

Quasiparticles in the superconducting state of high- T_c metals

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We consider the behavior of quasiparticles in the superconducting state of high- T_c metals within the framework of the theory of superconducting state based on the fermion condensation quantum phase transition. We show that the behavior coincides with the behavior of Bogoliubov quasiparticles, whereas the maximum value of the superconducting gap and other exotic properties are determined by the presence of the fermion condensate. If at low temperatures the normal state is recovered by the application of a magnetic field suppressing the superconductivity, the induced state can be viewed as Landau Fermi liquid. These observations are in good agreement with recent experimental facts.

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The Landau Fermi Liquid (LFL) theory explains the major part of the low-temperature properties of Fermi liquids [1]. The LFL theory has demonstrated that the low-energy elementary excitations of a Fermi liquid look like the spectrum of an ideal Fermi gas and can be described in terms of Landau quasiparticles (LQ) with an effective mass M^* , charge e and spin $1/2$. As well, the LFL theory gives theoretical grounds for the BCS (Bardeen, Cooper, and Schrieffer) theory [2] of conventional superconductivity which accounts for many of fundamental properties of superconductors. In turn, the BCS theory is based on the notion of quasiparticles which represent elementary excitations of superconducting electron liquid and are called Bogoliubov quasiparticles (BQ). In the case of high- T_c metals, when the understanding of their striking behavior remains among the main problems of the condensed matter physics, a number of primary ideas of the LFL theory and BCS theory has been called in question. Therefore, there exists a fundamental question about whether or not a theory of high- T_c metals can be developed in terms of LQ and BQ.

It was reported recently that the full energy dispersion of single-particle excitations and the corresponding coherence factors as a function of momentum were measured on high- T_c cuprate ($\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$, $T_c=108$ K) by using high-resolution angle-resolved photoemission spectroscopy [3]. All the observed features qualitatively and quantitatively agree with the behavior of BQ predicted from BCS theory. This observation suggests that the superconducting state of high- T_c cuprate

is BCS-like and implies the basic validity of BCS formalism in describing the superconducting state [3]. On the other hand, such properties as the pairing mechanism, the maximum value of the superconducting gap Δ_1 , the high density of states, and other exotic properties are beyond BCS theory.

Striking experimental facts on the transport properties of the normal state induced by applying a magnetic field greater than the upper critical field B_c were obtained in a hole doped cuprates at overdoped concentration ($\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$) [4] and at optimal doping concentration ($\text{Bi}_2\text{Sr}_2\text{CuO}_{6+\delta}$) [5]. These data have clearly shown that there are no any sizable violation of the Wiedemann-Franz (WF) law. Measurements for strongly overdoped non-superconducting $\text{La}_{1.7}\text{Sr}_{0.3}\text{CuO}_4$ have demonstrated that the resistivity ρ exhibits T^2 behavior, $\rho = \rho_0 + \Delta\rho$ with $\Delta\rho = AT^2$, and the WF law is verified to hold perfectly [6]. Since the validity of the WF law is a robust signature of LFL, these experimental facts demonstrate that the observed elementary excitations cannot be distinguished from LQ. Thus these experimental observations impose strong constraints for models describing the electron liquid of the high-temperature superconductors. For example, in the cases of a Luttinger liquid [7], spin-charge separation (see e.g. [8]), and in some solutions of $t - J$ model [9] a violation of the WF law was predicted.

In this Letter, we consider the superconducting state of high- T_c metals within the framework of the theory of superconducting state based on the fermion condensation quantum phase transition (FCQPT) [10–12]. We show that the superconducting state is BCS-like, the ele-

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mentary excitations are BQ, and the primary ideas of the LFL theory and BCS theory are valid. At temperatures $T \rightarrow 0$, the normal state recovered by the application of a magnetic field larger than the critical field B_c can be viewed as LFL induced by the magnetic field. In this state, the WF law is held and the elementary excitations are LQ.

At $T < T_c$, the thermodynamic potential Ω of an electron liquid is given the equation (see, e.g. [13])

$$\Omega = E_{gs} - \mu N - TS, \quad (1)$$

where N is the number of particles, S denotes the entropy, and μ is the chemical potential. The ground state energy $E_{gs}[\kappa(\mathbf{p}), n(\mathbf{p})]$ of an electron liquid is a functional of the order parameter of the superconducting state $\kappa(\mathbf{p})$ and of the quasiparticle occupation numbers $n(\mathbf{p})$. Here we assume that the electron system is two-dimensional, while all results can be transported to the case of three-dimensional system. This energy is determined by the known equation of the weak-coupling theory of superconductivity

$$E_{gs} = E[n(\mathbf{p})] + \int \lambda_0 V(\mathbf{p}_1, \mathbf{p}_2) \kappa(\mathbf{p}_1) \kappa^*(\mathbf{p}_2) \frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^4}. \quad (2)$$

Here $E[n(\mathbf{p})]$ is the Landau functional determining the ground-state energy of a normal Fermi liquid. The quasiparticle occupation numbers

$$n(\mathbf{p}) = v^2(\mathbf{p})(1 - f(\mathbf{p})) + u^2(\mathbf{p})f(\mathbf{p}), \quad (3)$$

and

$$\kappa(\mathbf{p}) = v(\mathbf{p})u(\mathbf{p})(1 - 2f(\mathbf{p})), \quad (4)$$

where the coherence factors $v(\mathbf{p})$ and $u(\mathbf{p})$ are obeyed the normalization condition

$$v^2(\mathbf{p}) + u^2(\mathbf{p}) = 1. \quad (5)$$

The distribution function $f(\mathbf{p})$ of BQ defines the entropy

$$S = -2 \int [f(\mathbf{p}) \ln f(\mathbf{p}) + (1 - f(\mathbf{p})) \ln(1 - f(\mathbf{p}))] \frac{d\mathbf{p}}{4\pi^2}. \quad (6)$$

We assume that the pairing interaction $\lambda_0 V(\mathbf{p}_1, \mathbf{p}_2)$ is weak and produced, for instance, by electron-phonon interaction. Minimizing Ω with respect to $\kappa(\mathbf{p})$ and using the definition $\Delta(\mathbf{p}) = -\delta\Omega/\kappa(\mathbf{p})$, we obtain the equation connecting the single-particle energy $\varepsilon(\mathbf{p})$ to the superconducting gap $\Delta(\mathbf{p})$,

$$\varepsilon(\mathbf{p}) - \mu = \Delta(\mathbf{p}) \frac{1 - 2v^2(\mathbf{p})}{2v(\mathbf{p})u(\mathbf{p})}. \quad (7)$$

The single-particle energy $\varepsilon(\mathbf{p})$ is determined by the Landau equation

$$\varepsilon(\mathbf{p}) = \frac{\delta E[n(\mathbf{p})]}{\delta n(\mathbf{p})}. \quad (8)$$

Note that $E[n(\mathbf{p})]$, $\varepsilon[n(\mathbf{p})]$, and the Landau amplitude

$$F_L(\mathbf{p}, \mathbf{p}_1) = \frac{\delta E^2[n(\mathbf{p})]}{\delta n(\mathbf{p})\delta(\mathbf{p}_1)} \quad (9)$$

implicitly depend on the density x which defines the strength of F_L . Minimizing Ω with respect to $f(\mathbf{p})$ and after some algebra, we obtain the equation for the superconducting gap $\Delta(\mathbf{p})$

$$\Delta(\mathbf{p}) = -\frac{1}{2} \int \lambda_0 V(\mathbf{p}, \mathbf{p}_1) \frac{\Delta(\mathbf{p}_1)}{E(\mathbf{p}_1)} (1 - 2f(\mathbf{p})) \frac{d\mathbf{p}_1}{4\pi^2}. \quad (10)$$

Here the excitation energy $E(\mathbf{p})$ of BQ is given by

$$E(\mathbf{p}) = \frac{\delta(E - \mu N)}{\delta f(\mathbf{p})} = \sqrt{(\varepsilon(\mathbf{p}) - \mu)^2 + \Delta^2(\mathbf{p})}. \quad (11)$$

The coherence factors $v(\mathbf{p})$, $u(\mathbf{p})$, and the distribution function $f(\mathbf{p})$ are given by the ordinary relations

$$\begin{aligned} v^2(\mathbf{p}) &= \frac{1}{2} \left(1 - \frac{\varepsilon(\mathbf{p}) - \mu}{E(\mathbf{p})} \right), \\ u^2(\mathbf{p}) &= \frac{1}{2} \left(1 + \frac{\varepsilon(\mathbf{p}) - \mu}{E(\mathbf{p})} \right), \end{aligned} \quad (12)$$

$$f(\mathbf{p}) = \frac{1}{1 + \exp(E(\mathbf{p})/T)}. \quad (13)$$

Equations (7)–(13) are the conventional equations of the BCS theory [2, 13], determining the superconducting state with BQ and the maximum value of the superconducting gap $\Delta_1 \sim 10^{-3}\varepsilon_F$ provided that one assumes that system in question has not undergone FCQPT.

Now we turn to a consideration of superconducting electron liquid with the fermion condensate (FC) which takes place after the FCQPT point. If $\lambda_0 \rightarrow 0$, then the maximum value of the superconducting gap $\Delta_1 \rightarrow 0$, as well as the critical temperature $T_c \rightarrow 0$, and Eq. (7) reduces to the equation [10, 11, 14]

$$\varepsilon(\mathbf{p}) - \mu = 0, \quad \text{if } 0 < n(\mathbf{p}) < 1; p_i \leq p \leq p_f. \quad (14)$$

At $T \rightarrow 0$, Eq. (14) defines a new state of electron liquid with FC [10, 15] which is characterized by a flat part of the spectrum in the $(p_f - p_i)$ region and has a strong impact on the system's properties up to temperature T_f [10, 11, 16]. Apparently, the momenta p_i and p_f have to

satisfy $p_i < p_F < p_f$, where p_F is the Fermi momentum. When the Landau amplitude $F_L(p = p_F, p_1 = p_F)$ as a function of the density x is sufficiently small, the flat part vanishes, and at $T \rightarrow 0$ Eq. (14) has the only trivial solution $\varepsilon(p = p_F) = \mu$, and the quasiparticle occupation numbers are given by the step function, $n(\mathbf{p}) = \theta(p_F - p)$ [10]. At some critical density $x = x_{FC}$ the amplitude becomes strong enough so that Eq. (14) possesses the solution corresponding to a formation of the flat part of spectrum, that is FC is created [17, 18]. Note, that a formation of the flat part of the spectrum has been recently confirmed in Ref. [19].

Now we can study the relationships between the state defined by Eq. (14) and the superconductivity. At $T \rightarrow 0$, Eq. (14) defines a particular state of a Fermi liquid with FC, for which the modulus of the order parameter $|\kappa(\mathbf{p})|$ has finite values in the $(p_f - p_i)$ region, whereas $\Delta_1 \rightarrow 0$ in this region. Observe that $f(\mathbf{p}, T \rightarrow 0) \rightarrow 0$, and it follows from Eqs. (3) and (4) that if $0 < n(\mathbf{p}) < 1$ then $|\kappa(\mathbf{p})| \neq 0$ in the region $(p_f - p_i)$. Such a state can be considered as superconducting, with an infinitely small value of Δ_1 , so that the entropy of this state is equal to zero. It is obvious that this state being driven by the quantum phase transition disappears at $T > 0$ [11]. Any quantum phase transition, which takes place at temperature $T = 0$, is determined by a control parameter other than temperature, for example, by pressure, by magnetic field, or by the density of mobile charge carriers x . The quantum phase transition occurs at a quantum critical point. At some density $x \rightarrow x_{FC}$, when the Landau amplitude F_L becomes sufficiently weak, and $p_i \rightarrow p_F \rightarrow p_f$, Eq. (14) determines the critical density x_{FC} at which FCQPT takes place leading to the formation of FC [10, 11]. It follows from Eq. (14) that the system becomes divided into two quasiparticle subsystems: the first subsystem in the $(p_f - p_i)$ range is characterized by the quasiparticles with the effective mass $M_{FC}^* \propto 1/\Delta_1$, while the second one is occupied by quasiparticles with finite mass M_L^* and momenta $p < p_i$. The density of states near the Fermi level tends to infinity, $N(0) \propto M_{FC}^* \propto 1/\Delta_1$ [11].

If $\lambda_0 \neq 0$, then Δ_1 becomes finite. It is seen from Eq. (10) that the superconducting gap depends on the single-particle spectrum $\varepsilon(\mathbf{p})$. On the other hand, it follows from Eq. (7) that $\varepsilon(\mathbf{p})$ depends on $\Delta(\mathbf{p})$ provided that at $\Delta_1 \rightarrow 0$ Eq. (14) has the solution determining the existence of FC. Let us assume that λ_0 is small so that the particle-particle interaction $\lambda_0 V(\mathbf{p}, \mathbf{p}_1)$ can only lead to a small perturbation of the order parameter $\kappa(\mathbf{p})$ determined by Eq. (14). Upon differentiation both parts of Eq. (7) with respect to the momentum p ,

we obtain that the effective mass $M_{FC}^* = d\varepsilon(p)/dp|_{p=p_F}$ becomes finite [11]

$$M_{FC}^* \sim p_F \frac{p_f - p_i}{2\Delta_1}. \quad (15)$$

It follows from Eq. (15) that the effective mass and the density of states $N(0) \propto M_{FC}^* \propto 1/\Delta_1$ are finite and constant at $T < T_c$ [11, 14]. As a result, we are led to the conclusion that in contrast to the conventional theory of superconductivity the single-particle spectrum $\varepsilon(\mathbf{p})$ strongly depends on the superconducting gap and we have to solve Eqs. (8) and (10) in a self-consistent way. On the other hand, let us assume that Eqs. (8) and (10) are solved, and the effective mass M_{FC}^* is determined. Now one can fix the dispersion $\varepsilon(\mathbf{p})$ by choosing the effective mass M^* of system in question equal to M_{FC}^* and then solve Eq. (10) as it is done in the case of the conventional theory of superconductivity [2]. As a result, one observes that the superconducting state is characterized by BQ with the dispersion given by Eq. (11), the coherence factors v, u are given by Eq. (12), and the normalization condition (5) is held. We are led to the conclusion that the observed features agree with the behavior of BQ predicted from BCS theory. This observation suggests that the superconducting state with FC is BCS-like and implies the basic validity of BCS formalism in describing the superconducting state. It is exactly the case that was observed experimentally in high- T_c cuprate $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ [3].

Consider other differences between the conventional superconducting state and the superconducting state with FC. We consider the case when $T_c \ll T_f$. This means that the order parameter $\kappa(\mathbf{p})$ is slightly perturbed by the pairing interaction because the particle-particle interaction $\lambda_0 V$ is small comparatively to the Landau amplitude F_L and the order parameter $\kappa(\mathbf{p})$ is governed mainly by F_L [10]. We can solve Eq. (10) analytically taking the Bardeen-Cooper-Schrieffer approximation for the particle-particle interaction: $\lambda_0 V(\mathbf{p}, \mathbf{p}_1) = -\lambda_0$ if $|\varepsilon(\mathbf{p}) - \mu| \leq \omega_D$, i.e. the interaction is zero outside this region, with ω_D being the characteristic phonon energy. As a result, the maximum value of the superconducting gap is given by [14]

$$\begin{aligned} \Delta_1 &\simeq \frac{\lambda_0 p_F (p_f - p_F)}{2\pi} \ln(1 + \sqrt{2}) \simeq \\ &\simeq 2\beta \varepsilon_F \frac{p_f - p_F}{p_F} \ln(1 + \sqrt{2}). \end{aligned} \quad (16)$$

Here, the Fermi energy $\varepsilon_F = p_F^2/2M_L^*$, and the dimensionless coupling constant β is given by the relation $\beta = \lambda_0 M_L^*/2\pi$. Taking the usual values of β as $\beta \simeq 0.3$, and assuming $(p_f - p_F)/p_F \simeq 0.2$, we get from Eq. (16)

a large value of $\Delta_1 \sim 0.1\varepsilon_F$, while for normal metals one has $\Delta_1 \sim 10^{-3}\varepsilon_F$. Now we determine the energy scale E_0 which defines the region occupied by quasiparticles with the effective mass M_{FC}^*

$$E_0 = \varepsilon(\mathbf{p}_f) - \varepsilon(\mathbf{p}_i) \simeq 2 \frac{(p_f - p_i)p_F}{M_{FC}^*} \simeq 2\Delta_1. \quad (17)$$

We have returned back to the Landau Fermi liquid theory since high energy degrees of freedom are eliminated and the quasiparticles are introduced. The only difference between LFL, which serves as a basis when constructing the superconducting state, and Fermi liquid after FCQPT is that we have to expand the number of relevant low energy degrees of freedom by introducing a new type of quasiparticles with the effective mass M_{FC}^* given by Eq. (15) and the energy scale E_0 given by Eq. (17). Therefore, the dispersion $\varepsilon(\mathbf{p})$ is characterized by two effective masses M_L^* and M_{FC}^* and by the scale E_0 , which define the low temperature properties including the line shape of quasiparticle excitations [11, 14], while the dispersion of BQ is given by Eq. (11). We note that both the effective mass M_{FC}^* and the scale E_0 are temperature independent at $T < T_c$, where T_c is the critical temperature of the superconducting phase transition [14]. Obviously, we cannot directly relate these new LFL quasiparticle excitations with the quasiparticle excitations of an ideal Fermi gas because the system in question has undergone FCQPT. Nonetheless, the main basis of the Landau Fermi liquid theory survives FCQPT: the low energy excitations of a strongly correlated liquid with FC are quasiparticles.

As it was shown above, properties of these new quasiparticles are closely related to the properties of the superconducting state. We may say that the quasiparticle system in the range $(p_f - p_i)$ becomes very “soft” and is to be considered as a strongly correlated liquid. On the other hand, the system’s properties and dynamics are dominated by a strong collective effect having its origin in FCQPT and determined by the macroscopic number of quasiparticles in the range $(p_f - p_i)$. Such a system cannot be perturbed by the scattering of individual quasiparticles and has features of a “quantum protectorate” [11, 20, 21].

At $T_c < T$, the order parameter κ vanishes, and the behavior of system in question can be viewed as the behavior of an anomalous electron Fermi liquid, or strongly correlated liquid, with the resistivity being a linear function of temperature, while the effective mass behaves as $M_{FC}^* \propto 1/T$ [11, 16]. Obviously, at this regime one observes strong deviations from the LFL behavior and cannot expect the WF law is held.

As any phase transition, FCQPT is related to the order parameter, which induces a broken symmetry. As we have seen, the order parameter is the superconducting order parameter $\kappa(\mathbf{p})$, while Δ_1 being proportional to the coupling constant (see Eq. (16)) can be small. Therefore, the existence of such a state, that is electron liquid with FC, can be revealed experimentally. Since the order parameter $\kappa(\mathbf{p})$ is suppressed by the critical magnetic field B_c , when $B_c^2 \sim \Delta_1^2$. If the coupling constant $\lambda_0 \rightarrow 0$, the weak critical magnetic field $B_c \rightarrow 0$ will destroy the state with FC converting the strongly correlated Fermi liquid into LFL. In this case the magnetic field plays a role of the control parameter determining the effective mass [22]

$$M_{FC}^* \propto \frac{1}{\sqrt{B}}. \quad (18)$$

Equation (18) shows that by applying a magnetic field B the system can be driven back into LFL with the effective mass M_{FC}^* which is finite and independent of the temperature. This means that the low temperature properties depend on the effective mass in accordance with the LFL theory. At $T > T^*$, the system possesses the behavior of the strongly correlated liquid. Here $T^* \propto \sqrt{B}$ is the temperature at which the transition from LFL to the strongly correlated liquid takes place. Such a behavior was observed experimentally in the heavy-electron metal YbRh_2Si_2 [23]. If λ_0 is finite, the critical field is also finite, and Eq. (18) is valid at $B > B_c$. In that case, the effective mass M_{FC}^* is finite and temperature independent at $T < T_c$, and low temperature elementary excitations of the system can be described in terms of LQ. Thus, system is driven back to LFL and has the LFL behavior induced by the magnetic field at least at $T < T_c$. While the low energy elementary excitations are characterized by M_{FC}^* and cannot be distinguished from LQ. As a result, at $T \rightarrow 0$, the WF law is held in accordance with experimental facts [4, 5].

The existence of FCQPT can also be revealed experimentally because at densities $x > x_{FC}$, or beyond the FCQPT point, the system should be LFL at sufficiently low temperatures [18]. Recent experimental data have shown that this liquid exists in heavily overdoped non-superconducting $\text{La}_{1.7}\text{Sr}_{0.3}\text{CuO}_4$ [6]. It is remarkable that up to $T = 55$ K the resistivity exhibits the T^2 behavior and the WF law is verified to within the experimental resolution [6].

In summary, we have shown that the superconducting state with FC is characterized by BQ. The behavior of these BQ agrees with the behavior of BQ predicted from BCS theory and suggests that the superconducting

state with FC is BCS-like and implies the basic validity of BCS formalism in describing the superconducting state. Although the maximum value of the superconducting gap and other exotic properties are determined by the presence of the fermion condensate. We have also demonstrated that the low temperature transport properties of high- T_c metals observed in optimally doped and overdoped cuprates by the application of a magnetic field higher than the critical field can be explained within the framework of the fermion condensation theory of high- T_c superconductivity. The quasiparticles are LQ and the WF law is held. The recent experimental observations of BQ in the superconducting state and verifications of the WF law in heavily overdoped, overdoped and optimally doped cuprates clearly favor the existence of FC in high- T_c metals.

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