Pis'ma v ZhETF, vol.60, iss.8, pp.578 - 582

QUANTUM OSCILLATIONS OF A NEW TYPE IN TWO-DIMENSIONAL ELECTRON SYSTEMS IN THE VICINITY OF THE PERCOLATION THRESHOLD

S.I.Dorozhkin[1)]

Institute of Solid State Physics RAS
Chernogolovka, Moscow district, 142432, Russia

Submitted 29 September 1994

A new type of conductivity quantum oscillations is predicted in two-dimensional electron systems at carrier concentrations close to the zero-magnetic-field percolation metal-insulator transition. The off-diagonal conductivity σ_{xy} changes nonmonotonically with magnetic field. In particular, these oscillations may be responsible for the reentrant transitions between a quantum-Hall effect state and an insulating state, which were recently observed in several two-dimensional electron systems. Influence of the Zeeman splitting on the oscillations is predicted.

It is generally accepted (see, for example, Ref. [1]) that in a high magnetic field H the energy spectrum of a two-dimensional electron system consists of magnetic levels broadened by a disorder in a sample. Degeneracy of each level is eH/hc. (Here e is the electron charge, c is the velocity of light and h is the Planck constant). In the presence of long-range potential fluctuations only (in comparison with a magnetic length $l = (\hbar c/eH)^{1/2}$) magnetic levels are inhomogeneously broadened, being locally sharp. Most of electronic states of a level are localized, only their small number in the vicinity of a level center being extended. The extended states of a magnetic level (i) provide quantized off-diagonal conductivity $\sigma_{xy} = e^2/h$ when occupied, and (ii) give rise to a non-zero diagonal conductivity σ_{xx} when they coincide with the Fermi level. Increasing of a magnetic field at a fixed carrier concentration results in a decrease of a number of the occupied magnetic levels. Then a conductivity σ_{xy} monotonically decreases changing step by step between the quantized values $\sigma_{xy} = ie^2/h$, where i is the number of magnetic levels under the Fermi level. These steps are periodic in the inverse magnetic field. In accord with the generally accepted notation the states of a two-dimensional electron system with quantized non-zero σ_{xy} are the integer quantum Hall effect states and that with $\sigma_{xy} = 0$ is an insulating state. (In both types of states $\sigma_{xx} = 0$). Transitions between different states occur via a metal phase [2] and are accompanied by peaks in the diagonal conductivity σ_{xx}.

In this paper we first consider magnetic quantization for noninteracting electrons in the case of a regular long-range potential modulation in the vicinity of the zero-magnetic-field percolation metal-insulator transition. We show that in this case a new type of quantum oscillations should appear. These oscillations result in nonmonotonic dependence of σ_{xy} on the magnetic field and, in particular, describe reentrant insulator - quantum Hall effect transitions. Such transitions were observed recently in two-dimensional electron systems of Si-MOSFETs [3-6], GaAs/AlGaAs heterostructures [7,8] and Si/SiGe heterostructures [9]. Then we argue that our results for the regular potential can also be valid for commonly used samples.

[1)]E-mail: dorozh@issp.ac.ru

To illustrate a possibility of the oscillations of a new type we start with an exactly solvable model of a long-range potential which looks like a chess-board with potential $V = 0$ and $V = 2V_0$ in white and black squares, correspondingly. The potential V varies between 0 and $2V_0$ within the transition regions which are narrow in comparison with the square size (see inset in Fig.1) but still much larger than the magnetic length. In principle, such potential can be easily produced by modern lithographic technique. In the chosen potential a zero-magnetic-field percolation threshold occurs at the Fermi energy $E_f^0 = V_0$. The extended states of the inhomogeneously broadened magnetic levels $E_n^{\pm} = (n + 1/2)\hbar\omega_c \pm g\mu H/2 + V$ are located at energies $E_{pn}^{\pm} = (n+1/2)\hbar\omega_c \pm g\mu H/2 + V_0$ [1]. Here n is an integer, $\omega_c = eH/m^*c$ is the cyclotron frequency (m^* is a carrier effective mass), g is the g-factor, and μ is the Bohr magneton. In the case of narrow transition regions we can neglect their influence on the density of states which is then approximated for each magnetic level by two peaks at energies $E_{n1}^{\pm} = (n + 1/2)\hbar\omega_c \pm g\mu H/2$ and $E_{n2}^{\pm} = E_{n1}^{\pm} + 2V_0$. Below we shall refer to these peaks as magnetic sublevels. Degeneracy of each sublevel is $eH/2hc$. Then the position of the Fermi level at the zero temperature can be easily calculated as a function of the magnetic field. The energies of the lower magnetic sublevels, those of the extended states as well as the Fermi energy are shown in Figs. 1*a* and 2*a* versus the cyclotron energy $\hbar\omega_c$, which is proportional to the magnetic field. All energies are presented in the units of V_0 . The zero-magnetic-field state is an insulator in Fig.1 ($E_f^0 < V_0$) and a metal in Fig.2. Fig.1 and Fig.2 correspond also to different Zeeman splitting (it equals to the cyclotron splitting in Fig.1 and it is zero in Fig.2). For the used value of E_f^0/V_0 we are interested in energies less than $2V_0$ and magnetic sublevels corresponding to the black squares of the chess-board are not shown. Besides, the energies of the two levels with different spins coincide except the lowest level of Fig.1. The value of σ_{xy} (Figs. 1*b* and 2*b*) was calculated as the number of the extended states under the Fermi level multiplied by e^2/h. Our results for σ_{xy} give sequences of jumps between two different σ_{xy} values as a function of the magnetic field. In particular, they describe the reentrant quantum Hall effect – insulator transition. These jumps originate from the crossing of the extended states by the Fermi level and should be accompanied by peaks in σ_{xx}. Comparison of Fig.1 and Fig.2 demonstrates the role of the spin splitting. For example, in systems with $g = 0$ (Fig.2) the transition from the zero-magnetic-field insulating state to a quantum Hall effect state is impossible. It is rather clear that the above considered effects exist only in the vicinity of the percolation threshold in the zero magnetic field. At $E_f^0 >> V_0$ we obtain usual picture of oscillations.

It is necessary to mention that the steepness of the potential in the intermediate regions is not of crucial importance for our results. It is demonstrated by Fig.3 where conductivity σ_{xy} is shown for potential $V = V_0(1 - \cos(2\pi x/a)\cos(2\pi y/a))$. Here a is a space scale of the potential variation ($a >> l$).

We can not expect existence of such regular potentials in commonly used samples. Nevertheless, we believe that the above considered effects can also exist in such samples. Indeed, the most important point for our consideration is small fluctuations of the bottoms of different potential minima in comparison with the total amplitude of potential fluctuations ($2V_0$ in our case). It has been shown in Ref. [10] that namely such potential should exist in heterostructures with thick spacer layers in the vicinity of a percolation metal-insulator transition. In this case the screening is nonlinear and the sample is divided into regions occupied

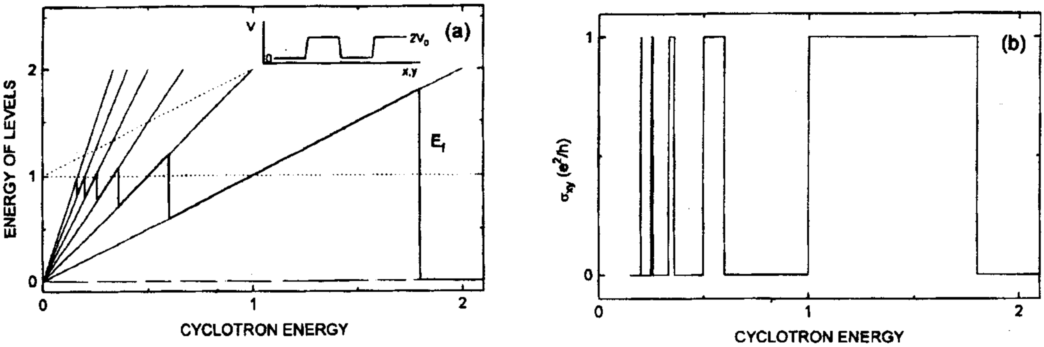

Fig.1. (*a*) Fermi energy E_f (wide solid line) versus the cyclotron energy. Lower magnetic sublevels are shown by narrow solid lines and the extended states are shown by dotted lines. The chosen value of the Fermi energy $E_f^0 = 0.9$ corresponds to the zero-magnetic-field insulating state. All the energies are given in the units of V_0 ; $g\mu H = \hbar\omega_c$. Inset: variation of the model potential with coordinates x, y in the plane of the electron system. (*b*) Calculated off-diagonal conductivity σ_{xy} versus the cyclotron energy

by electrons (electron "lakes") where potential fluctuations are screened, and those where electrons are absent and the amplitude of potential fluctuations is large. Moreover, in this case the transition regions are narrow in comparison with the electron "lake" size and the above presented consideration of magnetic quantization in the "lakes" is directly applicable. The point dependent on the total form of the potential fluctuations is the value of the percolation energy. Note that in systems with steep potential in the transition regions the self-consistent potential should be weakly dependent on the magnetic field. Indeed, the electron space distribution practically does not change with the magnetic field when the Fermi level coincides with one of the magnetic sublevels. Additional peculiarities in the screening are possible only when the exactly integer number of sublevels is occupied. But this can modify the dependence of the Fermi energy on the magnetic field only in the vicinity of the Fermi level jumps between magnetic sublevels. (We expect these jumps to be broadened).

Short-range disorder as well as finite temperature smear oscillations of the Fermi level and those of σ_{xy}. In particular, transitions between different quantum Hall effect states should occur through an intermediate metal state.

Up to date a nonmonotonic dependence of σ_{xy} on a magnetic field was observed only for transitions between different quantum Hall effect states and an insulating state. In increasing magnetic field the following sequences of σ_{xy} have been reported (in units of e^2/h): (i) (6,0,2,0,1,0) in electron channels of Si MOSFETs [3−6] where each Landau level consists of four levels corresponding to two spin orientations and two different valleys of the Si bulk energy spectrum, (ii) (0,2,0) in electron channels of GaAs/AlGaAs heterostructures [7,8], (iii) (3,0,1) in *p*-channels of Si/SiGe heterostructures [9]. No repetition of transitions between two different quantum Hall effect states was observed till now. Our results definitely show that the reentrant insulator – quantum Hall effect transitions can be naturally explained in terms of the percolation effects. Experimental results supporting the percolation mechanism of the metal-insulator transitions in Si MOSFETs have been recently

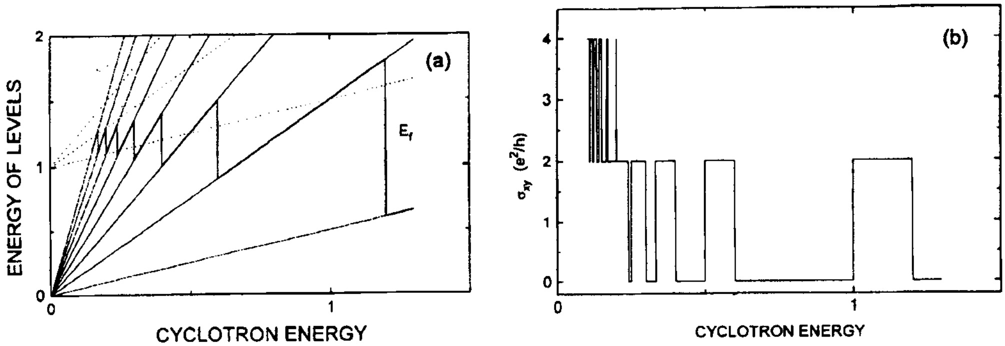

Fig.2. The same as in Fig.1 for another set of parameters: $g = 0$, $E_f^0 = 1.2$. The zero-magnetic-field state is a metal

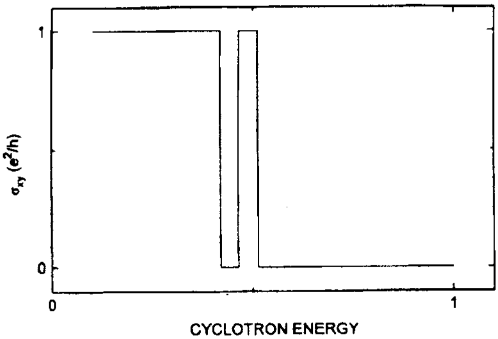

Fig.3. Variation of the conductivity σ_{xy} in the magnetic field for potential V of the form $V = V_0(1 - \cos(2\pi x/a)\cos(2\pi y/a))$; $g\mu H = \hbar\omega_c$, $E_f^0 = 1.06$

published in Ref. [2]. Our model explains the following experimental facts: (i) the reentrant behavior is observed only at carrier concentrations in the vicinity of the metal-insulator transition at $H = 0$, (ii) reentrant behavior is not observed for sweeping carrier concentration. Moreover, we predict that reentrant behavior should be much easier observed in systems with a large Zeeman splitting which is in agreement with Ref. [9]. Particularly, we predict that in electron channels of Si MOSFETs modulation of the phase boundary of low density insulating state reported in Refs. [2, 3, 6] will increase in amplitude in a tilted magnetic field.

Finally, in samples with regular modulation of the potential we made predictions of the reentrant transitions between different quantum Hall effect states and between these states and the insulating state. We argued that the same effects can exist in commonly used samples with long-range potential fluctuations. The latter statement is confirmed by recent observations of the reentrant quantum Hall effect - insulator transitions.

The author appreciates the valuable discussions with V.T.Dolgopolov, S.V.Iordansky, and V.F.Gantmakher. This work was supported in part by grant No 1010-CT-93-0051(INTAS) and by grant N 1-027 of the Russian program Physics of Solid State Nanostructures.

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