Pis'ma v ZhETF, vol. 78, iss. 6, pp. 850 – 853

Physics of the insulating phase in the dilute two-dimensional electron gas

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Submitted 20 August 2003

We propose to use the radio-frequency single-electron transistor as an extremely sensitive probe to detect the time-periodic ac signal generated by sliding electron lattice in the insulating state of the dilute two-dimensional electron gas. We also propose to use the optically-pumped NMR technique to probe the electron spin structure of the insulating state. We show that the electron effective mass and spin susceptibility are strongly enhanced by critical fluctuations of electron lattice in the vicinity of the metal-insulator transition, as observed in experiment.

PACS: 73.20.–r, 73.21.Fg

Detecting electron lattice with the single-electron transistor The metal-insulator transition (MIT) in the two-dimensional electron gas (2DEG) attracts considerable interest [1, 2]. In this paper, we focus on physics of the insulating phase. The great majority of experiments are transport measurements, and only few are thermodynamic. Dultz and Jiang [3] measured compressibility κ of the 2DEG as a function of carrier concentration n and found that it tends to vanish in the insulating phase, i.e. the phase is incompressible. The experimental dependence $\kappa(n)$ was semi-quantitatively reproduced within both the scenario of electron localization (E-LOC) in disordered potential [4] and the scenario of electron lattice (E-LAT) formation [5]. We use the term E-LAT to denote any state with local periodic modulation of electron density. The Wigner crystal (WC) and the charge-density wave (CDW) are the limiting cases of E-LAT, where the modulation amplitude is comparable to n and is much less than n, respectively. For simplicity, we call the carriers electrons, even though they may actually be holes.

Ilani et al. [6, 7] measured compressibility locally, using the single-electron transistor (SET) as a microscopic probe. They found that $\mu(n)$ has a series of quasi-random jumps, which become very strong in the insulating phase. These jumps were interpreted as single-electron charging events [6, 7] within the E-LOC scenario. Alternatively, the jumps can be interpreted as a manifestation of E-LAT [7]. When the average carrier concentration n is changed by the back gate, the period l of E-LAT must adjust, because it is proportional to the average distance between electrons $a = 1/\sqrt{n}$. However, because E-LAT is pinned by impurities, it cannot adjust its period continuously. Instead, E-LAT accumulates stress until it overcomes the pinning force and then makes a sudden local rearrangement of the lattice, which results in a jump of the local potential. Both the E-LOC and E-LAT scenarios are plausible, and it is difficult to decide between them on the basis of the known experimental data. Here we propose a modification of the experiments [6, 7], which may help to distinguish between the two scenarios.

In Ref. [8], Pudalov et al. observed a very nonlinear current-voltage (I-V) relation in the insulating phase of the 2DEG in Si-MOSFET. Almost no current I flows until electric field reaches the threshold field $\mathcal{E}_t$, and then I sharply surges at $\mathcal{E} > \mathcal{E}_t$, accompanied by the broad-band noise. Pudalov et al. interpreted their findings in terms of collective sliding of E-LAT depinned by the strong electric field $\mathcal{E} > \mathcal{E}_t$, which produces the large current I and generates the broad-band noise due to the local slip-stick motion. The I-V nonlinearity was found to be extremely sharp, with the differential conductivity increasing by the factor of 10^6, in the samples with the highest mobility and rounded in the samples with poor mobility [9]. These results suggest that the transition to the insulating state is not driven by disorder, as assumed by the E-LOC theories, but by E-LAT formation. The I-V nonlinearity was also observed in GaAs samples [10]. It was shown that the MIT deduced from the temperature dependence of resistivity is the same one as deduced from the I-V nonlinearity [11].

We propose to combine the SET experiment with the nonlinear I-V experiments. Suppose a strong pulling

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electric field $\mathcal{E} > \mathcal{E}_t$ is applied, and E-LAT slides. Then the SET would register a time-periodic ac signal with the frequency $\nu = v/l$ produced by E-LAT of the spatial period l, which slides with the velocity v. This effect is nothing but the narrow-band noise (NBN), well-known for CDW in the quasi-one-dimensional (Q1D) conductors [12]. Unlike in the Q1D conductors, attempts to observe the NBN in regular transport measurements in the 2DEG failed thus far [13]. We propose that the SET is a better tool for detecting the NBN, because of its very high sensitivity and because it is a local, microscopic probe, unlike the macroscopic current leads. In the experiment [7], the SET was situated at the distance $d = 400\,\mathrm{nm}$ from the 2DEG. This distance is comparable to the average distance between the carriers $a = 1/\sqrt{n} = 100\,\mathrm{nm}$ in the experiment [7] performed on p-GaAs with the typical hole concentration in the insulating phase $n = 10^{10}\,\mathrm{cm}^{-2}$. Because d and $l \sim a$ are comparable, the SET should experience a noticeable time-dependent signal when the periodically-modulated electron charge density slides past the SET. Reducing d and bringing the SET closer to the 2DEG would further increase sensitivity.

Let us estimate the frequency $\nu = v/l$ of the ac signal. The E-LAT period l is of the order of the average distance between the carriers $l \sim a = 1/\sqrt{n}$. The sliding velocity v is related to the current density $j = I/W = env$, where I is the total current, and W is the transverse width of the sample. Thus we find

$$\nu \simeq \frac{I}{e}\frac{1}{\sqrt{n}W} \simeq \left[6\,\frac{\mathrm{MHz}}{\mathrm{pA}}\right]\frac{I}{\sqrt{n}W}. \qquad (1)$$

For a crude estimate of the current density in the sliding regime, we use the data from Ref. [10] $j = I/W \simeq 0.4\,\mathrm{nA}/0.4\,\mathrm{mm} = 1\,\mathrm{nA/mm}$. (The data from Ref. [8] give a similar estimate.) Substituting these numbers into Eq. (1), we find $\nu \simeq 600\,\mathrm{kHz}$. The frequency scale is similar to that of the Q1D CDW [12]. Unfortunately, the frequency range of a typical SET is limited to less than 1 kHz. Thus, it is necessary to use the radio-frequency SET (RF-SET) [14], which can operate from dc to 100 MHz. With this experimental setup, it should be possible to detect the ac signal at the frequency ν.

Eq. (1) shows that ν is proportional to the current I carried by the sliding E-LAT, and the slope of that dependence is proportional to $1/\sqrt{n}$. An experimental observation of this effect would be the definitive proof of the existence of E-LAT in the dilute 2DEG. Periodicity in time is the direct consequence of periodicity in space, and the E-LOC scenarios cannot produce a periodic ac signal from the dc current. Although disorder destroys the long-range order of E-LAT [15, 16], the local periodicity is preserved and would produce the NBN peak in the Fourier spectrum. On the other hand, even if the RF-SET will not find a time-periodic signal, the measured time series would provide important microscopic information about electron conduction, such as the variable-range hopping. For example, uncorrelated single-electron hops would generate the Poisson stochastic process in the simplest case.

Probing spin order with the optically pumped NMR. Besides the question of charge ordering in the insulating state of the MIT, there is a question of spin ordering in that state. One of the great tools for obtaining information about electron spins is the nuclear magnetic resonance (NMR). In the quantum Hall regime, the optically-pumped NMR measurements on the ^{71}Ga nuclei in n-GaAs detected formation of skyrmions in the electron spin configuration for small deviations from the filling factor $\nu = 1$ [17]. In the $\nu = 1$ state, electrons are spontaneously spin-polarized and produce a significant effective magnetic field on the nuclei via the hyperfine interaction. Thus, the NMR line of the nuclei in contact with the 2DEG experiences the measurable Knight shift proportional to the spontaneous spin polarization of electrons [17, 18].

We propose to use a suitable modification of the same method to study the spin properties of the 2DEG in the insulating state in zero effective magnetic field. A magnetic field is needed for NMR, but we want to eliminate its effect on electrons. This can be achieved by engineering a situation where the electron g-factor is zero. For example, this is the case for a magnetic field parallel to the [100] surface of p-GaAs [19, 20]. It is also possible to achieve $g = 0$ by applying hydrostatic pressure [21].

For the Wigner crystal, different types of spin ordering were proposed theoretically: ferromagnetic [22], antiferromagnetic [22], and various exotic orderings [15]. In the ferromagnetic state, the NMR line should experience a measurable Knight shift, detection of which would be the proof of spontaneous spin polarization of electrons. In the antiferromagnetic state, the NMR line would broaden, because the nuclei experience a staggered hyperfine field from the electrons. This method is routinely used to detect formation of spin-density waves (SDW) in Q1D conductors [12]. On the other hand, when a strong electric field $\mathcal{E} > \mathcal{E}_t$ is applied, it forces SDW or E-LAT to slide. Then the nuclei experience the time-averaged hyperfine magnetic field produced by electrons, and the NMR line becomes narrow again [23] (the so-called motion narrowing). An observation of these effects in NMR would provide a great deal of information about spin ordering of electrons in the insulating state and would put the ongoing theoretical discussion of the subject on a firm experimental ground.

Enhancement of the effective mass and spin susceptibility. Experiments [24–28, 2] consistently show that the electron effective mass m^* and the effective spin susceptibility χ^* strongly increase when $n \to n_c$ from the metallic side, where n_c is the critical density of the MIT. This phenomenon has a simple explanation within the E-LAT scenario. The theory was developed in Refs. [29–31] and here we only briefly summarize the main physical idea.

The experiments [8, 9] show that the threshold field $\mathcal{E}_t$ and the thermal activation gap of resistivity continuously vanish at $n \to n_c$. Thus, the phase transition between the metallic phase at $n > n_c$ and the insulating phase at $n < n_c$ is of the second order. More precisely, it was found to be slightly of the first order [32], as expected by the symmetry reasons for a triangular or hexagonal lattice [33]. These results are in qualitative agreement with the self-consistent Hartree-Fock calculations [5], which show that E-LAT continuously evolves from the CDW limit to the WC limit with the decrease of $n < n_c$.

Assuming that the system has a tendency to form E-LAT with the wave vector $q_c \sim 1/a$, we can write the charge response function $S_0(q) = S(q, \omega = 0)$ in the following form [33] in the vicinity of the phase transition for $n > n_c$:

$$S_0(q) \simeq \frac{C_1}{n - n_c + (q - q_c)^2}, \tag{2}$$

where C_1 is a constant. Electrons can interact via exchange of the critical fluctuations (2). This interaction manifests itself in the Landau interaction function $f(\theta) \propto S_0(|\mathbf{p}_1 - \mathbf{p}_2|)$, where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the interacting electrons, and θ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$. Substituting this formula in the Landau equation for the effective mass m^* [34], we find

$$\frac{1}{m^*} = \frac{1}{m} - C_2 \int \frac{\cos\theta \, d\theta}{n - n_c + (|\mathbf{p}_1 - \mathbf{p}_2| - q_c)^2}, \tag{3}$$

where $C_2 \propto C_1$ is another constant. Taking into account that $|\mathbf{p}_1| = |\mathbf{p}_2| = p_F$, where p_F is the Fermi momentum, and assuming that $q_c \approx 2p_F$, we see that the integral in Eq. (3) is peaked around $\theta = \pi$, where $\cos\theta < 0$. Because of the Fermi statistics, the exchange interaction originating from the positive Coulomb repulsion is negative, so $C_2 < 0$. Thus, the interaction term in Eq. (3) causes an increase in the effective mass m^*, and this enhancement grows when $n \to n_c$.

To be specific and taking $q_c = 2p_F$, we obtain

$$\frac{1}{m^*} = \frac{1}{m} - C_2 \int \frac{\cos\theta \, d\theta}{n - n_c + p_F^2(1 + \cos\theta)^2/4}. \tag{4}$$

The integral in Eq. (4) diverges as $(n - n_c)^{-1/2}$ near $\theta \approx \pi$, and m^* diverges even earlier [29–31]:

$$\frac{m}{m^*} = 1 - \frac{C_3}{\sqrt{n - n_c}}, \tag{5}$$

where $C_3 > 0$ is another constant. However, these divergences should not be taken too literally, because they would be cut off by the weakly first-order character of the phase transition [32]. Thus, m^* increases steeply when $n \to n_c$, but does not necessarily go to infinity. The spin susceptibility is also enhanced via the standard relation $\chi^* = g^* m^*$. The qualitative agreement between the theory and experiment gives an argument in favor of the E-LAT scenario for the MIT in the 2DEG.

This theory is also applicable to other system experiencing transition for a liquid to a crystalline phase. Such a transition is observed in the 2D He-3, and the experiment [35] finds a very strong enhancement of m^* in the vicinity of the transition. Notice that there is no disorder in liquid He-3, so the E-LOC scenario is irrelevant in this case.

Conclusions. We propose to use the radio-frequency single-electron transistor (RF-SET) [14] as an extremely sensitive probe [6, 7] to detect the time-periodic ac signal generated by sliding E-LAT at $\mathcal{E} > \mathcal{E}_t$ in the insulating state of the 2DEG. An observation of this narrow-band-noise effect would be the definitive proof of E-LAT formation in the dilute 2DEG. We also propose to use the optically-pumped NMR technique [17] to probe the electron spin structure of the insulating state, which may have ferromagnetic, antiferromagnetic, or exotic types of spin ordering. NMR can be performed in a magnetic field without disturbing the electron spins in a situation where the electron g-factor is engineered to be zero [19, 21]. Within the Landau theory of Fermi liquids, we show that critical fluctuations of E-LAT near the MIT produce a strong enhancement of the effective mass m^* and spin susceptibility χ^* [29–31] in qualitative agreement with the experiments in the 2DEG [24–28], as well as in the 2D He-3 [35]. This is an argument in favor of the E-LAT scenario.

Although we concentrated on physics of the 2DEG in zero magnetic field, the same ideas also apply to the Wigner crystal in a non-zero magnetic field perpendicular to the 2DEG [36, 37].

The authors are grateful to S. Das Sarma for critical reading of the manuscript and many valuable comments, and to V. M. Galitski for discussions. The authors thank the Kavli Institute for Theoretical Physics for the opportunity to start this collaboration. VAK thanks the Condensed Matter Theory Center for arranging his visit to the University of Maryland. VMY is supported by the NSF Grant # DMR-0137726, and VAK by the NSF Grant # PHY-0140316, by the McDonnell Center for Space Sciences, and by the Grant # NS-1885.2003.2 from the Russian Ministry of Industry and Science.

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1. E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).
2. S. V. Kravchenko and M. P. Sarachik, cond-mat/0309140.
3. S. C. Dultz and H. W. Jiang, Phys. Rev. Lett. **84**, 4689 (2000).
4. J. Shi and X. C. Xie, Phys. Rev. Lett. **88**, 086401 (2002).
5. S. Orozco, R. M. Méndez-Moreno, and M. Moreno, Phys. Rev. **B67**, 195109 (2003).
6. S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. **84**, 3133 (2000).
7. S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, Science **292**, 1354 (2001).
8. V. M. Pudalov, M. D'Iorio, S. V. Kravchenko, and J. W. Campbell, Phys. Rev. Lett. **70**, 1866 (1993).
9. V. M. Pudalov, J. Phys. IV France **12**, Pr9-331 (2002).
10. J. Yoon, C. C. Li, D. Shahar et al., Phys. Rev. Lett. **82**, 1744 (1999).
11. A. A. Shashkin, S. V. Kravchenko, and T. M. Klapwijk, Phys. Rev. Lett. **87**, 266402 (2001).
12. G. Grüner, *Density Waves in Solids*, Addison-Wesley, New York, 1994.
13. Recently the NBN has been observed in the 2DEG in a high magnetic field in the quantum Hall regime, albeit with the frequency ν several orders of magnitude lower than given by Eq. (1): K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **90**, 226803 (2003).
14. R. J. Schoelkopf, P. Wahlgren, A. A. Kozhevnikov et al., Science **280**, 1238 (1998).
15. S. Chakravarty, S. Kivelson, C. Nayak, and K. Voelker, Philosophical Magazine **B79**, 859 (1999); K. Voelker and S. Chakravarty, Phys. Rev. **B64**, 235125 (2001).
16. B. Spivak, Phys. Rev. **B64**, 085317 (2001); **67**, 125205 (2003).
17. S. E. Barrett, G. Dabbagh, L. N. Pfeiffer et al., Phys. Rev. Lett. **74**, 5112 (1995).
18. P. Khandelwal, N. N. Kuzma, S. E. Barrett et al., Phys. Rev. Lett. **81**, 673 (1998).
19. M. Rahimi, M. R. Sakr, S. V. Kravchenko et al., Phys. Rev. **B67**, 081302 (2003).
20. R. Winkler, S. J. Papadakis, E. P. De Poortere, and M. Shayegan, Phys. Rev. Lett. **85**, 4574 (2000).
21. W. Kang, J. B. Young, S. T. Hannahs et al., Phys. Rev. **B56**, R12776 (1997); H. Cho, J. B. Young, W. Kang, K. L. Campman et al., Phys. Rev. Lett. **81**, 2522 (1998).
22. X. Zhu and S. G. Louie, Phys. Rev. **B48**, 13661 (1993).
23. E. Barthel, G. Kriza, G. Quirion et al., Phys. Rev. Lett. **71**, 2825 (1993).
24. T. Okamoto, K. Hosoya, S. Kawaji, and A. Yagi, Phys. Rev. Lett. **82**, 3875 (1999).
25. A. A. Shashkin, S. V. Kravchenko, V. T. Dolgopolov, and T. M. Klapwijk, Phys. Rev. Lett. **87**, 086801 (2001); Phys. Rev. **B66**, 073303 (2002); cond-mat/0302004.
26. S. A. Vitkalov, M. P. Sarachik, and T. M. Klapwijk, Phys. Rev. **B65**, 201106 (2002).
27. V. M. Pudalov, M. E. Gershenson, H. Kojima et al., Phys. Rev. Lett. **88**, 196404 (2002).
28. O. Prus, Y. Yaish, M. Reznikov et al., Phys. Rev. **B67**, 205407 (2003).
29. V. R. Shaginyan, Pis'ma ZhETF **77**, 104 (2003) [JETP Lett. **77**, 99 (2003)].
30. V. M. Galitski and V. A. Khodel, cond-mat/0308203.
31. V. A. Khodel, V. R. Shaginyan, and M. V. Zverev, Pis'ma ZhETF **65**, 242 (1997) [JETP Lett. **65**, 253 (1997)].
32. V. M. Pudalov and S. T. Chui, Phys. Rev. **B49**, 14062 (1994).
33. S. A. Brazovskii, ZhETF **68**, 175 (1975) [Sov. Phys. JETP **41**, 85 (1975)].
34. E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics*, Part 2, Butterworth-Heinemann, Oxford, 1999.
35. A. Casey, H. Patel, J. Nyéki et al., Phys. Rev. Lett. **90**, 115301 (2003).
36. H. W. Jiang, R. L. Willett, H. L. Stormer et al., Phys. Rev. Lett. **65**, 633 (1990); V. J. Goldman, M. Santos, M. Shayegan, and J. E. Cunningham, Phys. Rev. Lett. **65**, 2189 (1990); F. I. B. Williams, P. A. Wright, R. G. Clark et al., Phys. Rev. Lett. **67**, 1630 (1991); Y. P. Li, T. Sajoto, L. W. Engel et al., Phys. Rev. Lett. **66**, 3285 (1991); H. W. Jiang, H. L. Stormer, D. C. Tsui et al., Phys. Rev. **B44**, 8107 (1991).
37. B. G. A. Normand, P. B. Littlewood, and A. J. Millis, Phys. Rev. **B46**, 3920 (1992); X. Zhu, P. B. Littlewood, and A. J. Millis, Phys. Rev. **B50**, 4600 (1994).