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Trapping of plasmons in ion holes

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We present analytical and numerical studies of a new electron plasma wave interaction mechanism which reveals trapping of Langmuir waves in ion holes associated with non-isothermal ion distribution functions. This Langmuir-ion hole interaction is a unique kinetic phenomenon, which is governed by two second non-linear differential equations in which the Langmuir wave electric field and ion hole potential are coupled in a complex fashion. Numerical analyses of our nonlinearly coupled differential equations exhibit trapping of localized Langmuir wave envelops in the ion hole which is either standing or moving with sub- or super ion thermal speed. The resulting ambipolar potential of the ion hole is essentially negative, giving rise to bipolar slow electric fields. The present investigation thus offers a new Langmuir wave contraction scenario that has not been rigorously explored in plasma physics.

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More than three decades ago, Hasegawa [1], Karpman [2, 3] and Zakharov [4] presented an elegant description of strong electromagnetic and Langmuir wave turbulence in which high-frequency photons and plasmons interact nonlinearly with low-frequency ion-acoustic waves via the ponderomotive force arising due to the spatial gradient of the high-frequency wave intensity. This nonlinear interaction is typically described by the two-fluid and Poisson-Maxwell equations, and the governing equations admit the localization of photon and plasmon wave packets, leading to the formation of envelope light and Langmuir wave solitons (also referred to cavitons) [5 – 8]. The latter are composed of electron/ion density depression which traps photon and Langmuir wave envelops. Moreover, Yan'kov [9] studied the response of kinetic untrapped ions in the Langmuir envelope soliton theory, and predicted the formation of sub ion thermal small-amplitude negative potential wells in plasmas. On the other hand, Mokhov and Chukbar [10] found a Langmuir envelope soliton accompanied with small-amplitude negative potential well created by localized Langmuir wave electric field in a quasi-neutral plasma with non-isothermal ions whose temperature is much smaller than the electron thermal temperature. In two and three dimensions, one encounters photon self-focusing, Langmuir wave collapse [4, 11]. The formation of cavitons has been observed in the ionosphere [12] as well as in several laboratory experiments [13 – 15].

In this Letter, we present for the first time a new Langmuir turbulent state in the presence of ion phase-space vortices [16 – 19] that are associated with density holes and bipolar electric fields in collisionless plasmas. Ion phase-space vortices are natural products of ion-beam driven two-stream instabilities, and they play a very important role in laboratory experiments [20 – 22] as well as in the near-earth plasma environment [23 – 25]. They are described by a wide class of Bernstein-Greene-Kruskal solutions to the Vlasov-Poisson equations. In the following, we show that nonlinearly coupled Langmuir waves and fully nonlinear ions holes admit a new class of solutions. Specifically, we demonstrate the existence of standing and sub-ion thermal ion holes that trap Langmuir wave envelops.

We consider an unmagnetized electron-ion plasma in the presence of Langmuir waves and large amplitude ion holes. At equilibrium, we have $n_{e0} = n_{i0} = n_0$, where n_{j0} is the unperturbed number density of the particle species j (j equals e for electrons and i for ions). The Langmuir wave frequency is $\omega = \left(\omega_{pe}^2 + 3k^2V_{Te}^2\right)^{1/2}$, where $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency, n_e is number density of electrons, e is the magnitude of the electron charge, m_e is the electron mass, k is the wavenumber, $V_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal speed, and T_e is the electron temperature. Large amplitude Langmuir waves interacting nonlinearly with ion holes generate Langmuir wave envelope whose electric field E evolves slowly (in comparison with the electron plasma wave period) according to a nonlinear Schrödinger equation

$$2i\omega_p\left(\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial x}\right)E + 3V_{Te}^2\frac{\partial^2 E}{\partial x^2} + \omega_p^2\left(1 - \frac{n_e}{n_0}\right)E = 0, \tag{1}$$

where $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the unperturbed electron plasma frequency and $v_g = 3kV_{Te}^2/\omega_p$ is the group velocity of the Langmuir waves. We note that (1) is derived by combining the electron continuity and momentum equations as well as by using Poisson's equation with fixed ions, and by retaining the arbitrary large electron number density variation n_e associated with ion holes in the presence of the Langmuir wave ponderomotive force. Assuming that the phase speed of ion holes is much smaller than the electron thermal speed, we readily obtain from the inertialess electron equation of motion the electron number density in the presence of the ponderomotive force of Langmuir waves. The result is

$$n_e = n_0 \exp[\tau(\phi - W^2)], \qquad (2)$$

where $\tau = T_e/T_i$, T_i is the ion temperature, $\phi = e\varphi/T_i$, $W^2 = |E|^2/16\pi n_0 T_i$, and φ is the electrostatic potential of the ion hole. We note that the W-term in Eq.(2) comes from the averaging of the nonlinear term $m_e \mathbf{v}_{he} \cdot \nabla \mathbf{v}_{he}$ over the Langmuir wave period $2\pi/\omega_{pe}$, where $\mathbf{v}_{he} \approx -\hat{\mathbf{x}} eE/m_e\omega_{pe}$ is the electron quiver velocity in the Langmuir wave electric field.

If the potential has a maximum $\phi_{\max} > 0$, then there exist in general trapped ions where $\phi < \phi_{\max}$, while at the point where $\phi = \phi_{\max}$ there are no trapped ions. Similar to Schamel [16], we chose at this point a displaced Maxwellian distribution for the free ions. The ion distribution function associated with ion holes can then be obtained by solving the ion Vlasov equation for free and trapped ions, which have speeds larger and smaller than $[2(\phi_{\max} - \phi)]^{1/2}$, respectively. The electric potential will turn out to be essentially negative, with only a small-amplitude positive maximum $\phi_{\max}$ compared to the large-amplitude negative potential well with a minimum at $\phi_{\min} \equiv -\psi$. Thus, the potential is restricted by $-\psi \le \phi \le \phi_{\max}$, where ψ plays the role of the amplitude. Integrating the sum of the free and trapped ion distribution functions over velocity space, we obtain the ion number density [17]

$$n_i = n_0\, b \exp\left(-\frac{M^2}{2}\right) \times \left[I(\phi_{\max} - \phi) + K\left(\frac{M^2}{2}, \phi_{\max} - \phi\right) + \frac{2}{\sqrt{\pi|\alpha|}} W_D(\sqrt{\alpha(\phi - \phi_{\max})})\right], \qquad (3)$$

where $M = V/V_{Ti}$ is the Mach number, V is the ion hole speed, $V_{Ti} = (T_i/m_i)^{1/2}$ is the ion thermal speed, m_i is the ion mass, and α is a (negative) parameter which determines the number of trapped ions. The normalization constant b is chosen so that when $\phi = 0$, the total density of ions is n_0. Furthermore, we have denoted [17] $I(x) = \exp(x)[1 - \mathrm{erf}(\sqrt{x})]$, $K(x,y) = (2/\sqrt{\pi}) \int_0^{\pi/2} \sqrt{x}\cos\theta \exp(-y\tan^2\phi + x\cos^2\theta) \times \mathrm{erf}(\sqrt{x}\cos\theta)\, d\theta$, and the Dawson integral $W_D(x) = \exp(-x^2)\int_0^x \exp(t^2)\, dt$. A plateau in the resonant region is given by $\alpha = 0$, and $\alpha < 0$ corresponds to a vortex-like excavated trapped ion distribution. For positive α, we use [26] $W_D(ix) = i(\sqrt{\pi}/2)\exp(x^2)\mathrm{erf}(x)$ (where $i = \sqrt{-1}$) and replace the term $(2/\sqrt{\pi|\alpha|})W_D[\sqrt{\alpha(\phi - \phi_{\max})}]$ in Eq.(3) by $(1/\sqrt{\alpha})\exp[-\alpha(\phi - \phi_{\max})]\mathrm{erf}[\sqrt{-\alpha(\phi - \phi_{\max})}]$; we note especially that $M = 0$, $\alpha = 1$ leads to a Boltzmann distribution $n_i = n_0 \exp(-\phi)$ for the ion density. The Langmuir wave ponderomotive force acting on ions is weaker by the electron to ion mass ratio in comparison with that acting on electrons, and therefore it is ignored in Eq.(3). The electron ponderomotive force is transmitted to ions via the ambipolar potential ϕ, which is determined from Poisson's equation

$$\tau\lambda_{De}^2 \frac{\partial^2\phi}{\partial x^2} = \frac{n_e}{n_0} - \frac{n_i}{n_0}, \qquad (4)$$

where $\lambda_{De} = (T_e/4\pi n_0 e^2)^{1/2}$ is the electron Debye length.

We are interested in quasi-steady state solutions of Eqs.(1)–(4), which are fully nonlinear. We insert $E(x,t) = W(\xi)\exp\{i[X(x) + T(t)]\}$ and $\phi(x) = \phi(\xi)$, where $\xi = x - Vt$ and $W(x)$, $X(x)$, $T(x)$ are assumed to be real, into Eqs.(1)–(4) and obtain a coupled set of the nonlinear equations

$$3\frac{\partial^2 W}{\partial\xi^2} - (\lambda - 1)W - W\exp[\tau(\phi - W^2)] = 0, \qquad (5)$$

and

$$\tau\frac{\partial^2\phi}{\partial\xi^2} - \exp[\tau(\phi - W^2)] + b\exp\left(-\frac{M^2}{2}\right) \times \left[I(-(\phi - \phi_{\max}) + K\left(\frac{M^2}{2}, -(\phi - \phi_{\max})\right) + \frac{2}{\sqrt{\pi|\alpha|}} W_D(\sqrt{\alpha(\phi - \phi_{\max})})\right] = 0, \qquad (6)$$

where ξ is normalized by λ_{De} and $\lambda = 2\omega_p^{-1}(dT/dt) - 3k^2\lambda_{De}^2(1 - V^2/v_g^2)$ represents a nonlinear frequency shift. The system of Eqs.(5) and (6) admits the first integral in the form of a Hamiltonian

$$H(W,\phi,\lambda,M) = 3\left(\frac{\partial W}{\partial \xi}\right)^2 - \frac{\tau}{2}\left(\frac{\partial \phi}{\partial \xi}\right)^2 - \\ -(\lambda-1)W^2 + \frac{1}{\tau}\{\exp[\tau(\phi - W^2)]-1\} + \\ + b\exp\left(-\frac{M^2}{2}\right)\Big[P(\phi_{\max}-\phi,\alpha) + \\ + h\left(\frac{M^2}{2},0,\phi_{\max}-\phi\right) - 1\Big] - H_0 = 0, \quad (7)$$

where in the unperturbed state ($|\xi| = \infty$) we have used the boundary conditions $W=0$, $\phi=0$, $\partial W/\partial\xi = 0$, $\partial\phi/\partial\xi = 0$. The constant H_0 is chosen so that $H = 0$ at $|\xi| = \infty$. The auxiliary functions are defined as $P(x,y) = I(x) + 2\sqrt{x/\pi}(1-y^{-1}) + (2/y\sqrt{\pi|y|})W_D(\sqrt{-xy})$ and $h(x,a,b) = \int_a^b K(x,y)\,dy$.

Because we are interested in symmetric solutions defined by $W(\xi) = W(-\xi)$ and $\phi(\xi) = \phi(-\xi)$, the appropriate boundary conditions at $\xi = 0$ are $W = W_0$, $\phi = -\psi$, $\partial W/\partial\xi = 0$, and $\partial\phi/\partial\xi = 0$. Hence, from Eq.(7) we have

$$(\lambda-1)W_0^2 - \frac{1}{\tau}\{\exp[\tau(-\psi - W_0^2)]-1\} - \\ - b\exp\left(-\frac{M^2}{2}\right)\Big[P(\phi_{\max}+\psi,\alpha) + \\ + h\left(\frac{M^2}{2},0,\phi_{\max}+\psi\right) - 1\Big] - H_0 = 0, \quad (8)$$

which shows how the maximum values of W_0 and ψ are related to M, $\phi_{\max}$ and λ for given values of τ and α. A practical application of the Hamiltonian (7) is to check the correctness of any numerical scheme used to solve Eqs.(5) and (6), while Eq.(8) depicts the parameter regimes for the existence of trapped plasmons in ion holes.

In the absence of the Langmuir waves, ion holes are governed by the energy integral [27]

$$\frac{1}{2}\left(\frac{\partial \phi}{\partial \xi}\right)^2 + \Psi(\phi, M) = 0 \quad (9)$$

where the Sagdeev potential for our purposes with $\phi_{\max} = 0$ is [17]

$$\Psi(\phi,M) = -\frac{1}{\tau}\left\{\frac{1}{\tau}\{\exp[\tau\phi]-1\} + \exp\left(-\frac{M^2}{2}\right)\times \right. \\ \left. \times\left[P(-\phi,\alpha) + h\left(\frac{M^2}{2},0,-\phi\right) - 1\right]\right\}. \quad (10)$$

Equation (9), which is obtained from Eq.(7) in the limit of vanishing Langmuir wave electric fields, determines the profile of ion holes. The latter exist provided that $\Psi(\phi)$ is negative between zero and $\pm\phi_0$. Multivalued solutions of $\Psi(0)$ are ensured provided that $\partial^2\Psi/\partial^2\phi = 0$, while at $\phi = \phi_0(-\phi_0)$, we must have $\partial\Psi/\partial\phi > 0 (< 0)$. The condition $\Psi(\phi_0, M) = 0$ gives a relation between ϕ_0 and M for given values of α and τ. It turns out that ion holes without Langmuir waves have only a negative potential, as pre-assumed earlier.

We have carried out numerical studies of the equations governing ion holes with and without Langmuir waves for $\tau = 0.1$ and $\alpha = -1.0$. First, we consider small-amplitude Langmuir waves which are not strong enough to modify the ion hole, but which can be linearly trapped in the density well of the hole. Accordingly, for $W^2 \ll 1$ Eq.(5) turns into a linear eigenvalue problem of the form $3(d^2W/d\xi^2) + [1 - \exp(\phi) - \lambda]W = 0$, with the eigenvalue λ and the corresponding eigen-function W, and where ϕ is obtained by assuming $W = 0$ in the solution of Eq.(6); see the numerical solution of Eq.(6) in the form of ion density profiles and the associated ambipolar potentials, respectively, in the upper and lower panels of Fig.1. The eigenvalue problem will have a continuous spectrum for $\lambda < 0$, corresponding to "free particles" (in the language of quantum mechanics,) and a point spectrum for $\lambda > 0$, corresponding to "trapped particles". We have investigated numerically the cases corresponding to four different Mach numbers displayed in Fig.1, and found the corresponding positive eigenvalues listed in the second column of the table below, where each eigenvalue λ is associated to a bell-shaped eigenfunction W. Only one positive eigenvalue was found for each case, and thus these cases only admit the ground states for waves to be linearly trapped.

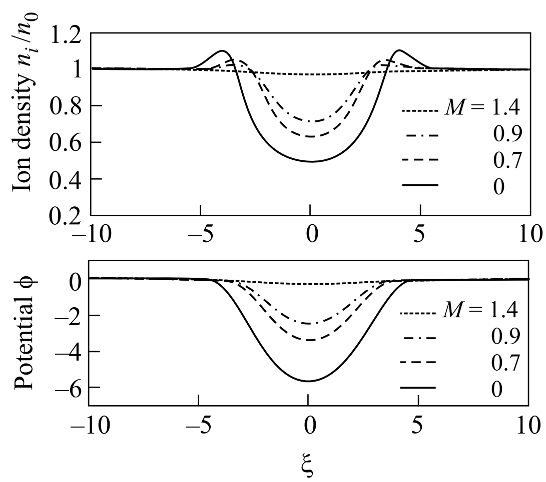

Fig.1. Ion holes without Langmuir waves ($W = 0$) for different Mach numbers M, with $\tau = 0.1$ and $\alpha = -1.0$

Small amplitude problem		Finite amplitude problem	
M	λ	M	λ
1.4	0.0013	1.4	0.1013
0.9	0.0463	0.9	0.1463
0.7	0.0772	0.7	0.1772
0.0	0.1906	0.0	0.2906

Next, we studied the presence of finite-amplitude Langmuir waves in the ion hole, in which the fully nonlinear system of equations (5) and (6) has to be solved numerically. The numerical solutions reveal that the ion hole deepened and widened, admitting the eigenvalue λ to be larger. We investigated the special case with a nonlinear shift of 0.1 of λ as listed in the fourth column in the table above, and found solutions for all cases except for $M = 1.4$; the numerical solutions are depicted in Fig.2. We can see from Figs.1 and 2 that the presence of trapped finite-amplitude Langmuir waves makes the ion density depletion both deeper and wider, and the same holds for the ambipolar potential well. The deepening of the ambipolar negative potential well is a feature closely related to the strongly non-isothermal trapped ion distribution function. For this case, the electrostatic potential had small-amplitude maxima $\phi_{\max}$ of the order $\approx 10^{-3}$ on each side of the ion hole, this maximum of the potential increased with increasing M.

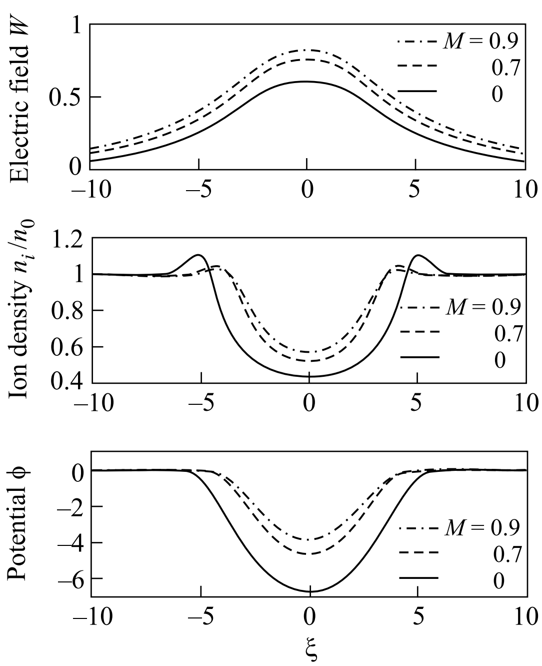

Fig.2. Ion holes in the presence of Langmuir waves for different Mach numbers M, with $\tau = 0.1$ and $\alpha = -1.0$

In order to investigate the conditions for existence of ion holes in the presence of strong Langmuir fields, we numerically solved Eq.(8) for ψ as a function of M; see Fig.3. We used the same parameters $\tau = 0.1$ and $\alpha = -1.0$ as above. Here, we assumed the Langmuir field to be given as an external parameter (say $W_0 = 0.8$) and with a nonlinear shift that follows $\lambda(M) = 0.3 - 0.14\,M$, as obtained approximately from the table above. This overestimates slightly the Langmuir field W_0 for small M and underestimates slightly the field for the highest M; see the upper panel in Fig. 2. We assumed a maximum potential of $\phi_{\max} = 0.003$. We found that for this set of parameters, the solution had an upper bound $M = 1.25$ for the existence of localized solutions, which is clearly smaller than the existence in the absence of the Langmuir fields. In a more exact mapping of the existence of ion holes one needs to explore more carefully the relationships between different parameters in Eq.(8), possibly by solving the system of equations (5) and (6) for different cases. Furthermore, the stability of the time-dependent system is not explored here, but could be studied by direct simulations of the Vlasov-Poisson system.

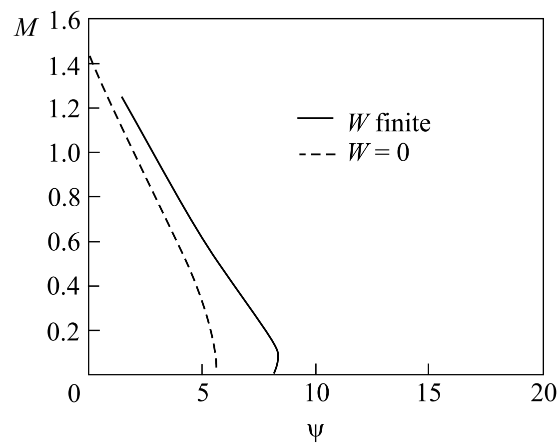

Fig.3. Numerical solutions of Eq.(8), depicting $\psi(= -\phi_{\min})$ vs M for W_0 and $W_0 = 0.8$, with $\tau = 0.1$ and $\alpha = -1.0$. We see that the ion hole loaded with the Langmuir wave electric fields has an upper bound on the Mach number which is smaller than that without the Langmuir wave fields

It should be stressed that the properties of the present Langmuir envelope solitons significantly differ from those based on Zakharov's model [4] which utilizes the fluid ion response for driven (by the Langmuir wave ponderomotive force) ion-acoustic perturbations and yield subsonic density depression accompanied with a positive localized ambipolar potential structure. Furthermore, consideration of a Boltzmann ion density distribution, viz. $n_i = n_0 \exp(-\phi)$, would correspond to the case $M = 0$ and $\alpha = 1$ in Eq.(6). Here, as shown in Fig.4, we have a localized Langmuir wave electric field

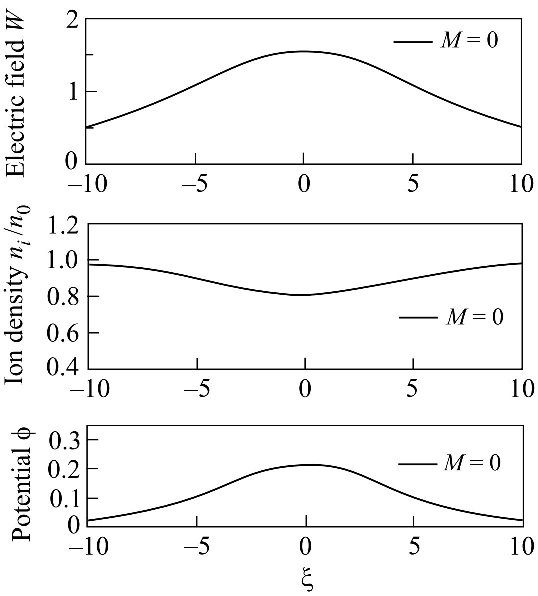

Fig.4. A Langmuir caviton with a Boltzmann ion distribution for $M = 0$, $\lambda = 0.1$, $\tau = 0.1$ and $\alpha = 1.0$

envelope trapped in a standing ion density cavity. The corresponding slow ambipolar potential is positive and localized.

In the numerical solutions of Eqs.(5) and (6), the second derivatives were approximated by a second-order centered difference scheme [28], and the values of W and ϕ were set to zero at the boundaries of the computational domain at $\xi = \pm 40$. The resulting nonlinear system of equations was solved iteratively. We used 2000 sampling points to resolve the solution.

In summary, we have presented the first analytical and numerical studies of a novel nonlinear plasma state in which the Langmuir waves interact with fully nonlinear ion holes. It is found that Langmuir waves have a dramatic effect on the ion hole in that the formation of envelope Langmuir solitons (Langmuir waves trapped in ion hole) becomes an eigenvalue problem, and only discrete eigenstates are allowed. Self-trapped Langmuir waves in ion hole are found to be either standing or moving with sub or super ion thermal speed. Ion cavity loaded with the Langmuir waves is typically wider, and are accompanied with negative localized ambipolar potential. Physically, the broadening of the ion hole and the enhancement of negative ambipolar potential occur because the ponderomotive force of the Langmuir waves locally expels electrons, which pull ions along to maintain the local charge neutrality. The deficit of ions in plasmas, in turn, produces more negative potential within the ion hole that is now widened and enlarged to trap the localized Langmuir wave electric field envelope. Hence, the properties of the ion holes in the presence of Langmuir waves are significantly different from ion holes without the Langmuir waves, or cavitons involving the fluid [2, 3] or a Boltzmann ion response. In conclusion, we stress that the present localized structures are outside the realm of the two-fluid model as they involve a trapped ion vortex state which can be dealt within the framework of a kinetic description only. We have thus solved one of the fundamental problems of the nonlinear plasma physics which has potential applications in space and laboratory plasmas that are driven by electron and ion beams.

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