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$D=3$ CHERN – SIMONS – HIGGS SUSY SYSTEM WITH ANOMALOUS MAGNETIC MOMENTUM

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We investigate D=3 Chern – Simons – Higgs SUSY system with anomalous magnetic momentum and effects of spontaneous gauge and supersymmetry breaking.

Low - dimensional models of field theories intensively investigated at present are sufficiently attractive for many reasons. Apart from a pure pedagogical interest the dimensional lowering gives an intristic simplification of the problem and allows one to achive faster understanding of principal questions, moreover, the models of such kind are able to have concrete physical application as , for example, in the case of the fractional quantum Hall effect.

The questions connected with particle gauge interaction having anomalous magnetic momentum in $D=3$ space – time dimensions and the construction of the superfield Chern – Simons theory have been discussed earler in series of papers (see, for example, [1–3]). This paper is an attempt to throw the bridge between such sort formulations and supersymmetry breaking in the supersymmetric Chern – Simons field theory with anomalous magnetic momentum.

Suggested master action of this type theory has the following form:

$$S=\int d^3x\, d^2\theta\, (\bar{\tilde{\nabla}}_\alpha\bar{\Phi}\nabla^\alpha\Phi - \mathrm{Im}\Phi\bar{\Phi} - \frac{1}{2}\bar{W}_\alpha W^\alpha + \frac{1}{4}k\bar{\Gamma}_\alpha W^\alpha) \qquad (1)$$

Here and further we shall use the notation given in ref. [2], viz.:

$$\Phi(x,\theta)=\phi(x)+i\bar{\theta}\psi(x)+i\bar{\theta}\theta F(x)$$

where $\phi(x), \psi^\alpha(x), F(x)$ are the complex fields in the $D=3$ Wess – Zumino superfield multiplet;

$$\nabla^\alpha\Phi=(D^\alpha+e\Gamma^\alpha)\Phi,\quad \tilde{\nabla}^\alpha\bar{\Phi}=(D^\alpha-e\Gamma^\alpha)\bar{\Phi};$$

$$D^\alpha=(\partial/\partial\bar{\theta}_\alpha)+i(\gamma^l\theta)^\alpha\partial_l;\bar{D_\alpha}=\varepsilon_{\alpha\beta}D^\beta$$

are the spinorial covariant derivatives;

$$\Gamma^\alpha(x,\theta)=\bar{\theta}_\beta(\gamma^m)^{\beta\alpha}v_m+i\bar{\theta}\theta\lambda^\alpha$$

is the vector gauge supermultiplet in the WZ gauge;

$$W^\alpha=-\frac{i}{2}\bar{D^\beta}D^\alpha\Gamma_\beta$$

is the irreducible submultiplet of gauge fields of the theory.

Thus, action (1) contains into itself the kinetic and mass terms of the Wess – Zumino multiplet interacting with the gauge Maxwell field, the kinetic term of gauge field and the superfield analog of Chern – Simons term.

After integrating over grassmanian variables action (1) turns into the following:

$$S = \int d^3x\,(\bar{F}F - (\partial_m - iev_m)\phi(\partial^m + iev^m)\bar{\phi} + i\bar{\psi}_\alpha(\gamma^m)^\alpha{}_\beta(\partial_m - iev_m)\psi^\beta +$$

$$+ e(\bar{\psi}_\alpha\lambda^\alpha\phi + \bar{\lambda}_\alpha\psi^\alpha\bar{\phi}) + m(\bar{F}\phi + F\bar{\phi}) - \frac{i}{2}m\bar{\psi}_\alpha\psi^\alpha - \quad (2)$$

$$-\frac{i}{2}\bar{\lambda}_\alpha(\gamma^m)^\alpha{}_\beta\partial_m\lambda^\beta - \frac{1}{4}f_{mn}f^{mn} + \frac{i}{4}k\bar{\lambda}_\beta\lambda^\beta + \frac{1}{2}k\varepsilon^{kln}v_k\partial_l v_n)$$

where $f_{mn} = \partial_m v_n - \partial_n v_m$.

For investigation of the spontaneous $U(1)$ gauge symmetry breaking let us introduce the term describing the matter field self – interaction

$$-i\frac{1}{2}g(\Phi\bar{\Phi})^2 \quad (3)$$

Then after eliminating the auxiliary fields F and $\bar{F}$ we get a system with the following depending on ϕ part of potential:

$$V(\phi) = |\phi|^2(\frac{1}{2}g|\phi|^2 + m)^2 \quad (4)$$

having a minimum under nonzero vacuum expectation of $|\phi|^2$

$$< |\phi|^2 > = -2m/g \equiv \mu \quad (5)$$

The form of potential (4) is in some sense typical for the $D = 3$ Chern – Simons – Higgs system and was reproduced earler in ref.[2] starting from self – dual requirements.

The presence in the theory of anomalous magnetic momentum modifies the interaction scheme from minimal, the field – current type, to unminimal. In this connection the vector – potential $v_m \rightarrow v_m + l\varepsilon_{mnl}f^{nl}$, where ε_{mnl} is the Levi – Chivita tensor, and, consequently, the vector covariant derivative $D_m = \partial_m - iev_m$ turns into $\tilde{D}_m = \partial_m - iev_m - il\varepsilon_{mnl}f^{nl}$. The key moment of our resoning will be a consideration of the theory with modified covariant vector derivative of the form

$$\tilde{\partial}_m = \partial_m - il\varepsilon_{mnl}f^{nl} \quad (6)$$

and the standard vector – potential v_m. In this connection it is very important to note that the superalgebra does not change, i.e.

$$\{\bar{\tilde{D}}_\alpha, \tilde{D}^\beta\} = 2i(\gamma^m)_\alpha{}^\beta\tilde{\partial}_m \quad (7)$$

where $\tilde{D}^\beta = \partial/\partial\bar{\theta}_\beta + i(\gamma^m\theta)^\beta\tilde{\partial}_m$

After corresponding modification action (2) has the following form:

$$S = \int d^3x\,\{\phi D_m \bar{D^m}\bar{\phi} + (il\phi f_m \bar{D^m}\bar{\phi} + h.c.) + l^2\phi f_m f^m\bar{\phi} +$$

$$+i\bar{\psi}_\alpha(\gamma^m)^\alpha{}_\beta D_m\psi^\beta - \frac{i}{2}m\bar{\psi}_\alpha\psi^\alpha + l\bar{\psi}_\alpha(\gamma^m)^\alpha{}_\beta f_m\psi^\beta - \frac{i}{2}\bar{\lambda}_\alpha(\gamma^m)^\alpha{}_\beta\partial_m\lambda^\beta +$$

$$+\frac{i}{4}k\bar{\lambda}_\alpha\lambda^\alpha - \frac{1}{4}f_{mn}f^{mn} + \frac{1}{2}k\varepsilon^{mnk}v_k\partial_m v_n + V(\psi,\phi)\} \quad (8)$$

where $f_m = \varepsilon_{mnk}f^{nk}$, $D_m = \partial_m - iev_m$ and

$$V(\phi,\psi) = |\phi|^2(\frac{1}{2}g|\phi|^2 + m)^2 - \frac{1}{4}ig(\psi_\alpha\psi^\alpha\bar{\phi}^2 - 4\bar{\psi}_\alpha\psi^\alpha\phi\bar{\phi} + \bar{\psi}_\alpha\bar{\psi}^\alpha\phi^2)$$

Spontaneous $U(1)$ symmetry breaking for the value of superpotential

$$V(\phi,\psi)|_{min} = V(\mu,0)$$

reduces to the following effective action for the vector gauge field:

$$S = \int d^3x\,[-(\frac{1}{4} - \frac{l^2}{2}\mu)f_{mn}{}^2 + (2le^2\mu + \frac{1}{2}k)\varepsilon^{mnl}v_m\partial_n v_l + \mu e^2 v_m v^m] \quad (9)$$

Conclusions made from this are analogous to those of ref.[3] , viz.: if Chern – Simons term is absent in the original action it arises as a result of spontaneous $U(1)$ symmetry breaking; it has two critical points of the parameter μ that drastically changes the dynamics of vector field. One of them is $\mu = 1/2l^2$ where the Maxwell kinetic term vanishes. Under $\mu > 1/2l^2$ this term obtains the irregular sign and leads to the unitarity breaking in this sector. If Chern – Simons term exists in the action before spontaneous breaking it has other critical point $\mu = -k/4le^2$ where CS term vanishes and usual action for the massive vector field remains.

Now consider the spontaneous supersymmetry breaking in the theory of such kind. It appears more naturally in the case of $N = 2$ SUSY where it is possible to introduce the "chiral" superfields and consider the formulation of Ogievetsky – Sokatchev type with unconstrained prepotentials. Such trick is difficultly realised for $N = 1$ theories, but, as is shown in ref.[4], $N = 2$ SUSY action may be copy out in terms of $N = 1$ superfields in the following form:

$$S = S_0 + \int d^3x\,d^2\theta\,\bar{\theta}\theta(\frac{1}{4}kB^2 + \xi B) \quad (10)$$

where S_0 is the action (1). Presence of the additional field B, which may be introduced in the case of $N = 1$ by hands, is a consequence of hidden $N = 2$ SUSY and allows to enter into the action the Fayet – Iliopoulos term.

From the equation of motion over B

$$\frac{1}{2}kB + \xi = 0 \quad (11)$$

we obtain that B has the nonzero vacuum expectation

$$< B > = -2\xi/k \quad (12)$$

and it leads to the non-linear transformation law for the vector field superpartner which becomes the Goldstone fermion (see,for example,ref.[5]).

It is important to note that in the breaking SUSY additional part of the action (10) the constant k standing at the Chern – Simons enters since by introducing of this part by hand the coefficient before B^2 may be chosen arbitrary. However choosing namely the constant k allows one to make sharp accordance between spontaneous symmetry breaking and Chern – Simons term presence: *under* $k \to 0$ *the spontaneous breaking SUSY phase does not exist.*

Of course, this conclusion is true only for a classical description. In the quantum case a possibility of dynamical SUSY breaking arises due to the quantum fluctuation of coupling fields over the vacuum state and the compensation of some terms in an effective action due to the play of constants. These and others principal questions require more detail and futher investigation.

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