Pis'ma v ZhETF, vol. 78, iss. 6, pp. 801 – 803

Comment on the proper QCD string dynamics in a heavy-light system

A. V. Nefediev[1)]

Institute of Theoretical and Experimental Physics, 117218 Moscow, Russia

Submitted 9 July 2003

The string correction to the inter-quark interaction at large distances is derived using the field theory approach to a heavy-light quark-antiquark system in the modified Fock-Schwinger gauge.

PACS: 11.10.Ef, 12.38.Aw, 12.39.Ki

Quantum chromodynamics at large distances is believed to be a string theory with the effective extended object – the QCD string – formed by nonperturbative gluons, which plays an important role in hadronic phenomenology. It was demonstrated in a number of approaches that the account for the proper dynamics of the QCD string strongly affects hadronic spectra and is necessary to explain the correct Regge trajectory slopes [1 – 3], to resolve puzzles with the identification of new states [4], and so on. At large inter-quark distances this dynamics can be encoded in the so-called string correction [5, 2], well known in the theory of the straight-line Nambu-Goto string with the tension σ and with massive quarks at the ends. The Lagrangian of this system is

$$L=-M\sqrt{\dot{x}_1^2}-m\sqrt{\dot{x}_2^2}-\sigma\int_0^1 d\beta\sqrt{(\dot{w}w')^2-\dot{w}^2w'^2},$$
$$w_\mu(t,\beta)=\beta x_{1\mu}(t)+(1-\beta)x_{2\mu}(t), \tag{1}$$

which leads to the centre of mass. Hamiltonian, at large inter-quark distances $r=|\mathbf{x}_1-\mathbf{x}_2|$, for $M\gg m$ [2]:

$$H\approx M+m+\frac{\mathbf{p}^2}{2m}+\sigma r-\frac{\sigma\hat{L}^2}{6m^2r}+\dots, \tag{2}$$

where, for the sake of convenience, we synchronised the quark times, $x_{10}=x_{20}\equiv x_0$, and fixed the reparametrisation invariance of the Lagrangian (1) by the laboratory frame condition $t=x_0$. The nonperturbative spin-orbit interaction comes from the area law for the Wilson loop [6],

$$V_{so}=-\boldsymbol{\sigma}\mathbf{L}/4m^2r, \tag{3}$$

and should be added to the Hamiltonian (2). The expansion in Eq. (2) is valid for $m\gg\sqrt{\sigma}$. Perturbative Coulomb interaction as well as extra spin-dependent terms due to the latter can be taken into account, and the resulting model appears rather successful in describing hadronic spectra (see, for example, [7], where the Hamiltonian (2) supplied by the perturbative interaction, but without the string correction, was used). A more sophisticated approach based on the einbein field formalism [8] is also well known in the literature [2]. This method possesses several advantages as compared to the Hamiltonian (2) since, in this case, the corresponding Hamiltonian is given in terms of the effective dynamically generated quark masses μ's, given by the extremal values of the corresponding einbeins [9]. For light quarks such a dynamical mass appear of order of the interaction scale, $\mu\sim\sqrt{\sigma}$, that is, much larger than the current quark mass – the latter can be even put to zero.

Recently, another approach to heavy-light systems was suggested, based on the Schwinger-Dyson series for a light quark in presence of a static antiquark [10, 11]. Namely, the Schwinger-Dyson equation, in Euclidean space-time,

$$(-i\hat{\partial}_x-im)S(x,y)-i\int d^4zM(x,z)S(z,y)=$$
$$=\delta^{(4)}(x-y), \tag{4}$$

was derived in the modified Fock–Schwinger gauge [12],

$$A_4(x_4,\mathbf{0})=0,\quad \mathbf{x}\mathbf{A}(x_4,\mathbf{x})=0, \tag{5}$$

where the self–energy part $M(x,z)$ and the light-quark Green's function (also playing the role of the $q\bar{Q}$ Green's function) are given by [10]

$$-iM(x,z)=K_{\mu\nu}(x,z)\gamma_\mu S(x,z)\gamma_\nu,$$
$$S(x,y)=\frac{1}{N_C}\langle\psi^\beta(x)\psi^+_\beta(y)\rangle. \tag{6}$$

The interaction kernel $K_{\mu\nu}$ can be expressed in terms of the irreducible field strength correlator $\langle F^a_{\mu\nu}(x)F^b_{\lambda\rho}(y)\rangle$ [13],

[1)] e-mail: nefediev@heron.itep.ru

$$\langle F^a_{\mu\nu}(x)F^b_{\lambda\rho}(y)\rangle = \delta^{ab}\frac{2N_C}{N_C^2-1}\times$$
$$\times D(x_0-y_0,|\mathbf{x}-\mathbf{y}|)(\delta_{\mu\lambda}\delta_{\nu\rho}-\delta_{\mu\rho}\delta_{\nu\lambda})+\Delta^{(1)}, \quad (7)$$

where the second term $\Delta^{(1)}$ is a full derivative and does not contribute to confinement. As we are interested in the long-range force, we consider only the term proportional to $D(x-y)$ in (7) which, in contrast to $\Delta^{(1)}$, contributes to the area law with the string tension

$$\sigma = 2\int_0^\infty d\tau\int_0^\infty d\lambda D(\tau,\lambda). \quad (8)$$

Finally, for the kernel $K_{\mu\nu}$ in the gauge (5), one has ($\tau = x_4 - y_4$) [10, 11]:

$$K_{44}(\tau,\mathbf{x},\mathbf{y}) = (\mathbf{x}\mathbf{y})\int_0^1 d\alpha\int_0^1 d\beta D(\tau,|\alpha\mathbf{x}-\beta\mathbf{y}|),$$
$$K_{i4}(\tau,\mathbf{x},\mathbf{y}) = K_{4i}(\tau,\mathbf{x},\mathbf{y}) = 0,$$
$$K_{ik}(\tau,\mathbf{x},\mathbf{y}) =$$
$$= ((\mathbf{x}\mathbf{y})\delta_{ik}-y_ix_k)\int_0^1 \alpha d\alpha\int_0^1 \beta d\beta D(\tau,|\alpha\mathbf{x}-\beta\mathbf{y}|). \quad (9)$$

Using a consequent expansion of Eq. (4) for a large quark mass m ($m \gg \sqrt{\sigma}$ and $mT_g \gg 1$ [11, 14], where T_g is the gluonic correlation length) one can derive the inter-quark interaction which is in agreement with the Eichten–Feinberg–Gromes results [15, 16]. Then, applying the Foldy–Wounthuysen (FW) transformation to the resulting interaction, it is easy to derive a Hamiltonian of the heavy-light system, at $r \gg T_g$, in the form [11, 14]:

$$H_{FW} = M + m + \frac{\mathbf{p}^2}{2m} + \sigma r - \frac{\sigma\mathbf{L}}{4m^2r} + \ldots, \quad (10)$$

where the ellipsis denotes terms $O(\sigma r/mT_g)$ suppressed in the limit $mT_g \gg 1$ [14, 17].

The Hamiltonian (10) coincides with the Hamiltonian of the quantum-mechanical quark-antiquark system connected by the Nambu-Goto string supplied by the nonperturbative spin-dependent interaction given by Eqs. (2), (3). In the meantime, an important ingredient mentioned above – the string correction – is still missing in the formula (10). The aim of the present paper is to resolve this inconsistency and, therefore, to complete matching of the two approaches: one, based on the quantum mechanical string model, and the other, following from the field theoretical treatment of the heavy-light quark-antiquark system.

Following the path integral ideology, we consider the trajectory of the quark, $\mathbf{r}(t)$, such that the two consequent positions of the latter are $\mathbf{x} = \mathbf{r}(t_1)$ and $\mathbf{y} = \mathbf{r}(t_2)$. Therefore for close t_1 and t_2 one can use the expansion:

$$\mathbf{y} = \mathbf{r}(t_2) = \mathbf{r}(t_1+\tau) \approx \mathbf{r}(t_1) + \dot{\mathbf{r}}(t_1)\tau = \mathbf{x} + \frac{\mathbf{p}}{m}\tau, \quad (11)$$

where $\mathbf{p}$ is the momentum of the quark. Due to the rotational invariance the function D from Eq. (7) actually depends on the certain combination of its arguments, $D(\tau,\lambda) = D(\tau^2+\lambda^2)$. In our case $\tau = t_2 - t_1$ and $\lambda = |\alpha\mathbf{x}-\beta\mathbf{y}|$, so that, with the help of the expansion (11), one easily finds:

$$\tau^2+\lambda^2 = \tau^2 + \left[(\alpha-\beta)\mathbf{r} + \alpha\tau\frac{\mathbf{p}}{m}\right]^2 =$$
$$= \left[1+\frac{\alpha^2p^2}{m^2}\right](\tau-\tau_0)^2 + \frac{(\alpha-\beta)^2}{1+\frac{\alpha^2p^2}{m^2}}\left[r^2+\alpha^2\frac{L^2}{m^2}\right], \quad (12)$$

where $\mathbf{r} \equiv \mathbf{x}$, $\mathbf{L}$ is the angular momentum, $\mathbf{L} = [\mathbf{r}\times\mathbf{p}]$, and the constant $\tau_0 = \alpha(\beta-\alpha)(\mathbf{r}\mathbf{p})/m(1+\alpha^2p^2/m^2)$ can be excluded using an appropriate shift of the time variable τ, so we omit it below.

The confining spin-independent interaction, at large inter-quark distances and in the limit $mT_g \gg 1$, is given by the formula [10, 11, 14]:

$$V_{conf}(r) = \gamma_\mu\frac{1+\gamma_0}{2}\gamma_\nu\int_0^\infty d\tau K_{\mu\nu}(\tau,\mathbf{x},\mathbf{y})_{|\mathbf{y}\to\mathbf{x}}. \quad (13)$$

Using the relations (9), one can easily calculate that

$$\int_0^\infty d\tau K_{00}(\tau,\mathbf{x},\mathbf{y})_{|\mathbf{y}\to\mathbf{x}} = r^2\int_0^\infty d\tau\int_0^1 d\alpha\int_0^1 d\beta\times$$
$$\times D\left(\tau\sqrt{1+\frac{\alpha^2p^2}{m^2}},(\alpha-\beta)\sqrt{\frac{r^2+\frac{\alpha^2L^2}{m^2}}{1+\frac{\alpha^2p^2}{m^2}}}\right)\underset{r\gg T_g}{\approx}$$
$$\underset{r\gg T_g}{\approx} r\left(2\int_0^\infty d\tau'\int_0^\infty d\lambda D(\tau',\lambda)\right)\times$$
$$\times\int_0^1\frac{d\alpha}{\sqrt{1+\frac{\alpha^2L^2}{m^2r^2}}} \approx \sigma r - \frac{\sigma L^2}{6m^2r}, \quad (14)$$

and, similarly,

$$\int_0^\infty d\tau K_{ik}(\tau,\mathbf{x},\mathbf{y})_{|\mathbf{y}\to\mathbf{x}} \approx (\delta_{ik}-n_in_k)\left(\frac{1}{3}\sigma r-\frac{\sigma L^2}{10m^2r}\right), \quad (15)$$

where the definition of the string tension (8) was used, as well as the case $n=0$ of the general formula

$$\int_0^1 d\alpha\int_0^1 d\beta f(\alpha,\beta)(\alpha-\beta)^n D(\tau,|\alpha-\beta|a) \underset{a\gg 1}{\approx}$$

$$\underset{a\gg 1}{\approx} \frac{2}{a^{n+1}}\int_0^\infty d\lambda D(\tau,\lambda)\int_0^1 d\alpha f(\alpha,\alpha),$$

which holds for an arbitrary function $f(\alpha,\beta)$, provided $f(\alpha,\alpha)\neq 0$.

Therefore, the confining interaction (13), in the limit $mT_g\gg 1$, becomes

$$V_{\rm conf}(r)=\left(\frac{5}{6}+\frac{1}{6}\gamma_0\right)\sigma r-\left(\frac{11}{60}-\frac{1}{60}\gamma_0\right)\frac{\sigma L^2}{m^2r}, \quad (16)$$

or, after the FW rotation this corresponds to the confining potential

$$V_{\rm conf}^{\rm FW}(r)=\sigma r-\frac{\sigma L^2}{6m^2r}. \quad (17)$$

The first term of the interaction (16), (17) was obtained in refs. [10, 11, 14, 17], whereas the second term was missing due to the immediate substitution of $\mathbf{y}=\mathbf{x}$ in the formula (13), which holds up to the order $1/m$, but, as demonstrated above, fails in the next order, $1/m^2$. As a result, the string correction was lost, although a more accurate expansion of the correlator D, performed in this paper, allows one to reproduce the confining potential, including its part due to the proper string dynamics. Therefore we conclude that indeed the string correction accompanies the linear confinement potential whatever approach is used to derive the latter, provided the string picture of confinement is adopted. Meanwhile the suggested approach is rather inconvenient for further investigations of the confining interaction in the approach of the Schwinger-Dyson nonlinear equation (4). On the other hand, a promising step is made in the paper [18] where a contour gauge is introduced which generalises the gauge condition (5) for the case of an arbitrary trajectory of the heavy particle. Formally, Eqs. (4), (6) remain valid, though the kernel of the interaction becomes contour-dependent. In the meantime, the form of the contour depends on the heavy particle trajectory, that is, it is defined dynamically, and the problem becomes selfconsistent. Consequent expansion of the aforementioned contour around the straight-line form may provide a way of systematic account for the $(1/m)^n$ and $(1/M)^n$ corrections in this approach.

Useful discussions with Yu. A. Simonov and Yu. S. Kalashnikova are acknowledged. This work is supported by INTAS, via grants OPEN # 2000-110 and YSF # 2002-49, by the grant # NS-1774.2003.2, and by the Federal Programme of the Russian Ministry of Industry, Science and Technology # 40.052.1.1.1112.

1. M. G. Olsson, Phys. Rev. **D55**, 5479 (1997).
2. A. Yu. Dubin, A. B. Kaidalov, and Yu. A. Simonov, Phys. Lett. **B323** (1994) 41; ibid **B343**, 310 (1995).
3. V. L. Morgunov, A. V. Nefediev, and Yu. A. Simonov, Phys. Lett. **B459**, 653 (1999).
4. A. V. Nefediev and Yu. S. Kalashnikova, Phys. Lett. **B492**, 91 (2000), ibid **B530**, 117 (2002); A. V. Nefediev, Yu. S. Kalashnikova, and Yu. A. Simonov, Phys. Rev. **D64**, 014037 (2001).
5. J. Merlin and J. Paton, J. Phys. **G11**, 439 (1985).
6. Yu. A. Simonov, Nucl. Phys. **B324**, 67 (1989).
7. D. R. Stanley and D. Robson, Phys. Lett. **B45**, 235 (1980); P. Cea, G. Nardulli, and G. Preparata, Z. Phys. **C16**, 135 (1982), Phys. Lett. **B115**, 310 (1982); J. Carlson, John B. Kogut, and V. R. Pandharipande, Phys. Rev. **D27**, 233 (1983); N. Isgur and S. Godfrey, Phys. Rev. **D32**, 189 (1985).
8. L. Brink, P. Di Vecchia, and P. Howe, Nucl. Phys. **B118**, 76 (1977); Yu. S. Kalashnikova and A. V. Nefediev, Yad. Fiz. **60**, 1529 (1997) (Phys. Atom. Nucl. **60**, 1389 (1997)).
9. Yu. A. Simonov, Nucl. Phys. **B307**, 512 (1988); Yu. A. Simonov and J. A. Tjon, Ann. Phys. (NY) **228**, 1 (1993).
10. Yu. A. Simonov, Yad. Fiz. **60**, 2252 (1997) (Phys. At. Nucl. **60**, 2069 (1997)).
11. N. Brambilla and A. Vairo, Phys. Lett. **B407**, 167 (1997); P. Bicudo, N. Brambilla, E. Ribeiro, and A. Vairo, Phys. Lett. **B442**, 349 (1998).
12. I. I. Balitsky, Nucl. Phys. **B254**, 166 (1985).
13. H. G. Dosch and Yu. A. Simonov, Phys. Lett. **B205**, 339 (1988).
14. Yu. S. Kalashnikova and A. V. Nefediev, Phys. Lett. **B414**, 149 (1997).
15. E. Eichten and F. L. Feinberg, Phys. Rev. **D23**, 2724 (1981).
16. D. Gromes, Z. Phys. **C26**, 401 (1984).
17. Yu. A. Simonov and J. A. Tjon, Phys. Rev. **D62**, 094511 (2000).
18. V. I. Shevchenko and Yu. A. Simonov, Phys. Lett. **B437**, 146 (1998).