q,\bar{q})$, computed within the Standard Model, via some "fragmentation amplitudes" $D_{\lambda_h \lambda_X;\lambda_q \lambda_{\bar{q}}}(q\bar{q} \to \Lambda + X)$. This generalizes the approach of Refs. [6,7] in that it also takes into account the non diagonal elements of the $\rho(h)$ matrix and allows for final state interactions among the $q\bar{q}$ pair; these final state interactions, normally neglected in the (successful) computation of jet cross-sections and angular distributions, might be important when more subtle quantities, like spin correlations, are involved. Our results reproduce the usual ones [6,7] when only the independent fragmentation of one quark and diagonal helicity density matrix elements are considered.

Our approach, a coherent fragmentation picture versus an incoherent one, was already proposed in a previous paper [10], for the production of vector mesons in the purely electromagnetic case (*e.g.*, $e^-e^+ \to D^* + X$, via one photon annihilation). A related experimental measurement has also been performed, with the finding of a very small value of $\rho_{1,-1}(D^*)$ [11]; the large errors, however, do not allow to draw any definite conclusion and we think that further tests of the importance of coherence effects, especially in the light of so many unexpected and subtle spin effects, are important.

For the production of spin 1/2 hadrons it turns out, however, that all non diagonal spin density matrix elements are strongly suppressed if the particles inside the jet are well collimated; more precisely, we find that $\rho_{\lambda_h \lambda'_h}(h)(h, S_h = 1/2) \sim 2p_T/(z\sqrt{s})$ for $\lambda_h \neq \lambda'_h$, where p_T is the transverse momentum of the hadron with respect to the jet (original quark) direction and $z\sqrt{s}/2$ is the hadron longitudinal momentum. For large energy spin 1/2 hadrons we recover the usual independent quark fragmentation results which seem to be well founded; surprisingly, this is not necessarily true for spin 1 vector particles [10].

In the next Section 2 we discuss our formalism and give the general expression of the matrix elements $\rho_{\lambda_h \lambda'_h}(h)$ in our scheme. In Section 3 we specialize to the case of spin $S = 1/2$, 1 hadrons; we also recall how to measure these matrix elements, *e.g.* via the angular distribution of the Λ decay products in the Λ rest frame. In the conclusions we discuss our results and the differences between different spin particles; we stress how clarifying measurements could be actually performed at LEP or LSC.

2. $e^-e^+ \to q\bar{q} \to h + X$ and the helicity density matrix of particle h

Following Ref. [10] we consider the spin state of a hadron h inclusively produced in e^-e^+ annihilation; such a process is supposed to proceed via the usual two-step process: first a $q\bar{q}$ pair is created ($e^-e^+ \to q\bar{q}$) which then annihilates into the observed hadron plus other unobserved particles ($q\bar{q} \to h + X$). Whereas the first process can be perturbatively computed within the Standard Model, the second one is essentially unknown and parameterized using phenomenological fragmentation functions. The helicity density matrix of the hadron h can then be written as

$$\rho_{\lambda_h \lambda'_h}(h) = \frac{1}{N_h} \sum_{q,X,\lambda_X,\lambda_q,\lambda_{\bar q},\lambda'_q,\lambda'_{\bar q}} D_{\lambda_h \lambda_X;\lambda_q \lambda_{\bar q}}\, \rho_{\lambda_q \lambda_{\bar q};\lambda'_q \lambda'_{\bar q}}(q,\bar q)\, D^*_{\lambda'_h \lambda_X;\lambda'_q \lambda'_{\bar q}} \tag{2.1}$$

where $\rho_{\lambda_q \lambda_{\bar q};\lambda'_q \lambda'_{\bar q}}(q,\bar q)$ is the helicity density matrix of the $q\bar q$ state:

$$\rho_{\lambda_q \lambda_{\bar q};\lambda'_q \lambda'_{\bar q}}(q,\bar q) = \frac{1}{N_{q\bar q}} \sum_{\lambda_{e^-},\lambda_{e^+},\lambda'_{e^-},\lambda'_{e^+}} M_{\lambda_q \lambda_{\bar q};\lambda_{e^-} \lambda_{e^+}}(e^-,e^+)\cdot$$

$$\cdot M^*_{\lambda'_q \lambda'_{\bar q};\lambda'_{e^-} \lambda'_{e^+}}. \tag{2.2}$$

The M's are the helicity amplitudes for the $e^- e^+ \to q\bar q$ process and the D's are the fragmentation *amplitudes* for the process $q\bar q \to h + X$; they are related to the usual fragmentation *functions* D_q^h by:

$$\sum_{X,\lambda_X,\lambda_h,\lambda_q,\lambda_{\bar q},\lambda'_q,\lambda'_{\bar q}} D_{\lambda_h \lambda_X;\lambda_q \lambda_{\bar q}}\, \rho_{\lambda_q \lambda_{\bar q};\lambda'_q \lambda'_{\bar q}}\, D^*_{\lambda_h \lambda_X;\lambda'_q \lambda'_{\bar q}} = D_q^h. \tag{2.3}$$

In Eq. (2.1) a sum is performed over all quark flavours q. In Eqs. (2.1) and (2.3) the $\sum_{X,\lambda_X}$ stays for the phase space integration and the sum over spins of all the unobserved particles, grouped into a state X. The normalization factors N_h and $N_{q\bar q}$ are such that $\mathrm{Tr}(\rho) = 1$; in particular

$$N_h = \sum_q D_q^h. \tag{2.4}$$

At last, the spin state of the initial $e^- e^+$ system is described by the helicity density matrix $\rho_{\lambda_{e^-} \lambda_{e^+};\lambda'_{e^-} \lambda'_{e^+}}(e^-,e^+)$. For unpolarized e^- and e^+ one has

$$\rho_{\lambda_{e^-} \lambda_{e^+};\lambda'_{e^-} \lambda'_{e^+}}(e^-,e^+) = \frac{1}{4}\, \delta_{\lambda_{e^-} \lambda'_{e^-}}\, \delta_{\lambda_{e^+} \lambda'_{e^+}}. \tag{2.5}$$

Eq. (2.1) differs from the usual independent quark fragmentation approach in that the hadronization process is $q\bar q \to h + X$ rather than $q \to h + X$; the (necessary) $q\bar q$ interactions are taken into account in the fragmentation amplitudes. Indeed, if one assumes the $D_{\lambda_h \lambda_X;\lambda_q \lambda_{\bar q}}$ to be independent of $\bar q$ and ignores the quantum numbers of $\bar q$ and all its fragmentation products (so that one can set $\lambda_{\bar q} = \lambda'_{\bar q}$ in $\rho(q,\bar q)$ and uses $\sum_{\lambda_{\bar q}} \rho_{\lambda_q \lambda_{\bar q};\lambda'_q \lambda_{\bar q}} = \rho_{\lambda_q \lambda'_q}(q)$), Eq. (2.1) gives

$$\rho_{\lambda_h \lambda'_h}(h) = \frac{1}{N_h} \sum_{q,X,\lambda_X,\lambda_q,\lambda'_q} D_{\lambda_h \lambda_X;\lambda_q}\, \rho_{\lambda_q \lambda'_q}(q)\, D^*_{\lambda'_h \lambda_X;\lambda'_q}. \tag{2.6}$$

Moreover, if the final hadron momentum is parallel to the quark one, angular momentum conservation in the forward direction requires, for each $D_{\lambda_h \lambda_X;\lambda_q}(q \to \to h + X)$,

$$\lambda_h + \lambda_X = \lambda_q. \tag{2.7}$$

Remembering also that, when neglecting quark masses, the helicity density matrix of a quark produced in $e^-e^+ \to q\bar{q}$ annihilation is diagonal ($\rho_{\lambda_q \lambda'_q}(q) = 0$ for $\lambda_q \neq \lambda'_q$) we have that only the diagonal terms of Eq. (2.6) survive, giving the usual probabilistic formula [5]

$$\rho_{\lambda_h \lambda_h}(h) = \frac{1}{N_h} \sum_{q,\lambda_q} \rho_{\lambda_q \lambda_q}(q) D^{h,\lambda_h}_{q,\lambda_q}, \tag{2.8}$$

where $D^{h,\lambda_h}_{q,\lambda_q} = \sum_{X,\lambda_X} |D_{\lambda_h,\lambda_X;\lambda_q}|^2$.

This needs not be true in general [10], as it can be seen from Eqs. (2.1,2). The $q\bar{q}$ helicity density matrix can be computed from the knowledge of $\rho(e^-, e^+)$ (Eq. (2.5), for unpolarized beams) and the center of mass annihilation amplitudes $M_{\lambda_q \lambda_{\bar{q}};\lambda_{e^-} \lambda_{e^+}}$. At lowest order in the perturbative Standard Model, neglecting quark masses and including both the weak (Z_0) and the electromagnetic (γ) contributions, they are given, for unpolarized initial leptons (l) and in terms of the usual Standard Model parameters [12], by

$$\begin{aligned} M_{\lambda_q \lambda_{\bar{q}};\lambda_{l^-} \lambda_{l^+}}(s,\theta) &= i\delta_{\lambda_{l^-},-\lambda_{l^+}} \delta_{\lambda_{\bar{q}},-\lambda_q} \cdot \\ \cdot\Big\{\Big[e_q e^2 - \frac{1}{4} g_Z^2 c_V^l c_V^q &\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}\Big](1 + 4\lambda_{l^-}\lambda_q \cos\theta) + \\ &+\frac{1}{4} g_Z^2 \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}\big[2c_V^l c_A^q(\lambda_{l^-} \cos\theta + \lambda_q) + \\ &+2c_A^l c_V^q(\lambda_{l^-} + \lambda_q \cos\theta) - c_A^l c_A^q(\cos\theta + 4\lambda_{l^-}\lambda_q)\big]\Big\}, \end{aligned} \tag{2.9}$$

where $\sqrt{s}$ is the l^-l^+ c.m. energy, θ the q production angle and e_q the quark charge. From Eqs. (2.2), (2.5) and (2.9) it can be seen that the only non zero elements of $\rho_{\lambda_q \lambda_{\bar{q}};\lambda'_q \lambda'_{\bar{q}}}(q,\bar{q})$ are

$$\begin{aligned} \rho_{+-;+-}(q\bar{q}) &= 1 - \rho_{-+;-+}(q\bar{q}), \\ \rho_{+-;-+}(q\bar{q}) &= \rho^*_{-+;+-}(q\bar{q}). \end{aligned} \tag{2.10}$$

Inserting Eq. (2.10) into Eq. (2.1) obtains

$$\begin{aligned} \rho_{\lambda_h \lambda'_h}(h) = \frac{1}{N_h} \sum_{X,\lambda_X} \Big\{ &\Big[D_{\lambda_h \lambda_X;+-} D^*_{\lambda'_h \lambda_X;+-} - D_{\lambda_h \lambda_X;-+} D^*_{\lambda'_h \lambda_X;-+}\Big] \rho_{+-;+-} + \\ &+ \Big[D_{\lambda_h \lambda_X;+-} D^*_{\lambda'_h \lambda_X;-+} + D_{\lambda_h \lambda_X;-+} D^*_{\lambda'_h \lambda_X;+-}\Big] \mathrm{Re}[\rho_{+-;-+}] + \\ &+i \Big[D_{\lambda_h \lambda_X;+-} D^*_{\lambda'_h \lambda_X;-+} - D_{\lambda_h \lambda_X;-+} D^*_{\lambda'_h \lambda_X;+-}\Big] \mathrm{Im}[\rho_{+-;-+}] + \\ &+ D_{\lambda_h \lambda_X;-+} D^*_{\lambda'_h \lambda_X;-+} \Big\} \end{aligned} \tag{2.11}$$

The above expression of $\rho_{\lambda_h \lambda'_h}(h)$ can be further simplified by exploiting the fact that the hadronization process ($q\bar{q} \to h + X$) is parity invariant; the parity relations for c.m. helicity amplitudes [13] then yield

$$\sum_{\lambda_X} D_{\lambda_h \lambda_X;\lambda_q \lambda_{\bar{q}}} D^*_{\lambda'_h \lambda_X;\lambda'_q \lambda'_{\bar{q}}} = (-1)^{2S_h - \lambda_h - \lambda'_h} D_{-\lambda_h \lambda_X;-\lambda_q -\lambda_{\bar{q}}} D^*_{-\lambda'_h \lambda_X;-\lambda'_q -\lambda'_{\bar{q}}}. \tag{2.12}$$

It appears from Eq. (2.11) that the non diagonal elements of $\rho(q\bar{q})$, usually neglected in the independent quark fragmentation scheme, contribute both to $\rho_{\lambda_h\lambda_h}(h)$ and $\rho_{\lambda_h\lambda'_h}(h)\,(\lambda_h \neq \lambda'_h)$; in particular the latter can be different from zero. However, even if the fragmentation amplitudes $D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}$ are essentially unknown, we know from experiment that the hadron production, at lowest perturbative order in e^-e^+ annihilation, proceeds via the creation of two collimated jets of particles, each of which retains the original q and $\bar{q}$ direction. The $D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}$ can then be regarded as center of mass helicity amplitudes for the process $q\bar{q} \to h + X$, *essentially in the forward direction*, that is with the hadron momentum h almost parallel to the quark one , q (and $\bar{\mathrm{q}}$ parallel to X).

From the well known forward behaviour of c.m. helicity amplitudes [13] then we have

$$D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}} D^*_{\lambda'_h\lambda_X;\lambda'_q\lambda'_{\bar{q}}} \sim \left(\sin\frac{\theta_h}{2}\right)^{|\lambda_h-\lambda_X-\lambda_q+\lambda_{\bar{q}}|+|\lambda'_h-\lambda_X-\lambda'_q+\lambda'_{\bar{q}}|} \tag{2.13}$$

where θ_h is the angle between the hadron momentum, $\mathbf{h} = z\mathbf{q} + \mathbf{p}_T$, and the quark momentum q, that is

$$\sin\theta_h = \frac{2p_T}{z\sqrt{s}}, \tag{2.14}$$

where we have used $|\mathbf{q}| = \sqrt{s}/2$.

The bilinear combinations of fragmentation amplitudes contributing to $\rho(h)$ are then not suppressed by powers of $(p_T/(z\sqrt{s}))$ only if the exponents in Eq. (2.13) are zero; which entails

$$\lambda_h - \lambda'_h = (\lambda_q - \lambda_{\bar{q}}) - (\lambda'_q - \lambda'_{\bar{q}}). \tag{2.15}$$

Eqs. (2.11)–(2.14) hold in general, for the production of any hadron h with spin S_h. In the next Section we specialize them to the cases $S_h = 1/2$ and $S_h = 1$.

3. $e^-e^+ \to q\bar{q} \to h + X$, $S_h = 1/2, 1$

From Eqs. (2.11) and (2.12) we have, for the helicity density matrix of hadrons with spin $S_h = 1/2$

$$\begin{aligned}\rho_{++}(S_h = 1/2) = \frac{1}{N_h}\sum_{X,\lambda_X,q} \Big\{ &|D_{+\lambda_X;-+}|^2 + \left[|D_{+\lambda_X;+-}|^2 - |D_{+\lambda_X;-+}|^2\right]\rho_{+-;+-} + \\ &+2\mathrm{Re}\left[D_{+\lambda_X;+-}D^*_{+\lambda_X;-+}\right]\mathrm{Re}[\rho_{+-;-+}] - \\ &-2\mathrm{Im}\left[D_{+\lambda_X;+-}D^*_{+\lambda_X;-+}\right]\mathrm{Im}[\rho_{+-;-+}]\Big\},\end{aligned} \tag{3.1}$$

$$\begin{aligned}\mathrm{Re}[\rho_{+-}(S_h = 1/2)] = \frac{1}{N_h}\sum_{X,\lambda_X,q} \Big\{ &\mathrm{Re}\left[D_{+\lambda_X;+-}D^*_{-\lambda_X;+-}\right](2\rho_{+-;+-} - 1) - \\ -\mathrm{Im}&\left[D_{+\lambda_X;+-}D^*_{-\lambda_X;-+} - D_{+\lambda_X;-+}D^*_{-\lambda_X;+-}\right]\mathrm{Im}[\rho_{+-;-+}]\Big\},\end{aligned} \tag{3.2}$$

$$\begin{aligned}\mathrm{Im}[\rho_{+-}(S_h = 1/2)] = \frac{1}{N_h}\sum_{X,\lambda_X,q} \Big\{ &\mathrm{Im}\left[D_{+\lambda_X;+-}D^*_{-\lambda_X;+-}\right] + \\ +\mathrm{Im}&\left[D_{+\lambda_X;+-}D^*_{-\lambda_X;-+} + D_{+\lambda_X;-+}D^*_{-\lambda_X;+-}\right]\mathrm{Re}[\rho_{+-;-+}]\Big\}.\end{aligned} \tag{3.3}$$

However, in this case, none of the non diagonal bilinear combinations appearing in the above equations satisfy Eq. (2.15). That is, in the $p_T/(z\sqrt{s}) \to 0$ limit, we recover the usual probabilistic independent quark fragmentation results (2.8)

$$\rho_{++}(S_h = 1/2) = \frac{1}{N_h} \sum_q \left[\rho_{++}(q) D^{h,+}_{q,+} + \rho_{--}(q) D^{h,+}_{q,-}\right],$$

$$\rho_{+-}(S_h = 1/2) = 0, \tag{3.4}$$

where we have used $\rho_{+-;+-}(q\bar{q}) = \rho_{++}(q)$. Notice that the corrections to Eq. (3.4) are of order $(p_T/(z\sqrt{s}))^2$ for ρ_{++} and $(p_T/(z\sqrt{s}))$ for ρ_{+-}.

The same conclusion does not hold for the production of vector particles, $S_h = 1$ [10]. In such case Eq. (2.15) can be satisfied by $\lambda_h = -\lambda'_h = 1$, $\lambda_q - \lambda_{\bar{q}} = \lambda'_{\bar{q}} - \lambda'_q = 1$ and, even in the $(p_T/(z\sqrt{s})) \to 0$ limit, one remains with non zero non diagonal matrix elements:

$$\mathrm{Re}[\rho_{1,-1}(S_h = 1)] = \frac{1}{N_h} \sum_{X,\lambda_X,q} D_{1\lambda_X;+-} D^*_{-1\lambda_X;-+} \mathrm{Re}[\rho_{+-;-+}],$$

$$\mathrm{Im}[\rho_{1,-1}(S_h = 1)] = \frac{1}{N_h} \sum_{X,\lambda_X,q} D_{1\lambda_X;+-} D^*_{-1\lambda_X;-+} \mathrm{Im}[\rho_{+-;-+}]. \tag{3.5}$$

The helicity density matrix elements can be measured by looking at the two-body decay of the hadron h in its helicity rest frame, $h \to A + B$. Let us consider as two most common examples the decays of a spin 1/2 hadron into a spin 1/2 + a spin 0 (e.g. $\Lambda \to p\pi^-, \Sigma^+ \to p\pi^0, \Lambda_c \to \Lambda\pi^+$) and of a spin 1 into two spin 0 (e.g. $\rho \to \pi\pi, K^* \to K\pi, D^* \to D\pi$). In the former case the normalized angular distribution of the final particle A (e.g. p, for $\Lambda \to p\pi$ decay) is given by

$$W(\theta_A, \varphi_A) = \frac{1}{2\pi}\Big\{\frac{1}{2} - \frac{1}{2}\alpha\cos\theta_A + \alpha\rho_{++}(S_h = 1/2)\cos\theta_A +$$
$$+\alpha\mathrm{Re}[\rho_{+-}(S_h = 1/2)]\sin\theta_A\cos\varphi_A -$$
$$-\alpha\mathrm{Im}[\rho_{+-}(S_h = 1/2)]\sin\theta_A\sin\varphi_A\Big\}, \tag{3.6}$$

where θ_A and φ_A are respectively the polar and azimuthal angle of particle A in the rest frame of the decaying spin 1/2 hadron; α is the known weak decay parameter (e.g., $\alpha = 0.642 \pm 0.013$ for $\Lambda \to p\pi^-$ decay).

In the case of a spin 1 $\to$ spin 0 + spin 0 decay one has

$$W(\theta_A, \varphi_A) = \frac{3}{4\pi}\Big\{\frac{1}{2}(1 - \rho_{0,0}) + \frac{1}{2}(3\rho_{0,0} - 1)\cos^2\theta_A -$$
$$-\frac{1}{\sqrt{2}}\sin 2\theta_A\cos\varphi_A\mathrm{Re}[\rho_{1,0} - \rho_{0,-1}] +$$
$$+\frac{1}{\sqrt{2}}\sin 2\theta_A\sin\varphi_A\mathrm{Im}[\rho_{1,0} - \rho_{0,-1}] -$$
$$-\sin^2\theta_A\cos 2\varphi_A\mathrm{Re}[\rho_{1,-1}] + \sin^2\theta_A\sin 2\varphi_A\mathrm{Im}[\rho_{1,-1}]\Big\}. \tag{3.7}$$

In the case in which the production process of the hadron h is parity invariant ($e^-e^+ \to h + X$ at $\sqrt{s} \ll M_Z$, when weak effects can be neglected) we have further parity relations between the matrix elements of $\rho(h)$. Eqs. (3.6) and (3.7) then simplify respectively to

$$W(\theta_A, \varphi_A) = \frac{1}{2\pi}\left\{\frac{1}{2} - \alpha \mathrm{Im}[\rho_{+-}] \sin\theta_A \sin\varphi_A\right\}, \tag{3.8}$$

$$W(\theta_A, \varphi_A) = \frac{3}{4\pi}\Big\{\frac{1}{2}(1 - \rho_{0,0}) + \frac{1}{2}(3\rho_{0,0} - 1)\cos^2\theta_A - \\ -\sqrt{2}\sin 2\theta_A \cos\varphi_A \mathrm{Re}[\rho_{1,0}] - \sin^2\theta_A \cos 2\varphi_A \rho_{1,-1}\Big\}. \tag{3.9}$$

A measurement of the spin density matrix elements of D^* mesons produced in e^-e^+ annihilation at $\sqrt{s} = 29$ GeV, according to Eq. (3.9), has been performed [11]. The data seem to be consistent with zero values of $\rho_{\lambda\lambda'}(D^*)$ $(\lambda \neq \lambda')$, but the large errors do not allow yet any definite conclusion.

4. Conclusions

The study of spin properties of hadrons produced in e^-e^+ annihilations should supply a rich and interesting ground for understanding the relationship between the spin of the constituents and that of the composite mesons and baryons. Such understanding is at the moment, after the so called "proton spin crisis" triggered by DIS experiments, rather confused and in need of much better theoretical and experimental information.

In the large energy e^-e^+ experiments which are being performed both at LEP and SLC, quarks are always produced, via electro-weak interactions, with a large, well known polarization; to what extent this is transferred to the final observed hadrons is an open question which should be addressed and hopefully answered in the near future. The usual approach is a simple probabilistic scheme, according to which a polarized quark fragments independently into the final hadron, and our ignorance on the hadronization process is hidden in phenomenological fragmentation functions which should be measured via inclusive $e^-e^+ \to h + X$ cross-sections.

Such a scheme is rather successful when measuring jet angular distributions and unpolarized cross-sections, in that the amount and directions of the original quarks reflect accurately into the amount and directions of the observed hadrons. This might not be true when more subtle quantities, like spin observables, are involved; indeed, many mysterious spin effects in several physical processes prove to be a challenge for the existing theories. The reason is that spin variables often involve subtle interference effects between different amplitudes rather than simple squared moduli, like unpolarized cross-sections.

In this paper we have considered a more general scheme, namely $e^-e^+ \to q\bar{q}$ and $q\bar{q} \to h + X$, as a two-step process to describe hadron production in e^-e^+ annihilations. The second part, $q\bar{q} \to h + X$, is treated as an effective two-body into two-body process which takes into account the $q\bar{q}$ interactions and the full spin states of the initial $q\bar{q}$ system; these can be easily computed within the perturbative Standard Model.

Our general result for the spin density matrix of the hadron h is given by Eq. (2.11) and it differs from the usual independent quark fragmentation; for example, it allows non zero non diagonal matrix elements. However, when exploiting further experimental information, like the narrowness of the observed jets, it turns out that many of the contributions to Eq. (2.11) are small and negligible at large energies. Surprisingly, whether one recovers the usual probabilistic results or not, depends on the spin of the final hadron h: for spin 1/2 particles this is indeed the case, whereas for spin 1 vector particles one can still remain with non diagonal matrix elements. Only a direct measurement of these matrix elements would definitely settle the question.

The fact that for spin 1/2 hadrons the independent fragmentation of one quark turns out to be a well founded model is an encouraging information in view of the proposed analysis of the Λ-hyperon polarization: to a good approximation a fast Λ produced in e^-e^+ annihilation is generated by a strongly polarized s-quark. Its independent fragmentation into the Λ should certainly determine the spin of the latter; if entirely, that is with the s-quark and the Λ polarization equal, or only partially, is something we do not know and we have to investigate. The answer depends on the details and subtleties of the Λ wave function and might not be trivial, as the proton spin crisis has taught us; having reached the conclusion that $q\bar{q}$ interactions should not affect the analysis is already a first reassuring step in the difficult and hard task in front of us.

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