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Quantum phase transition for the BEC–BCS crossover in condensed matter physics and CPT violation in elementary particle physics

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We discuss the quantum phase transition which separates a vacuum state with fully-gapped fermion spectrum from a vacuum state with topologically-protected Fermi points (gap nodes). In the context of condensed-matter physics, such a quantum phase transition with Fermi point splitting may occur for a system of ultracold fermionic atoms in the region of the BEC–BCS crossover, provided the Cooper pairing occurs in the non-s-wave channel. For elementary particle physics, the splitting of Fermi points may lead to CPT violation, neutrino oscillations, and other phenomena.

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There are two major schemes for the classification of states in condensed matter physics and relativistic quantum field theory: classification by symmetry and by universality classes.

For the first classification method, a given state of the system is characterized by a symmetry group H which is a subgroup of the symmetry group G of the relevant physical laws (see, e.g., Ref. [1] for symmetry classification of superconducting states). The thermodynamic phase transition between equilibrium states is usually marked by a change of the symmetry group H. The subgroup H is also responsible for topological defects, which are determined by the nontrivial elements of the homotopy groups $\pi_n(G/H)$; cf. Ref. [2].

The second classification method deals with the ground states of the system at zero temperature ($T=0$), i.e., it is the classification of quantum vacua. The universality class determines the general features of the quantum vacuum, such as the linear response and the energy spectrum of fermionic excitations. For translation-invariant systems in which momentum is a well-defined quantity, these features of the fermionic quantum vacuum are determined by momentum-space topology. For (3+1)–dimensional systems, there are only three basic universality classes of fermionic vacua [3]: (i) vacua with fully-gapped fermionic excitations; (ii) vacua with fermionic excitations characterized by Fermi points (the excitations behave as massless Weyl fermions close to the Fermi points); (iii) vacua with fermionic excitations characterized by Fermi surfaces. [Fermi points $\mathbf{p}_n$ are points in 3-momentum space at which the energy vanishes, $E(\mathbf{p}_n)=0$, and similarly for Fermi surfaces S_n, with $E(\mathbf{p})=0$ for $\mathbf{p}\in S_n$.]

It may happen that by changing some parameter q of the system we transfer the vacuum state from one universality class to another, without changing its symmetry group H. The point q_c, where this zero-temperature transition occurs, marks the quantum phase transition. For $T\neq 0$, the phase transition is absent, as the two states belong to the same symmetry class H. Hence, there is an isolated singular point $(q_c,0)$ in the (q,T) plane. Two examples of a quantum phase transition are: 1. the Lifshitz transition in crystals, at which the Fermi surface changes its topology or shrinks to a point; and 2. the transition between states with different values of the Hall (or spin-Hall) conductance in (2+1)–dimensional systems.

In this Letter, we discuss the quantum phase transition between a vacuum with fully-gapped fermionic excitations and a vacuum with Fermi points. At the transition point $q=q_c$, a topologically-trivial Fermi point emerges from the fully-gapped state. This marginal Fermi point then splits into two or more topologically-nontrivial Fermi points (see Figure). The topologically-protected Fermi points give rise to anomalous properties of the system in the low-temperature regime; cf. Sec. 7.3.2 of Ref. [4] and Part IV of Ref. [3].

These effects may occur in a system of ultracold fermionic atoms in the region of the BEC–BCS crossover in a non-s–wave Cooper channel. Superfluidity in the BEC regime and the BEC–BCS crossover has been observed for ^{40}K and ^{6}Li atoms [5 – 9]. In these exper-

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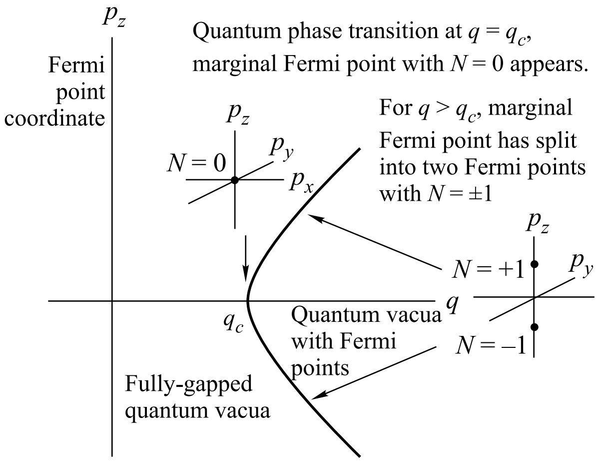

Quantum phase transition at $q = q_c$ between a fully-gapped vacuum and a vacuum with topologically-protected Fermi points (gap nodes). At $q = q_c$, there appears a marginal Fermi point with topological charge $N = 0$ (inset at the top). For $q > q_c$, the marginal Fermi point has split into two Fermi points characterized by nonzero topological invariants $N = \pm 1$ (inset on the right). For a system of ultracold fermionic atoms qualitatively described by Hamiltonians (1) and (9), the critical parameter is $q_c = 0$ [note that eight Fermi points emerge for the case of Hamiltonian (9)]. For Dirac fermions with CPT violation in Hamiltonian (6), the parameter q is chosen as $q \equiv |\mathbf{b}|$ and the critical parameter is $q_c = M$

iments, a magnetic-field Feshbach resonance was used to control the interactions in the s-wave channel. For the case of s-wave pairing, there are fully-gapped vacua on both sides of the crossover and there is no quantum phase transition. If, however, the pairing occurs in a non-s-wave channel, a quantum phase transition may be expected between the fully-gapped state and the state with Fermi points. It was reported recently [10, 11] that three p–wave Feshbach resonances were found for ^{6}Li atoms. This suggests the possibility of future observations of non-s-wave pairing and of the quantum phase transition associated with the splitting of Fermi points.

Here, we will discuss two examples of such a transition, using for simplicity p–wave spin-triplet pairing and their possible analogs in relativistic quantum field theory. We will argue in the following that a similar quantum phase transition characterized by Fermi point splitting may occur for the Standard Model of elementary particle physics [12], but refer the reader to Refs. [13–15] for further details. In fact, condensed-matter physics provides us with a broad class of quantum field theories not restricted by Lorentz invariance, which allows us to consider many problems in the relativistic quantum field theory of the Standard Model from a more general perspective. Just as for nonrelativistic systems, the basic properties of relativistic quantum field theories (including quantum anomalies) are determined by momentum-space topology, which classifies the relativistic vacua according to the same three universality classes.

Since we are only interested in effects determined by the topology and the symmetry of the fermionic Green's function $G(p)$, we do not require a special form of the Green's function and can choose the simplest one with the required topology. First, consider the Bogoliubov–Nambu Hamiltonian which qualitatively describes fermionic quasiparticles in the axial state of p–wave pairing. This Hamiltonian can be applied to both the Bardeen–Cooper–Schrieffer (BCS) and Bose–Einstein condensation (BEC) regimes, and also to superfluid ^{3}He-A [4]. Specifically, the Bogoliubov–Nambu Hamiltonian is given by:

$$H = \\ = \begin{pmatrix} |\mathbf{p}|^2/2m - q & c_\perp \mathbf{p} \cdot (\widehat{\mathbf{e}}_1 + i\,\widehat{\mathbf{e}}_2) \\ c_\perp \mathbf{p} \cdot (\widehat{\mathbf{e}}_1 - i\,\widehat{\mathbf{e}}_2) & -|\mathbf{p}|^2/2m + q \end{pmatrix}, \qquad (1)$$

and $G^{-1}(i\omega, \mathbf{p}) = i\omega - H(\mathbf{p})$, with $\hbar = 1$. Considered are fermionic atoms of mass m with a given direction of the atomic spin, assuming that only these atoms experience the Feshbach resonance. The orthonormal triad $(\widehat{\mathbf{e}}_1, \widehat{\mathbf{e}}_2, \widehat{\mathbf{l}} \equiv \widehat{\mathbf{e}}_1 \times \widehat{\mathbf{e}}_2)$ and the maximum transverse speed $c_\perp$ of the quasiparticles characterize the order parameter in the axial state of triplet superfluid. The unit vector $\widehat{\mathbf{l}}$ corresponds to the direction of the orbital momentum of the Cooper pair or the diatomic molecule. We further assume that the parameter q is controlled by the magnetic field in the vicinity of the Feshbach resonance.

The energy spectrum of these Bogoliubov–Nambu fermions is

$$E^2(\mathbf{p}) = \left(\frac{|\mathbf{p}|^2}{2m} - q\right)^2 + c_\perp^2 \left(\mathbf{p} \times \widehat{\mathbf{l}}\right)^2. \qquad (2)$$

The BCS regime occurs for $q > 0$, with the parameter q playing the role of a chemical potential. In this regime, there are two Fermi points, i.e., points in 3-momentum space with $E(\mathbf{p}) = 0$. For the energy spectrum (2), the Fermi points are $\mathbf{p}_1 = p_F\,\widehat{\mathbf{l}}$ and $\mathbf{p}_2 = -p_F\,\widehat{\mathbf{l}}$, with Fermi momentum $p_F = \sqrt{2mq}$.

For a general system, be it relativistic or nonrelativistic, the stability of the a-th Fermi point is guaranteed by the topological invariant N_a, which can be written as a surface integral in frequency-momentum space. In terms of the fermionic propagator $G = G(p_0, p_1, p_2, p_3)$, for $p_\mu = (\omega, \mathbf{p})$, the topological invariant is [3]

$$N_a \equiv \frac{1}{24\pi^2}\epsilon_{\mu\nu\rho\sigma}\ \mathrm{tr}\oint_{\Sigma_a} dS^\sigma \times$$
$$\times\ G\frac{\partial}{\partial p_\mu}G^{-1}\ G\frac{\partial}{\partial p_\nu}G^{-1}\ G\frac{\partial}{\partial p_\rho}G^{-1}, \qquad (3)$$

where Σ_a is a three-dimensional surface around the isolated Fermi point $p_{\mu a} = (0, \mathbf{p}_a)$ and 'tr' stands for the trace over the relevant spin indices.

For the case considered, the trace in Eq. (3) is over the Bogoliubov-Nambu spin and the two Fermi points $\mathbf{p}_1$ and $\mathbf{p}_2$ have nonzero topological charges $N_1 = +1$ and $N_2 = -1$. The density of states in this gapless regime is given by $\nu(E) \propto E^2$. At $q = 0$, these two Fermi points merge and form one topologically-trivial Fermi point with $N = 0$. This intermediate state, which appears at the point of quantum phase transition ($q_c = 0$), is marginal: the momentum-space topology is trivial and cannot protect the vacuum against decay into one of the two topologically-stable vacua. For $q < 0$, the marginal Fermi point disappears altogether and the spectrum becomes fully-gapped. In this topologically-stable fully-gapped vacuum, the density of states is drastically different from that in the topologically-stable gapless regime: $\nu(E) = 0$ for $E < |q|$. All this demonstrates that the quantum phase transition considered is of purely topological origin.

Note that if a single pair of Fermi points appears in momentum space, the vacuum state has nonzero internal angular momentum along $\hat{\mathbf{l}}$, i.e., this quantum vacuum has the property of an orbital ferromagnet. Later, we will discuss an example with multiple Fermi points, for which the total orbital momentum is zero and the vacuum state corresponds to an orbital antiferromagnet.

We now turn to elementary particle physics [12]. It appears that the vacuum of the Standard Model above the electroweak transition (vanishing fermion masses) is marginal: there is a multiply degenerate Fermi point $\mathbf{p} = 0$ with topological charge $N = 0$. It is therefore the intermediate state between two topologically-stable vacua, the fully-gapped vacuum and the vacuum with topologically-nontrivial Fermi points. In the Standard Model, this marginal Fermi point is protected by symmetries, namely the continuous electroweak symmetry (or the discrete symmetry discussed in Sec. 12.3.2 of Ref.[3]) and the CPT symmetry.

Explicit violation or spontaneous breaking of one of these symmetries transforms the marginal vacuum of the Standard Model into one of the two topologically-stable vacua. If, for example, the electroweak symmetry is broken, the marginal Fermi point disappears and the fermions become massive. This is known to happen with the quarks and electrically charged leptons below the electroweak transition. If, on the other hand, the CPT symmetry is violated, the marginal Fermi point splits into topologically-stable Fermi points. One can speculate that the latter happens for the Standard Model, in particular with the electrically neutral leptons, the neutrinos [13 – 15]. The splitting of Fermi points may also give rise to a CPT-violating Chern–Simons-like term in the effective gauge field action [16, 17], as will be discussed later.

Let us first consider this scenario for a marginal Fermi point describing a *single* pair of relativistic chiral fermions, that is, one right-handed fermion and one left-handed fermion. These are Weyl fermions with Hamiltonians $H_{\text{right}} = \boldsymbol{\sigma}\cdot\mathbf{p}$ and $H_{\text{left}} = -\boldsymbol{\sigma}\cdot\mathbf{p}$, where $\vec{\sigma}$ denotes the triplet of Pauli matrices and natural units are employed with $c = \hbar = 1$. Each of these Hamiltonians has a topologically-stable Fermi point $\mathbf{p} = 0$. The corresponding inverse Green's functions are given by

$$G^{-1}_{\text{right}}(i\omega, \mathbf{p}) = i\omega - \boldsymbol{\sigma}\cdot\mathbf{p},$$
$$G^{-1}_{\text{left}}(i\omega, \mathbf{p}) = i\omega + \boldsymbol{\sigma}\cdot\mathbf{p}. \qquad (4)$$

The positions of the Fermi points coincide, $\mathbf{p}_1 = \mathbf{p}_2 = 0$, but their topological charges (3) are different. For this simple case, the topological charge equals the chirality of the fermions, $N_a = C_a$ (i.e., $N = +1$ for the right-handed fermion and $N = -1$ for the left-handed one). The total topological charge of the Fermi point $\mathbf{p} = 0$ is therefore zero.

The splitting of this marginal Fermi point can be described by the Hamiltonians $H_{\text{right}} = \boldsymbol{\sigma}\cdot(\mathbf{p}-\mathbf{p}_1)$ and $H_{\text{left}} = -\boldsymbol{\sigma}\cdot(\mathbf{p}-\mathbf{p}_2)$, with $\mathbf{p}_1 = -\mathbf{p}_2 \equiv \mathbf{b}$ from momentum conservation. The real vector $\mathbf{b}$ is assumed to be odd under CPT, which introduces CPT violation into the physics. The 4×4 matrix of the combined Green's function has the form

$$G^{-1}(i\omega, \mathbf{p}) =$$
$$= \begin{pmatrix} i\omega - \boldsymbol{\sigma}\cdot(\mathbf{p}-\mathbf{b}) & 0 \\ 0 & i\omega + \boldsymbol{\sigma}\cdot(\mathbf{p}+\mathbf{b}) \end{pmatrix}. \qquad (5)$$

Equation (3) shows that $\mathbf{p}_1 = \mathbf{b}$ is the Fermi point with topological charge $N = +1$ and $\mathbf{p}_2 = -\mathbf{b}$ the Fermi point with topological charge $N = -1$.

Let us now consider the more general situation with both the electroweak and CPT symmetries broken. The Hamiltonian has then an additional mass term,

$$H = \begin{pmatrix} \boldsymbol{\sigma}\cdot(\mathbf{p}-\mathbf{b}) & M \\ M & -\boldsymbol{\sigma}\cdot(\mathbf{p}+\mathbf{b}) \end{pmatrix} =$$
$$= H_{\text{Dirac}} - \mathbf{I}_2 \otimes (\vec{\sigma}\cdot\mathbf{b})\,. \qquad (6)$$

This Hamiltonian is the typical starting point for investigations of the effects of CPT violation in the fermionic sector (see, e.g., Refs. [18, 19] and references therein). The energy spectrum of Hamiltonian (6) is

$$E_{\pm}^2(\mathbf{p}) = M^2 + |\mathbf{p}|^2 + q^2 \pm \pm 2\,q\sqrt{M^2 + \left(\mathbf{p}\cdot\widehat{\mathbf{b}}\right)^2}, \tag{7}$$

with $\widehat{\mathbf{b}} \equiv \mathbf{b}/|\mathbf{b}|$ and $q \equiv |\mathbf{b}| \geq 0$.

Allowing for a variable parameter q, one finds a quantum phase transition at $q_c = M$ between fully-gapped vacua for $q < M$ and vacua with two Fermi points for $q > M$. These Fermi points are given by

$$\mathbf{p}_1 = +\widehat{\mathbf{b}}\,\sqrt{q^2 - M^2}, \qquad \mathbf{p}_2 = -\widehat{\mathbf{b}}\,\sqrt{q^2 - M^2}. \tag{8}$$

Equation (3), now with a trace over the indices of the 4×4 Dirac matrices, shows that $\mathbf{p}_1$ is the Fermi point with topological charge $N = +1$ and $\mathbf{p}_2$ the Fermi point with topological charge $N = -1$ [see Figure for $\widehat{\mathbf{b}} = (0,0,1)$]. The magnitude of the splitting of the two Fermi points is given by $2\sqrt{q^2 - M^2}$. At the quantum phase transition $q_c = M$, the Fermi points with opposite charge annihilate each other and form a marginal Fermi point $\mathbf{p}_0 = 0$. The momentum-space topology of this marginal Fermi point is trivial (topological invariant $N = +1 - 1 = 0$).

The full Standard Model contains *eight* pairs of chiral fermions per family and a quantum phase transition can be characterized by the appearance and splitting of multiple marginal Fermi points. For systems of cold atoms, an example is provided by another spin-triplet p–wave state, the so-called α–phase with orbital antiferromagnetism. The Bogoliubov–Nambu Hamiltonian which qualitatively describes fermionic quasiparticles in the α–state is given by [1, 4]:

$$H = \begin{pmatrix} |\mathbf{p}|^2/2m - q & (\Sigma\cdot\mathbf{p})\;c_\perp/\sqrt{3} \\ (\Sigma\cdot\mathbf{p})^\dagger\;c_\perp/\sqrt{3} & -|\mathbf{p}|^2/2m + q \end{pmatrix}, \tag{9}$$

with $|\mathbf{p}|^2 \equiv p_x^2 + p_y^2 + p_z^2$ and $\Sigma\cdot\mathbf{p} \equiv \sigma_x p_x + \exp(2\pi i/3)\,\sigma_y p_y + \exp(-2\pi i/3)\,\sigma_z p_z$.

On the BEC side ($q < 0$), fermions are again fully-gapped, while on the BCS side ($q > 0$), there are eight Fermi points $\mathbf{p}_a$ ($a = 1,\ldots,8$), situated at the vertices of a cube in momentum space [1]. The fermionic excitations in the vicinity of these points are left- and right-handed Weyl fermions. In terms of the Cartesian unit vectors ($\widehat{\mathbf{x}}$, $\widehat{\mathbf{y}}$, $\widehat{\mathbf{z}}$), the four Fermi points with right-handed Weyl fermions ($C_a = +1$, for $a = 1,\ldots,4$) are given by

$$\begin{aligned} \mathbf{p}_1 &= p_F\;(+\widehat{\mathbf{x}} + \widehat{\mathbf{y}} + \widehat{\mathbf{z}})/\sqrt{3}, \\ \mathbf{p}_2 &= p_F\;(+\widehat{\mathbf{x}} - \widehat{\mathbf{y}} - \widehat{\mathbf{z}})/\sqrt{3}, \\ \mathbf{p}_3 &= p_F\;(-\widehat{\mathbf{x}} - \widehat{\mathbf{y}} + \widehat{\mathbf{z}})/\sqrt{3}, \\ \mathbf{p}_4 &= p_F\;(-\widehat{\mathbf{x}} + \widehat{\mathbf{y}} - \widehat{\mathbf{z}})/\sqrt{3}, \end{aligned} \tag{10}$$

while the four Fermi points with the left-handed Weyl fermions ($C_a = -1$, for $a = 5,\ldots,8$) have opposite vectors.

Since the quantum phase transition between the BEC and BCS regimes of ultracold fermionic atoms and the quantum phase transition for Dirac fermions with CPT violation are described by the same momentum-space topology, we can expect common properties. An example of such a common property would be the axial or chiral anomaly. For quantum anomalies in (3+1)–dimensional systems with Fermi points and their reduction to (2+1)–dimensional systems, see, e.g., Refs. [3, 20] and references therein.

One manifestation of the anomaly is the topological Wess–Zumino–Novikov–Witten (WZNW) term in the effective action. The general expression for the WZNW term is represented by the following sum over Fermi points (see, for example, Eq. (6a) in Ref. [21]):

$$S_{\rm WZNW} = (12\pi^2)^{-1}\sum_a N_a \times \times \int d^3x\;dt\;d\tau\;\mathbf{p}_a\cdot(\partial_\tau\mathbf{p}_a\times\partial_t\mathbf{p}_a). \tag{11}$$

Here, N_a is the topological charge of a-th Fermi point and $\tau\in[0,1]$ is an additional coordinate which parametrizes a disc, with the usual spacetime at the boundary $\tau = 1$.

In the Standard Model, Eq.(11) can be seen to give rise to an anomalous Chern–Simons-like action term in the gauge-field sector. Start, for simplicity, from the spectrum of a single electrically charged Dirac fermion (charge e) and again set $c = \hbar = 1$. In the presence of the vector potential $\mathbf{A}$ of a $U(1)$ gauge field, the minimally-coupled version of Hamiltonian (6) is

$$H = = \begin{pmatrix} \boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A} - \mathbf{b}) & M \\ M & -\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A} + \mathbf{b}) \end{pmatrix}. \tag{12}$$

The positions of the Fermi points for $q \equiv |\mathbf{b}| > M$ are then shifted due to the gauge field,

$$\mathbf{p}_a \equiv \mathbf{p}_a^{(0)} + e\mathbf{A} = \pm\widehat{\mathbf{b}}\;\sqrt{q^2 - M^2} + e\mathbf{A}, \tag{13}$$

with a plus sign for $a = 1$ and a minus sign for $a = 2$. This result follows immediately from Eq. (8) by the minimal substitution $\mathbf{p}_a \to \mathbf{p}_a - e\mathbf{A}$, consistent with the gauge principle. For relativistic quantum field theory and with different charges e_a at the different Fermi points, one has the general expression $\mathbf{p}_a = \mathbf{p}_a^{(0)} + e_a\mathbf{A}$.

Next, insert these Fermi points into formula (11) and assume the charges to be τ dependent, so that $\mathbf{p}_a = \mathbf{p}_a^{(0)} + e_a(\tau)\,\mathbf{A}$. Specifically, we use a parametrization for which the charges $e_a(\tau)$ are zero at the center of the disc, $e_a(0) = 0$, and equal to the physical charges at the boundary of the disc, $e_a(1) = e_a$. From Eq. (11), one then obtains the general form for the Abelian Chern–Simons-like term

$$S_{\text{CS-like}} = (24\pi^2)^{-1} \sum_a N_a\, e_a^2 \times \times \int d^3x\, dt\;\; \mathbf{p}_a^{(0)} \cdot (\mathbf{A} \times \partial_t \mathbf{A}). \qquad (14)$$

This result has the "relativistic" form

$$S_{\text{CS-like}} = \int d^4x\; k_\mu\, \epsilon^{\mu\nu\rho\sigma} A_\nu(x)\, \partial_\rho A_\sigma(x), \qquad (15)$$

with gauge field $A_\mu(x)$, Levi–Civita symbol $\epsilon^{\mu\nu\rho\sigma}$, and a purely spacelike "vector" k_μ,

$$k_\mu = (0, \mathbf{k}) = \left(0,\; (24\pi^2)^{-1} \sum_a \mathbf{p}_a^{(0)}\, e_a^2\, N_a\right). \qquad (16)$$

Note that only gauge invariance has been assumed in the derivation of Eq. (16). As shown in the Appendix of Ref. [13], the Chern–Simons vector (16) can be written in the form of a momentum-space topological invariant.

Returning to the case of a single Dirac fermion with charge e and using Eqs. (16) and (8), one finds that the CPT-violating Chern–Simons parameter $\mathbf{k}$ can be expressed in terms of the CPT-violating parameter $\mathbf{b}$ of the fermionic sector,

$$\mathbf{k} = \frac{e^2}{12\pi^2}\, \theta(q - M)\, \widehat{\mathbf{b}}\, \sqrt{q^2 - M^2}. \qquad (17)$$

This particular contribution to $\mathbf{k}$ comes from the splitting of a marginal Fermi point, which requires $|\mathbf{b}| \equiv q > > M$, as indicated by the step function on the right-hand side [$\theta(x) = 0$ for $x \le 0$ and $\theta(x) = 1$ for $x > 0$].

In the context of relativistic quantum field theory, the existence of such a nonanalytic contribution to $\mathbf{k}$ has also been found by Perez-Victoria [22] and Andrianov et al. [23] using standard regularization methods, but with a prefactor larger by a factor 3 and 3/2, respectively. The result (17), on the other hand, is determined by the general topological properties of the Fermi points [13] and applies to nonrelativistic quantum field theory as well. In condensed-matter quantum field theory, the result has been obtained without ambiguity, since the microphysics is known at all scales and regularization occurs naturally.

For the "ferromagnetic" quantum vacuum of Hamiltonian (6), the Chern–Simons vector $\mathbf{k}$ obtained from Eq.(16) by summation over all Fermi points (8) is nonzero and given by Eq. (17). For the "antiferromagnetic" α–phase vacuum of Hamiltonian (9), the vector $\mathbf{k}$ vanishes, because $e_a^2 = 1$ for the fermion charges $e_a = \pm 1$ and $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_4 = 0$ for the tetrahedron (10). A similar situation may occur for the Standard Model: antiferromagnetic splitting of the Fermi point without induced Chern–Simons-like term [13]. The antiferromagnetic splitting may, however, lead to other observable effects such as neutrino oscillations [14, 15].

In conclusion, one may expect quantum phase transitions in systems of ultracold atoms, provided the pairing occurs in the non-s-wave channel. The quantum phase transition separates an anomalous-free fully-gapped vacuum on the BEC side and a gapless superfluid state on the BCS side, which is characterized by Fermi points and quantum anomalies. This phenomenon is general and may occur in many different systems, including the vacuum of the relativistic quantum field theory relevant to elementary particle physics.

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