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Nonzeroth-order anomalous optical transparency in modulated metal films owing to excitation of surface plasmon polaritons: an analytical approach.

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The problem of anomalous light tunneling through periodically modulated metal films is examined in a purely analytical approach. The approach uses large magnitude of the dielectric permittivity of metals in the visible and near infrared (it is equivalent to that resulting in the Leontovich boundary conditions for semi-infinite problems). It is shown that the anomalous transparency recently discovered experimentally is caused by the excitation of single- or double-boundary surface plasmon polaritons due to film modulation. Dependencies of the resonance transparency on parameters of the problem are analyzed in detail, and the optimum parameters (optimal layer thickness and optimal modulation amplitude) corresponding to extreme values of the transmittance of both zero and nonzero diffraction orders have been found. Classifying the possible types of the resonances has allowed identifying special and nontrivial features of the effect. In particular, we predict strong nonzeroth-order anomalous transparency.

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Despite the great amount of papers on the photon-SPP (surface plasmon polariton) transformations in periodical structures that appeared over the past decades (it is sufficient to mention [1 – 3]), the experiment concerning transparency of metal films [4], which is caused by the effect was realized only recently. Upon this first observation of the extraordinary optical transmission through sub-wavelength hole arrays many theoreticians have contributed to explaining the effect [5 – 11]. But still the qualitative analytical picture of the light tunneling phenomenon is uncovered enough. The majority of the explanations are based upon numerical calculations (to the best of the author's knowledge, the only exceptions are [5, 9], but paper [5] contains only crude quantitative estimates, while paper [9] deals with the case of a strictly normal incidence onto the harmonically modulated film. Moreover, the second spatial field harmonic was not taken into account in the latter paper which prevented finding the true position of the resonance). Numerical approaches do not allow obtaining a deep intuitive insight into the problem.

While the principal channel for anomalous light transmittance for normal incidence and nonsymmetrical dielectric surrounding is in a non-specular direction, most papers on the subject examined the zeroth-order transmittance and reflectivity alone (an exception is the recent paper [12]). The nonzeroth-order transmittance can exceed the zeroth order value of a few tens that of zeroth-order. This means that the process of light tunneling in the case of "single-boundary-localized" (SB) excitation (when a SPP is localized at one face of the metal film only) can be in a no less degree effective than under the "double-boundary-localized" (DB) excitation (with SPP localized at both sides of the film, see the experiment of Ref. [13]).

This Letter suggests a thorough new analytical examination of the effect. We will discuss both the specular and non-specular transmittance. General results are obtained both for oblique and normal incidence. Simple estimates are given for 2D periodic structures and it is shown that they are similar to such for 1D gratings, contrary to ascertains of some writers, see Refs. [10, 14]. Our approach also makes it clear that the main resonance effects depend on the existence of the periodicity itself, being rather insensitive to the specific type of modulation (modulation of the dielectric permittivity [5, 9] or dielectric pillars [6, 12], cylindrical [11] or square [10] holes, relief corrugations [7]). However the type of modulation can influence polarization properties of the periodically modulated film as will be shown in a future paper.

Let a p (TM)-polarized monochromatic wave (time dependence of the form $\exp(-i\omega t)$ is omitted) of magnetic field amplitude, H^i, be incident onto a metal film surrounded by dielectric media with permittivities ε_σ,

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$\sigma = \pm$, from the medium with ε_-. Let the dielectric permittivity of the film be periodically modulated along the x direction, $\varepsilon(x) = \varepsilon(x + \Lambda)$, and the plane of incidence coincide with the xz plane, see Fig.1. Owing to the symmetry, only the y – component of the magnetic field is not zero. Seeking the solution in the form of a Fourier-Floquet expansion and taking into account the radiation conditions, we obtain in the dielectric media

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Fig.1. Geometry of the problem. Nonsymmetric dielectric surrounding of a metal film. A single SB metal-superstrate first-order resonance, $k_{1x} \simeq K^-$ is shown. The circles $\odot$ show that the magnetic field is guided along the y axis

$$H^\sigma(x,z) = \delta_{\sigma,-} H^i \exp(ik_{-|z} z + ik_x x) +$$
$$+ \sum_n H_n^\sigma \exp[ik_{\sigma|nz}(z - \delta_{\sigma,+} d) + ik_{nx} x)],$$
$$k_{nx} = k_x + ng, \quad k_{\sigma|nz} = \sigma\sqrt{\varepsilon_\sigma k^2 - k_{nx}^2}, \; g = 2\pi/\Lambda,$$
$$\mathrm{Re}, \mathrm{Im}\left\{\sqrt{\varepsilon_\sigma k^2 - k_{nx}^2}\right\} \geq 0, \quad \sigma = \pm,$$

where d is the film thickness and $H^\sigma(x,z)$ denote field strengths in the media with the dielectric permittivity ε_σ, i.e., for $z \geq d$ ($z \leq 0$) if $\sigma = +$ ($\sigma = -$), respectively.

The solution in the film can be represented as

$$H(x,z) = H^i \sum_{n,\sigma} h_n^\sigma(z) \exp[ik_{nx} x + \sigma \tilde{k} z] \,,$$

where $\tilde{k} = k\sqrt{-\varepsilon_0}$, $\mathrm{Re}\,\tilde{k} \geq 0$, and ε_0 denotes some mean value of $\varepsilon(x)$ with $\mathrm{Re}\,\varepsilon_0 < 0$. For simplicity we will neglect the dependence upon z of the transformation coefficients (TC) $h_n^\sigma(z)$ here and below. This approximation is equivalent to the one resulting in the impedance (Leontovich) boundary conditions for semi-infinite problems, cf. [15], and takes account of the large dielectric permittivity, $|\varepsilon(x)| \gg 1$, along with assumption of a small modulation[2)] , $|\varepsilon(x) - \varepsilon_0| \ll |\varepsilon_0|$. Besides, we neglect the impact of bulk modulation on the diffraction, that is the permittivity modulation is taken into account in the boundary conditions only. An analogous model was first applied in paper [17] to explaining the Wood anomalies.

Using continuity of the electric and magnetic field tangential components, we can express the TCs outside the metal, $R_m \equiv H_m^-/H^i$ and $T_m \equiv H_m^+/H^i$, in terms of the inner TCs, h_m^σ,

$$R_m = -\delta_{n,0} + \sum_\sigma h_m^\sigma, \quad T_m = \sum_\sigma h_m^\sigma \exp(\sigma\Phi), \qquad (1)$$

to obtain the following infinite set of linear equations, cf. Ref. [18],

$$\sum_{m,\sigma'=\pm} D_{nm}^{\sigma\sigma'} h_m^{\sigma'} = V_n^\sigma, \quad \sigma = \pm\,,$$
$$D_{nm}^{\sigma\sigma'} = [(\beta_{\sigma|n} + \sigma'\sigma\xi_0)\delta_{n,m} + \sigma\sigma'\tilde{\xi}_{n-m}]\times$$
$$\times[\delta_{\sigma,-} + \delta_{\sigma,+}\exp(\sigma'\Phi)], \quad V_n^\sigma = 2\beta_{-|0}\delta_{n,0}\delta_{\sigma,-},$$
$$\beta_{\sigma|n} = k_{\sigma|nz}/k\varepsilon_\sigma, \quad \Phi = \tilde{k}d,$$
$$\xi(x) \equiv 1/\sqrt{\varepsilon(x)} = \xi_0 + \sum_n \tilde{\xi}_n \exp(ingx), \quad \tilde{\xi}_0 = 0.$$

By equating to zero the determinant of diagonal in diffraction order $2-2$ submatrices of the $D_{nm}^{\sigma\sigma'}$ matrix, $\widehat{D}_n = \|D_{nn}^{\sigma\sigma'}\|$,

$$|\widehat{D}_n| \equiv d_n = b_{+|n}^+ b_{-|n}^+ \exp\Phi - b_{+|n}^- b_{-|n}^- \exp(-\Phi),$$

we obtain the SPP dispersion relation for the non-modulated film. In case the spatial field harmonics are far away from eigenmodes of the non-modulated film (non-resonance conditions), the coefficients $b_{\sigma|n}^{\sigma'}$ are of order of unity, $|b_{\sigma|n}^{\sigma'}| \simeq |\beta_{\sigma|n}| \sim 1$, whence the estimate for the determinant is $|d_n| \sim \exp\Phi'$. Under the non-resonance conditions, the matrix $D_{nm}^{\sigma\sigma'}$ is diagonal-dominated, that is elements of the off-diagonal submatrices $D_{nm}^{\sigma\sigma'} \propto \tilde{\xi}_{n-m}$ for $m \neq n$, and hence are small as compared with that of the diagonal ones, $D_{nn}^{\sigma\sigma'}$. On

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[2)] For an arbitrary film thickness the approximation is certainly valid under the restriction $|\varepsilon(x) - \varepsilon_0| \ll \sqrt{|\varepsilon_0|}$. But this condition, while being sufficient seems to be far from necessary, as it follows from comparison with straightforward numerical calculations, cf. [16]. Evidently, in the limit $d \to \infty$ the validity condition is much less restrictive, $|\varepsilon(x) - \varepsilon_0| \ll |\varepsilon_0|$. Moreover, the structure of the solution obtained is completely analogous to the one for the film with a corrugated relief, see forthcoming papers.

the contrary, under the resonance conditions, the determinant of $\widehat{D}_n$ decreases by several orders of magnitude. For thick films, $\exp(-\Phi') \ll 1$, and for the determinant to decrease significantly, the condition $|b^{+}_{\sigma|m}| \ll 1$ or $|\beta_{\sigma|m}| \lesssim |\xi_0|$ must hold. As for the diffraction problem, $\beta_{\sigma|m}$ can posses either purely real or purely imaginary values only; the minimum of $|b^{+}_{\sigma|m}|$ (namely, $|b^{+}_{\sigma|m}|_{\min} = \xi'_0$) is achieved for $\beta_{\sigma|m} = -i\xi''_0$, which yields a SPP dispersion relation for the boundary between the metal and the dielectric half-spaces[3]. The magnitude of the determinant at this point is $|d_m| \sim \sim \xi'_0 \exp\Phi'$. Denoting the resonance diffraction orders as r, r', etc., and the set of the resonance orders as $\mathcal{R}$, we can write the resonance condition as $\mathrm{Im}(b^{+}_{\sigma|r}) \simeq 0$, or in a more explicit form,

$$\begin{aligned} &\sqrt{\varepsilon_-}\sin\theta + r\kappa \simeq \tau K^{\sigma}, \quad \kappa \equiv g/k, \quad \tau = \pm, \\ &K^{\sigma} \equiv \sqrt{\varepsilon_{\sigma} + \varepsilon_{\sigma}^2 \xi_0''^2} \simeq \sqrt{\varepsilon_{\sigma}}, \quad \exp(-\Phi') \ll 1. \end{aligned} \tag{2}$$

Evidently, equation with $\tau = +(-)$ corresponds to the forward (backward) direction of SPP propagation relative to the incident wave; $\sigma = -(+)$ corresponds to SPP excitation at the metal-superstrate (substrate) interface.

Analogously, the condition $|b^{\sigma'}_{\sigma|N}| \gg |\xi_0|$ defines the nonresonance subset of diffraction orders, $\mathcal{N}$, where the integers, belonging to $\mathcal{N}$, are denoted as, N, M, etc. Accordingly, the matrix $\widehat{D}$ is decomposed into four submatrices with resonance (nonresonance) diffraction orders, $\widehat{R} = \|D^{\sigma\sigma'}_{rr'}\|$ ($\widehat{M} = \|D^{\sigma\sigma'}_{NN'}\|$); and two mixed matrices, $\widehat{U} = \|D^{\sigma\sigma'}_{rN}\|$, and $\widehat{L} = \|D^{\sigma\sigma'}_{Nr'}\|$. The corresponding right-hand sides have been denoted $\widehat{u} = \|V^{\sigma}_r\|$ and $\widehat{v} = \|V^{\sigma}_N\|$. Decomposing the submatrix $\widehat{M} = \|D^{\sigma\sigma'}_{NN'}\|$ into a block-diagonal and a nondiagonal matrices, we have

$$\widehat{M} = \widehat{A}(\hat{1} - \widehat{\nu}), \quad A^{\sigma\sigma'}_{NM} = \delta_{N,M} D^{\sigma\sigma'}_{NN}, \quad \widehat{\nu} = O(\widetilde{\xi}),$$

where the norm of the matrix $\widehat{\nu}$ is small as its elements are proportional to the small modulation amplitude, $\widehat{\nu} \sim \widetilde{\xi}$. Then $\widehat{M}^{-1} = \sum_{s=0}^{\infty} \widehat{\nu}^s \widehat{A}^{-1}$. As a result, we can solve the nonresonance subsystem for h^{σ}_N, $N \in \mathcal{N}$, in an explicit form as $\|h^{\sigma}_N\| = \widehat{M}^{-1}\left(\widehat{v} - \widehat{L}\,\|h^{\sigma}_r\|\right)$, and represent the resonance subsystem as

$$\sum_{r',\sigma'} \widetilde{D}^{\sigma\sigma'}_{rr'} h^{\sigma'}_{r'} = \widetilde{V}^{\sigma}_r, \quad r, r' \in \mathcal{R}, \tag{3}$$

where $\|\widetilde{D}^{\sigma\sigma'}_{rr'}\| \equiv \widehat{R} - \widehat{U}\widehat{M}^{-1}\widehat{L}$, $\|\widetilde{V}^{\sigma}_r\| \equiv \widehat{u} - \widehat{U}\widehat{M}^{-1}\widehat{v}$.

[3] For thin films the dispersion relation changes essentially. If $\varepsilon_+ = \varepsilon_-$ and $|\Phi| \ll 1$, there are two very distinct resonance points, $\beta_r \simeq -2i\xi''_0/\Phi'$ and $\beta_r \simeq -i\xi''_0\Phi'/2$, see Ref. [19].

For further analytical treatment we will consider this solution in the *main approximation*, taking into account the terms linear in $\widetilde{\xi}$, in $\widetilde{V}^{\sigma}_r$ and quadratic terms in $\widetilde{D}^{\sigma\sigma'}_{rr'}$. In this approximation it is sufficient to restrict the analysis by the zeroth order term in the series expansion $\widehat{M}^{-1} = \widehat{A}^{-1} + O(\widetilde{\xi})$. Then

$$\begin{aligned} \widetilde{D}^{\sigma\sigma'}_{rr'} &= D^{\sigma\sigma'}_{rr'} + \sigma' \sum_N \widetilde{\xi}_{r-N}\widetilde{\xi}_{N-r'} d_N^{-1} [q^{\sigma-}_N - q^{\sigma+}_N e^{\sigma'\Phi}], \\ \widetilde{V}^{\sigma}_r &= 2\beta_{-|0}\left(\delta_{r,0}\delta_{\sigma,-} - \widetilde{\xi}_r d_0^{-1} q^{\sigma-}_0\right), \\ q^{\sigma\sigma'}_N &= \delta_{\sigma,\sigma'}[b^{+}_{\overline{\sigma}|N} e^{\Phi} + b^{-}_{\overline{\sigma}|N} e^{-\Phi}] - 2\overline{\delta}_{\sigma,\sigma'}\beta_{\overline{\sigma}|N}. \end{aligned} \tag{4}$$

Here $\overline{\sigma} = -\sigma$, $\overline{\delta}_{\sigma,\sigma'} = 1 - \delta_{\sigma,\sigma'}$.

All the external TCs of Eq. (1) are expressed in terms of the inner TCs with resonance indices, h^{σ}_r. Thus, the solution of the resonance diffraction problem reduces to that of the resonance subsystem, Eq. (3). For instance, in the main approximation,

$$\begin{aligned} T_N &= \delta_{N,0} T^F_0 - \sum_{r,\sigma,\sigma'} \sigma\sigma' d_N^{-1} \times \\ &\times \left[b^{\sigma}_{-|N} \exp[(\sigma' + \sigma)\Phi] + b^{\overline{\sigma}}_{+|N}\right] \widetilde{\xi}_{N-r} h^{\sigma'}_r, \\ R_N &= \delta_{N,0} R^F_0 - \sum_{r,\sigma,\sigma'} \sigma\sigma' d_N^{-1} \times \\ &\times \left[b^{\sigma}_{-|N} \exp(\sigma'\Phi) + b^{\overline{\sigma}}_{+|N} \exp(\overline{\sigma}\Phi)\right] \widetilde{\xi}_{N-r} h^{\sigma'}_r, \end{aligned} \tag{5}$$

where R^F_0 and T^F_0 are the well-known transmittance and reflectivity coefficients corresponding to the nonmodulated film.

The energy flux in the dielectrics is determined by the energy TCs, giving a ratio of the z- components of the corresponding Poynting vectors to that of the incident wave,

$$\rho_m = |R_m|^2 \cdot \frac{\mathrm{Re}(\beta_{-|m})}{\beta_{-|0}}, \quad \tau_m = |T_m|^2 \cdot \frac{\mathrm{Re}(\beta_{+|m})}{\beta_{-|0}}.$$

When considering the simplest resonance ($|d_r| \ll \ll 1$ for the unique number r, in view of the condition $|b^{+}_{\sigma|r}| \ll 1$ for one or simultaneously both values of σ, the latter can hold for $\varepsilon_- = \varepsilon_+$) the resonance subsystem, (3), includes two equations for two main Fourier field amplitudes, h^{+}_r and h^{-}_r. The solution presented permits easy numerical calculations for films of arbitrary thicknesses, but to achieve a better insight into the problem and having comparison with the experiment in mind, we concentrate on the results for thick films, $\exp(-\Phi') \ll 1$. In the framework of the approximation (4) we obtain from Eq. (3)

$$h^{\sigma}_r \simeq \frac{2\sigma\widetilde{\xi}_r}{\widetilde{d}_r}\left[\delta_{\sigma,-}\widetilde{\beta}_{+|r} + \left(b^{\sigma}_{+|r} + 2\beta_{-|r}\right)e^{-2\Phi}\right], \tag{6}$$

where

$$\widetilde{d}_r \equiv \widetilde{\beta}_{+|r}\widetilde{\beta}_{-|r} - b^-_{+|r}b^-_{-|r}e^{-2\Phi},$$
$$\widetilde{\beta}_{\sigma|r} \equiv \beta_{\sigma|r} + \xi_0 + \sum_N \frac{a_{r-N}}{b^+_{\sigma|N}}, \quad a_m \equiv -\widetilde{\xi}_m\widetilde{\xi}_{-m}. \qquad (7)$$

Assuming absorption to be small, $\xi_0' \ll |\xi_0''|$, we restrict ourselves to modulation of the imaginary part of ξ only. Then $a_m = |\widetilde{\xi}_m|^2$. The denominator, $\widetilde{d}_r$, retains contributions of first-order resonance-to-nonresonance scattering processes and vice versa. The processes involving inhomogeneous waves result mainly in a shift of the resonance, while those involving homogeneous ones give rise to the widening. Also, consider the TC for the rth diffraction order. As will be seen below, it may be of great interest for the SB resonance, since with $\varepsilon_+ \neq \varepsilon_-$ the corresponding Fourier field component may represent an outgoing wave in one of the half-spaces. Using Eqs. (6) and (1), and considering a quasi-harmonic modulation, $|\widetilde{\xi}_N| \lessgtr |\widetilde{\xi}_{\pm r}|$, we have

$$T_r \simeq \frac{2\widetilde{\xi}_r}{\widetilde{d}_r}\left[\widetilde{\beta}_{+|r} + b^+_{+|r} + 2\beta_{-|r} + \right.$$
$$\left. + \left(b^-_{+|r} + 2\beta_{-|r}\right)e^{-2\Phi}\right]e^{-\Phi}.$$

The specular coefficients can be simplified as

$$\Delta R_0 \simeq -\frac{2a_r}{\beta_{-|0}\widetilde{d}_r}\left[\widetilde{\beta}_{+|r} + \left(4\beta_{-|r} + 2\beta_{+|r}\right)e^{-2\Phi}\right],$$
$$\Delta T_0 \simeq \frac{a_r}{\xi_0\widetilde{d}_r}\left(\beta_{-|r} + \xi_0 + \beta_{+|r}\right), \qquad (8)$$

where $\Delta R_0 \equiv (R_0 - R_0^F)/R_0^F$ and $\Delta T_0 \equiv (T_0 - T_0^F)/T_0^F$. We neglect ξ_0 in comparison with $\beta_{\sigma|0}$, which is valid for not too grazing incidence.

Let examine an essentially nonsymmetric situation, where the resonance condition is fulfilled for a sole number r and a fixed sign of σ, $|\beta_{\sigma|r}| \ll 1$, i. e. a SPP mode is excited at the one of metal-dielectric interfaces. While being the simplest example of SPP excitation, it may provide a very non-trivial light transmission. We choose the case of the metal-superstrate resonance, $\sigma = -$. Estimating the values $|\beta_{\sigma|r}|$ in the resonance vicinity as $|\beta_{-|r}| \simeq |\xi_0|$ and $|\beta_{+|r}| \simeq 1$ (see the discussion above Eq. (2)), we have for the minimal magnitude of the denominator $\widetilde{d}_r$

$$|\widetilde{d}_r|_{\min} \simeq \xi_0' + a_r + O(e^{-2\Phi'}). \qquad (9)$$

Then Eq. (7) results in $|\Delta T_0| \sim \dfrac{a_r}{|\xi_0|(\xi_0' + a_r)}$. $|\Delta T_0|$ increases with an increase of a_r to become of order one for the modulation amplitude $a_r \sim \xi_0'|\xi_0|$. The zeroth-order transmittance exhibits a saturation for $a_r \gtrsim \xi_0'$, see Fig.2, at the level $|T_0|_{\max} \sim |\xi_0|^{-1}|T_0^F|$, being enhanced by the factor $|\xi_0|^{-1}$ as compared with $|T_0^F| \simeq 4|\xi_0|\exp(-\Phi') \ll 1$. The maximal zeroth-order transmittance τ_0 at $\Phi' = 3$ (three skin-depths) is about several percent (for $|\xi_0| \simeq 0.1$), while in the experiment of Ebbesen et al. the maximal transmittance was about several percent for $\Phi' = 7$. Such an increase apparently may be due to different reasons: first, it is evident that the validity of all above formulas is restricted by the smallness of the modulation amplitude, and therefore we cannot suggest a rigorous estimate of the effect for hole array. Second, while measuring the transmitted light intensity in the experiment, the detector could fully register not only the zeroth-order channel, but other homogeneous outgoing channels as well (the details concerning position of the detector have not been reported, in spite of this being an essential point, hence we cannot make a positive judgement). Note that in spite of the low transmittance, the reflectivity may have deep resonance minima, and they appear to be better pronounced in the case of SPP excitation at the front surface ($\sigma = -$) than at the back ($\sigma = +$) which follows directly from Eq. (8) for $|\beta_{-|r}| \ll 1$ and $|\beta_{+|r}| \ll 1$, respectively, cf. also [8].

The transmittance can be increased in two possible ways. One is to adjust the parameters so that excitation of a DB SPP should exist, see below. The other relates to the specific case when the diffraction order of a SPP excited at one of the interfaces corresponds to a propagating wave in the opposite-side dielectric half-space, and we arrive at "nonzeroth-order, plasmon enhanced light transmittance". Consider the latter case in some detail.

Let a light wave be incident from the dielectric of lower optical density, $\varepsilon_- < \varepsilon_+$, and the diffraction order r correspond both to the homogeneous outgoing wave in the substrate and the SB SPP on the front interface, see Fig.1 for $r = 1$. In Fig.2 the ρ_0 minima and τ_1, τ_0 maxima correspond to excitation of a SPP; τ_0 possesses typical Fano profile. The rth-order transmitted wave becomes the principal channel for light tunneling, resulting in $T_r \simeq 4\widetilde{\xi}_r\exp(-\Phi)/\left[\widetilde{\beta}_{-|r} + 2\xi_0\exp(-2\Phi)\right]$. Since $\widetilde{\beta}_{-|r}$ includes a term proportional to $|\widetilde{\xi}_r|^2$, there exists an optimal modulation depth, $a_r = a_{\text{opt}} \sim \xi_0'$, corresponding to the highest transmittance (see Fig.3): $\tau_r|_{\max} \sim {\xi_0'}^{-1}\exp(-2\Phi')$ exceeds the maximum value of τ_0 by the factor ${\xi_0'}^{-1}$. This value of a_r differs only weakly from that corresponding to a totaly suppressed specular reflection for the case of diffraction at the interface between the metal and dielectric half-spaces, see [20]. This result is universal for different metals if we renormalize the modulation amplitude using its charac-

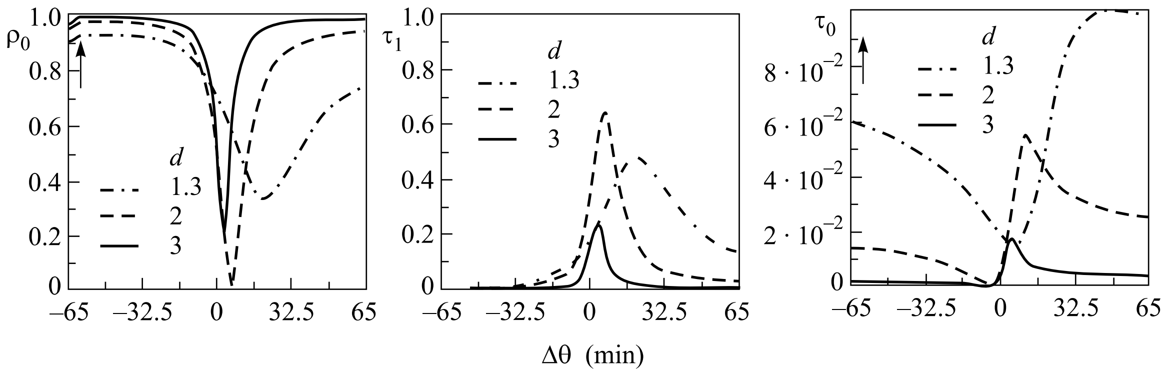

Fig.2. A single nonsymmetric metal-superstrate SB resonance for a sinusoidal grating. Dependences of homogeneous wave TCs on the angle of incidence, $\Delta\theta = \theta - \theta_{\rm i}$, $\theta_{\rm i} = 60^\circ$, for incidence from air onto an Ag film on the quartz substrate, $\varepsilon_+ = 2.31$. Wavelength $\lambda = 1.06\,\mu m$ ($\xi_0 = -0.133i + 0.00071$), spacing $\Lambda = 7.41\,\mu m$ ($\kappa = 0.143$), film thickness d is shown in skin-depths, $\widetilde{\xi}_{\pm 1} = 1.53i\sqrt{\xi_0'}$. The arrows in τ_0 and ρ_0 indicate the Rayleigh anomalies (branchpoints $\beta_{-|1} = 0$)

teristic value, $\sqrt{\xi_0'}$, so that $w = |\widetilde{\xi}_r|/\sqrt{\xi_0'}$ and $w_{\rm opt} \sim 1$. A close-to-total light transmittance, $\tau_r \sim 1$, takes place for film thicknesses of order $\Phi' \sim |\ln \xi_0'|/2$, see Fig.3.

A strong nonzeroth-order anomalous transmittance also can occur under the condition that the zeroth-order wave in the substrate is inhomogeneous ($\varepsilon_- > \varepsilon_+$, cf. Kretshmann geometry, [19]). Then for such an angle of incidence with which the zeroth diffraction order corresponds to SPP excitation on the far interface, $\sqrt{\varepsilon_-}\sin\theta \simeq K^+$, the Nth order diffracted waves with $|\sqrt{\varepsilon_-}\sin\theta + N\kappa| < \sqrt{\varepsilon_+}$, $N < 0$, $\kappa < 2\sqrt{\varepsilon_+}$ are homogeneous ones. The corresponding TCs can become of the same order as the τ_r of the previous case. Besides, for a long-spacing nonharmonic grating, $\kappa < \sqrt{\varepsilon_+}$, the transmitted energy flux is redistributed between these diffraction orders in accordance with the Fourier spectrum of the grating. The first observation of this effect (in the simplest case and for a dielectric grating deposited on the metal film) was reported in paper [12].

Let us briefly discuss more complicated SPP resonances. Along with the resonances related to the excitation of one SPP, double and fourfold resonances can occur under some specific conditions, which result in complex spectral and angular dependencies and in additional enhancement of the transmittance peaks (reflectance dips). Recall that for given values of ε_σ, the period Λ, the resonance condition of Eq. (2) defines a resonance curve in the $\lambda-\theta$ (vacuum wavelength and angle of incidence) in the region $-\sqrt{\varepsilon_-} < K^\sigma - \tau r\kappa < \sqrt{\varepsilon_-}$, which is enumerated by three numbers, σ, τ and the integer r. Then for a fixed θ (and a fixed period) there is a specific value of the wavelength and vice versa, for a fixed λ there is a specific value of angle of incidence corresponding to SPP excitation. In the generic case these curves corresponding to different values of r and σ do not intersect. Then we arrive at a *single* SB resonance (SBS).

Under specific conditions different resonance curves may intersect. An intersection of two curves specify the values of κ and θ corresponding to a *double* resonance. If these curves correspond to equal σ (and two different resonance orders, $r' \neq r$), then a simultaneous excitation of two SPPs on one of the interfaces holds. These SPPs are coupled due to the periodicity, and we arrive at a double SB resonance (SBD). SBDs do not essentially enhance the peak transmittance magnitudes (reflection minima) as compared with the single resonance, however result in complex wavelength and angle of incidence dependencies. Specifically, the inter-resonance modulation amplitude, $\widetilde{\xi}_{r-r'}$, can strongly effect the resonance behavior even for small magnitudes of order $\gtrsim \xi_0'$ (cf. Ref. [20], where a comprehensive examination of such resonances is performed for diffraction on metal-dielectric interfaces). Note that for symmetric film surroundings, $\varepsilon_+ = \varepsilon_-$, we arrive at the fourfold resonance.[4)]

An essential enhancement of the transmittance can be achieved during simultaneous excitation of SPPs on both interfaces, coupled due to the finite film thickness, i.e., when a double DB resonance (DBD) occurs. This corresponds to the intersection of two curves with different σ. Eqs. (6)–(8) describe the symmetric DBD as well, and the transmittance enhancement is caused by the fact that the minimal magnitude of the resonance

4) The *fourfold* DB resonance (DBF) can also exist under the condition that the ratio of the refraction indices of the surrounding dielectrics (or, more exactly, the values of K^+ and K^-) is equal to that of two integers resulting in simultaneous intersection of four resonance curves. Note that the threefold resonance is impossible.

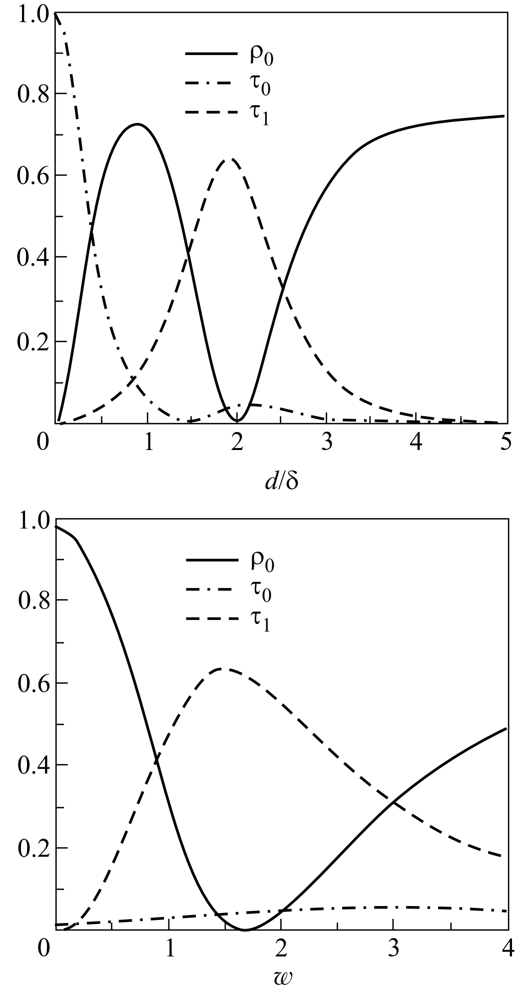

Fig.3. Modulation amplitude and film thickness dependencies of the transmittance and reflectivity (for $d = 2$, in terms of skin-depths $\delta = (k|\xi_0''|)^{-1}$, and for $w = 1.53$, $\widetilde{\xi}_{\pm 1} = iw\sqrt{\xi_0'}$), corresponding to a nonsymmetric, single metal-superstrate SB resonance. The angle of incidence is $\theta = 60°$, other parameters are the same as in Fig.2

denominator, $\widetilde{d}_r$, is squared in comparison with Eq. (9) while the nominator does not decrease like in the case of a SBD resonance. The greatest relative change of the zeroth-order transmittance is of order $|\Delta T_0| \sim \sim |\widetilde{\xi}_r|^2/|\widetilde{d}_r|_{\min} \sim |\widetilde{\xi}_r|^2(\xi_0' + |\widetilde{\xi}_r|^2 + \ldots)^{-2}$. Therefore, the zeroth-order transmittance (along with reflectance and absorbance) anomalies are expressed much stronger for the DB resonances. This fact is in agreement with experimental data and numerical calculations for normal incidence, cf. Refs. [13, 6]. The detailed analysis (to be published elsewhere) for this specific case is in agreement with results for harmonic modulation presented in the theoretical paper [9] (up to the shift of the resonance peak and dip positions due to scattering processes involving second order inhomogeneous diffracted waves, which we take into account).

The approach developed allows a natural generalization to 2D periodic structures, cf. [21] for the half-space problem. The main difference consists in the fact that we have to appeal to a 2D Fourier expansion, $\widetilde{\xi}(\mathbf{r}) = \sum_{n_1,n_2} \widetilde{\xi}_{n_1 n_2} \exp[(in_1\mathbf{g}_1 + in_1\mathbf{g}_1)\mathbf{r}]$, $\widetilde{\xi}_{00} = 0$, where $\mathbf{r} = (x, y)$ and $\mathbf{g}_1$, $\mathbf{g}_2$ are reciprocal vectors relating to minimum translations of the reciprocal lattice. Similar changes need to be performed for the Fourier-Floquet expansion of the electromagnetic field. Consequently, a resonance enhancement of the transmittance can be caused by excitation of a SPP in any diffraction order, now numbered by a pair of integers (r_1, r_2). The approximate resonance condition reads as $|\mathbf{k}_t + r_1\mathbf{g}_1 + r_2\mathbf{g}_2| \simeq K^{\pm}$, where $\mathbf{k}_t$ is the tangential component of the wave vector of the incident wave, $|\mathbf{k}_t| = k^- \sin\theta = \omega c^{-1}\sqrt{\varepsilon_-}\sin\theta$.[5)] The resonance contribution to the TCs depends on the incident polarization and orientation of the SPP excited. Specifically, it depends on the angle φ between the SPP propagation direction relative to the incident plane. No data on the polarization of the incident and transmitted light are given in the majority of experimental works (like the information concerning orientation of the incidence plane), exceptions are Refs. [14, 22][6)]. Therefore, in the generic geometry we can obtain a nontrivial transformation of the polarization of the zeroth-order transmitted wave, similar to that discussed for the resonance reflection, cf. [23, 24]. Simple estimates for the magnitudes of these coefficients are analogous to that presented above for the 1D grating.

We have shown that the principal point for the light tunneling enhancement is the existence of well-defined surface modes (SPPs) at the interfaces. A great effect can be caused both by the zeroth-order transmittance (which has been observed experimentally and discussed theoretically) and other diffraction orders as well. The latter may exceed the zeroth-order transmittance, τ_0, by factors about $\xi_0'^{-1}$ in case of SB resonance, while both zeroth- and nonzeroth-order transmittances under DB resonance diffraction on nonharmonic grating may be comparable.

[5)]In the majority of experimental works devoted to the problem, the double periodicity is realized by square hole arrays (i.e., $(\mathbf{g}_1 \cdot \mathbf{g}_2) = 0$, $g_1 = g_2$).

[6)]Simple calculations show that the zeroth-order TC angle dependence with (without) change of polarization can be approximated to $\sin 2\varphi$ ($\cos^2\varphi$) for the impedance modulation. For the relief modulation it is necessary to change the angle φ to ψ, where ψ is the angle between the incident plane and the vector $r_1\mathbf{g}_1 + r_2\mathbf{g}_2$.

The analytical approach has allowed us to perform a transparent analytical treatment and identify the role of different parameters. It shows as trivialities some results which seem non-trivial within other approaches. While the modulation is supposed to be small, it is in a sense arbitrary (defined by an arbitrary Fourier expansion), in contrast to numerical calculations where the spectral composition of modulation is fixed.

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