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Spin relaxation in the quantized Hall regime in the presence of disorder

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We study the spin relaxation (SR) of a two-dimensional electron gas in the quantized Hall regime and discuss the role of spatial inhomogeneity effects on the relaxation. The results are obtained for small filling factors ($\nu \ll 1$) or when the filling factor is close to an integer. In either case SR times are essentially determined by a smooth random potential. For small ν we predict a "magneto-confinement" resonance manifested in the enhancement of the SR rate when the Zeeman energy is close to the spacing of confinement sublevels in the low-energy wing of the disorder-broadened Landau level. In the resonant region the B-dependence of the SR time has a peculiar non-monotonic shape. If $\nu \simeq 2n + 1$, the SR is going non-exponentially. Under typical conditions the calculated SR times range from 10^{-8} to 10^{-6} s.

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1. For the relaxation of electron spins to occur, two conditions have to be fulfilled: the first one is the presence of an interaction mixing different spin states in the system studied; the second is the availability of a mechanism which makes the relaxation process irreversible. Both conditions can be realized in a rich variety of ways, and even in the case of two-dimensional (2D) electrons one finds a wide scatter of experimental data [1] and theoretical results [2 – 10] devoted to spin relaxation (SR) problems. Besides, in the magnetic field the SR process is actually the relaxation of the Zeeman energy $|g\mu_B B\Delta S_z|$ ($\mathbf{B} \parallel \hat{z}$, $\Delta S_z = S_z - S_0$ is the spin deviation of the S_z component from the equilibrium value S_0).

Theoretically, the relaxation problem of a flipped spin in a semiconductor heterostructure in high perpendicular magnetic field seems to be first formulated by Frenkel [4], who considered the relativistic part of the phonon field acting directly on the electron spin as the interaction mixing the electron spin states. One could estimate that in the case of another mechanism, exactly due to the spin-orbit (SO) coupling reformulated for the 2D case [2, 3], the relevant spin-flip transition matrix element is at least by an order of magnitude greater. However, the work of Ref. [4] caused some misunderstandings even almost a decade after its publication. (See the comment [9] and references therein).

Any properties of a 2D electron gas (2DEG) in the quantum Hall regime crucially depend on the filling factor $\nu = N/N_\phi$, where N and $N_\phi = L^2/2\pi l_B^2$ are the numbers of electrons and magnetic flux quanta, respectively ($L \times L$ is the 2DEG area, l_B is the magnetic length). In this Letter we consider the SR problem in two formulations.

First, we solve *the problem of one-electron spin-flip* in the presence of a random potential, and in so doing we study the SR in a lateral quantum dot in high magnetic field, where the effects of interplay between the localization and the Zeeman coupling are essential. We emphasize that we deal with a weak confinement but in the presence of strong B. Therefore well smaller energies are relevant (~ 1 K) than, e.g., in the case of usual interplay of Fock-Darwin states [11]. (The latter is based on competition of confinement and cyclotron energies which are $\gtrsim 10$ K). The most striking manifestation of the *"magneto-confinement"* effect occurs when the single-electron the Zeeman energy is close to the spacing of lowest-energy levels in a quantum dot or in a potential minimum. This results in *strong enhancement of the SR rate.* The studied one-electron problem is actual for small filling factors, $\nu \ll 1$, but in strong magnetic fields it may also be extended in terms of the Hartree-Fock approach to a vicinity of an even filling factor, when $\nu \simeq 2n$.

Second, we report on the results for the SR in a *strongly correlated* 2DEG when it presents a quantum Hall ferromagnet (QHF), i.e. when the filling factor ν is close to an odd integer ($\nu \simeq 2n + 1$). In this case we study a new mechanism of relaxation of the total 2DEG spin, where the cause of irreversibility is neither electron-phonon interaction [6] nor inter-spin-wave scattering [7] but a disorder. We will see that in real experimental regimes this disorder relaxation channel has significantly to prevail over the phonon one.

In both SR problems the temperature is assumed to be equal to zero which actually means that it is lower than the single-electron the Zeeman energy $\epsilon_Z = |g|\mu_B B$.

2. We consider a smooth random potential (SRP) as the disorder. The total single-electron Hamiltonian is thereby as follows: $\mathcal{H} = \hbar^2\hat{\mathbf{q}}^2/2m_e^* - \epsilon_Z\hat{\sigma}_z/2 + u(\mathbf{r}) + H_{SO} + U_{\text{e-ph}}$, where $\hat{\mathbf{q}} = -i\nabla + e\mathbf{A}/c$ and $\mathbf{r} = (x, y)$ are 2D vectors, $u(\mathbf{r})$ is the SRP field, the H_{SO} and $U_{\text{e-ph}}$ terms respond to the SO and electron-phonon interactions (see below). If the SRP is assumed to be Gaussian, then it is defined by the correlator $K(\mathbf{r}) = \langle u(\mathbf{r})u(0)\rangle$. We choose also $\langle u(\mathbf{r})\rangle = 0$ which means that the SRP energy is measured from the center of the Landau level. In terms of the correlation length Λ and Landau level width Δ, the correlator is

$$K(\mathbf{r}) = \Delta^2 \exp\left(-r^2/\Lambda^2\right). \quad (1)$$

In the realistic case $\Delta \sim 10\,\text{K}$, $\Lambda \sim 30 - 50\,\text{nm}$, therefore $\Delta \gtrsim \hbar^2/m_e^*\Lambda^2$. We study the case $\Delta \ll \hbar\omega_c$ (ω_c is the cyclotron frequency), and $\Lambda \gg l_B$. In the SRP field the electron drifts quasiclassically along an equipotential line. However before the spin-flip it relaxes to a SRP minimum. Estimates for this relaxation time (due to phonon emission without any spin-flip) yield the values not exceeding 1 ns.

For simplicity, we model the SRP in the vicinity of a minimum by a parabolic confinement potential $u = m_e^*\omega^2r^2/2$, and to describe the electron states we use the symmetric gauge basis ($\mathbf{A} = \mathbf{r}\times\mathbf{B}/2$):

$$|n, m, \sigma\rangle = \sqrt{\frac{n!}{(n+m)!}}\frac{e^{-im\varphi - r^2/4a^2}}{a\sqrt{2\pi}} \times \left(\frac{ir}{\sqrt{2}a}\right)^m L_n^m(r^2/2a^2)|\sigma\rangle. \quad (2)$$

(L_n^m is a Laguerre polynomial; note also that only the states with $n + m \geq 0$ are considered in the following.) For the length a it should be substituted $a = (\hbar/2m_e^*\Omega)^{1/2}$, where $\Omega = \sqrt{\omega^2 + \omega_c^2/4}$. The system thus becomes equivalent to a lateral quantum dot [8, 10], and we deal with the Fock-Darwin states [11] with energies $E_{n,m} = \hbar(2n+m+1)\Omega - \hbar\omega_c m/2$. The appropriate quantity is also the level spacing $\delta = \hbar(\Omega - \omega_c/2) \approx \hbar\omega^2/\omega_c$, concerning the $m \geq 0$ states which belong to the same number n. We calculate the *total rate* of the transition of an electron initially occupying the upper spin sublevel to *any final state* of the lower spin sublevel. At first sight we should consider the spin-flipped state $|0, 0, \downarrow\rangle$ as the initial one. However, a correction of the states due to the SO coupling has to be taken into account. We use the SO Hamiltonian specified for the (001) GaAs plane: $H_{SO} = \alpha\left(\hat{\mathbf{q}} \times \hat{\boldsymbol{\sigma}}\right)_z + \beta\left(q_y\hat{\sigma}_y - q_x\hat{\sigma}_x\right)$. This expression is a combination of the Rashba term [2] (with the coefficient α) and the crystalline anisotropy term [3, 5 – 10] ($\hat{\sigma}_{x,y,z}$ are the Pauli matrices). Assuming that α and β are small ($\alpha, \beta \ll \hbar\omega_c l_B$) we find after perturbative treatment the spin-orbitally corrected states before and after the spin flip:

$$|i\rangle = C_{1,1}|0,0,\downarrow\rangle + C_{2,1}|0,1,\uparrow\rangle + \frac{\beta}{\hbar\Omega\sqrt{2}a}|1,-1,\uparrow\rangle, \quad (3)$$

$$|f_m\rangle = C_{1,m}|0,m,\uparrow\rangle + C_{2,m}|0,m-1,\downarrow\rangle - -\frac{i\alpha}{\hbar\Omega\sqrt{2}a}|1,m-1,\downarrow\rangle + \frac{\beta\delta\sqrt{m+1}}{\hbar\Omega\sqrt{2}a(\delta+\epsilon_Z)}|1,m+1,\downarrow\rangle, \quad (4)$$

where $0 \leq m < \epsilon_Z/\delta$ (the Zeeman energy ϵ_Z has been neglected as compared to $\hbar\omega_c$). The coefficients $C_{i,m}$ are defined as follows: let $T = \alpha\delta\sqrt{2}/a\hbar\Omega(\epsilon_Z - \delta)$ and $P_m = 2\sqrt{1 + T^2m}$, then $C_{1,m} = \sqrt{1/2 + 1/P_m}$, and $C_{2,m} = i\,\text{sign}(\delta - \epsilon_Z)\sqrt{1/2 - 1/P_m}$. Here the resonance mixing of the "spin-up" and "spin-down" states (if $\epsilon_Z \simeq \delta$) has been properly taken into account. Note that the behaviour of the states (3), (4) in the vicinity of the resonance (namely in the interval $\Delta B/B \lesssim \alpha\sqrt{m_e^*/\hbar^3\omega_c} \lesssim 0.1$) is governed only by the Rashba SO mechanism.

In the resonance region $C_{1,1} \sim |C_{2,1}|$, and $|i\rangle$ is thereby a well hybridized spin state. The $|i\rangle \to |f_0\rangle$ transition then is not due to the SO coupling, and this should lead to the SR enhancement. The final state $|f_0\rangle$ is the almost "pure" spin-state. In fact we may always set $C_{1,m} = 1$ and $C_{2,m} = 0$ in the expression for $|f_m\rangle$. Indeed, though the spin hybridization of the $|f_1\rangle$ state is significant, it plays negligible role in the SR process because of vanishing of the relevant phonon momentum $(\epsilon_Z - \delta)/c_s$. Then we find the matrix element $\langle f_m|U_{\text{e-ph}}|i\rangle$ and, using the Fermi golden rule, obtain the SR rate within a certain SRP minimum,

$$1/\tau_\omega = \frac{2\pi}{\hbar}\sum_{m,\mathbf{k}}\left|\langle f_m\left|U_{\text{e-ph}}\right|i\rangle\right|^2\delta(\hbar c_s k - \epsilon_Z).$$

Here

$$U_{\text{e-ph}}(\mathbf{r}) = (\hbar/V)^{1/2}\sum_s \tilde{U}_s(\mathbf{q}, k_z)e^{i\mathbf{qr}},$$

where the index s labels phonon polarization, V is the sample volume, and $\tilde{U}_s$ is the renormalized (in the 2D layer) vertex which includes the deformation and piezoelectric fields created by the phonon [6, 12]. The summation over s involves averaging over directions of the polarization unit vector for both components of the electron-phonon interaction, and this may be reduced to $|\sum_s U_s|^2 = \pi\hbar c_s k/p_0^3\tau_A(\mathbf{k})$, where $\tau_A{}^{-1} = \tau_D^{-1} + 5\tau_p^{-1}p_0^2(q^2k_z^2 + q_x^2q_y^2)/k^6$ (see Ref. [6]). The nominal times for the deformation and piezoelectric interactions in GaAs are $\tau_D \approx 0.8$ ps, and $\tau_P \approx 35$ ps [6, 12]. The nominal momentum is $p_0 = 2.52\cdot 10^6$/cm [12]. (We also refer to Refs. [6, 12] for details concerning the meaning of these quantities and their expressions in terms of the GaAs material parameters.) Finally, in the general expression for $1/\tau_\omega$ we perform the summation and after a routine treatment arrive at the result

$$\frac{1}{\tau_\omega} = \frac{1}{8ap_0\tau_p}\int_0^1 \frac{d\xi}{\sqrt{1-\xi}} \sum_{0\le m<\epsilon_Z/\delta} \frac{b_m}{m!}\left[\mathcal{A}_m^2(\xi) + \mathcal{B}_m^2(\xi)\right]\times$$

$$\times\left(S_m + 5\xi - 35\xi^2/8\right)e^{-\xi b_m^2}\left(\frac{\xi b_m^2}{2}\right)^m, \qquad (5)$$

where

$$b_m = a\frac{\epsilon_Z - m\delta}{\hbar c_s}, \quad S_m = \frac{b_m^2\tau_p}{(ap_0)^2\tau_d},$$

$$\mathcal{A}_m = \left(\frac{2m}{b_m\sqrt{\xi}} - \sqrt{\xi}b_m\right)|C_{2,1}| + \frac{\alpha b_m}{\hbar\Omega a}\sqrt{\frac{\xi}{2}}\,\mathrm{sign}(\delta - \epsilon_Z),$$

$$\mathcal{B}_m = \frac{\beta b_m}{\hbar\Omega a}\sqrt{\frac{\xi}{2}}\left(1 - C_{1,1}\frac{\delta}{\delta + \epsilon_Z}\right).$$

The estimate for α and β depends on the effective layer width [2, 3]. The rate $1/\tau_\omega$ as a function of B at $\omega = 5$ K is shown in the inset of Figure for realistic parameters indicated in the capture.

At $\omega = 0$ (i.e. in the "clean" limit), the summation in the formula (5) is carried out over all numbers $m = 0, 1, \ldots, \infty$, and the expression (5) is reduced to

$$\frac{1}{\tau_0} = \mathcal{V}\int_0^1 \frac{\xi\, d\xi}{\sqrt{1-\xi}}e^{-b^2\xi/2}\left(S + 5\xi - 35\xi^2/8\right), \qquad (6)$$

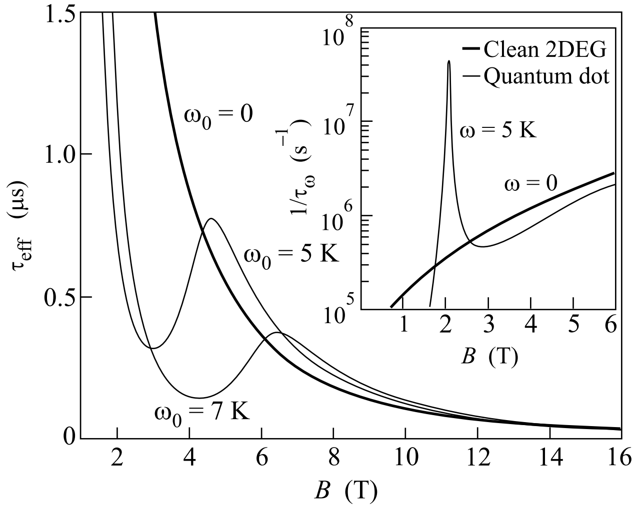
where

$$\mathcal{V} = \frac{\epsilon_Z{}^3(\alpha^2+\beta^2)}{4p_0\hbar^5\omega_c^2c_s^3\tau_p} \propto B, \quad b = \frac{\epsilon_Z l_B}{\hbar c_s} \propto \sqrt{B},$$

$$S = \frac{\tau_p\epsilon_Z^2}{\tau_D(\hbar c_s p_0)^2} \propto B^2.$$

The bold curves in the inset and in the main picture of Figure show the corresponding B-dependencies.

Note that if $\epsilon_Z = 0$, then at any $\mathbf{r}_0$ the projection $\mathcal{P}(\mathbf{r}_0) = \langle f_m|\delta(\mathbf{r}-\mathbf{r}_0)|i\rangle$ vanishes when calculated at a finite ω in the leading order in the SO constants. (It does not occur for the "clean" states, i.e. if in Eqs. (3) and (4) we pass to the $\omega \to 0$ limit before equating ϵ_Z to zero.) Such a vanishing is a manifestation of the general feature [8, 10]: at zero Zeeman energy the effects of the SO coupling in the leading order and of the orbital magnetic field are similar. In particular, in a quantum dot (where $\delta > \epsilon_Z$!), the first order SO approximation in the $\epsilon_Z \to 0$ limit results only in a small rotation of eigen states in the spin space. With decreasing B we get $\mathcal{P} \propto B^{3/2}$ (if $\epsilon_Z \ll \delta \ll \omega_c$) and obtain a sharper fall of the relaxation rate as compared to the "clean" case (6).



Calculations of SR time $\tau_{\rm eff}$ in a 2DEG are carried out for $\alpha = \beta/3 = 10^{-6}$ K·cm and $c_s = 3.37\cdot 10^5$ cm/s (other material parameters are given in the text). In the inset the position of the SR peak corresponds to the condition $\epsilon_Z = \delta$. The evolution of B-dependences of the spin relaxation time $\tau_{\rm eff}$ with the parameter ω_0 is shown in the main picture

In the presence of the SRP we have to carry out averaging $1/\tau_{\rm eff} = \int_0^\infty d\omega F(\omega)/\tau_\omega$, where the distribution function $F(\omega)$ is the probability for the confinement frequency to take a certain value ω. One may prove that in the case of a Gaussian potential $u(\mathbf{r})$ it should be chosen in the form

$$F(\omega) = \frac{2\omega}{\sqrt{\pi}\omega_0}\exp\left(-\omega^4/4\omega_0^4\right).$$

(The value ω^2 is proportional to the curvature $\nabla^2 u$, and a routine analysis yields $\omega_0 = 6^{1/4}(\Delta/m_e^*)^{1/2}/\Lambda$.) Calculating $1/\tau_{\rm eff}$ with this function, we obtain the final result (see Figure). As it has to be, in comparison with τ_ω the resonant behaviour of $1/\tau_{\rm eff}$ is smoothed, but at actual values of ω_0 it results in non-monotonic B-dependence of $\tau_{\rm eff}$. Beyond the resonance region the behaviour is as follows: (i) at small magnetic fields (when $\epsilon_Z \ll \hbar\omega_0^2/\omega_c \ll \omega_c$) only one final state $|f_0\rangle$ participates in the SR, and we find that $\tau_{\rm eff} \propto B^{-5}$; (ii) at high fields (when $\epsilon_Z \gg \delta_0$) there is a large but finite number of possible states $|f_m\rangle$ into which the confined spin-flipped electrons could relax, and the SR time is always longer than τ_0 but approaches this with increasing magnetic field. Note that exactly in this high-field regime the one-electron model becomes relevant for fillings $\nu \simeq 2n$. Then the total 2DEG spin is determined only by a small amount $|\nu - 2n|N_\phi$ of effectively "free" electrons/holes belonging to the $(n+1)$-st/n-th Landau level.

3. So, the problem has been solved when the total 2DEG spin is well smaller than N_ϕ. Now we study the opposite case: in the ground QHF state the spin numbers attain maximum, $S = S_z = N_\phi/2$. This case is also remarkable, since to the first order in the ratio $r_c = (e^2/\varepsilon l_B)/\hbar\omega_c$ the low-lying excitation are again exactly known: these are 2D spin waves or spin excitons (SEs). The most adequate description of the SE states is realized by spin-exciton creation

$$Q^\dagger_{\mathbf{q}} = \frac{1}{\sqrt{N_\Phi}} \sum_p e^{-ipq_x l_B^2} a^\dagger_{\downarrow\, p+\frac{q_y}{2}} a_{\uparrow\, p-\frac{q_y}{2}}$$

and annihilation $Q_{\mathbf{q}} = \left(Q^\dagger_{\mathbf{q}}\right)^\dagger$ operators [13]. In this definition $a_{\sigma p}$ stands for the Fermi annihilation operator corresponding to the $|n,p,\sigma\rangle = L^{-1/2}e^{ipy}\varphi_n(x+pl_B^2)$ state in the Landau gauge (φ_n is the nth harmonic oscilatory function). So the one-exciton state is $Q^\dagger_{\mathbf{q}}|0\rangle$, where $|0\rangle$ stands for the ground state. At small 2D momentum ($ql_B \ll 1$) the one-exciton state has the energy $\mathcal{E}_q = \epsilon_Z + (ql_B)^2/2M_n$ (now we need only this small momenta approximation, see also general expressions for the 2D magneto-excitons at integer filling factors in Refs. [14]). M_n is the SE mass at $\nu = 2n+1$, namely in the $r_c \to \infty$ limit: $1/M_0 = (e^2/\varepsilon l_B)\sqrt{\pi/8}$, $1/M_1 = 7/4M_0$,... Note that the sum $S_- = \sum_i \sigma_-^{(i)}$ lowering the spin number S_z by 1 (i labels the electrons), when considered to be projected onto the nth Landau level, is simply proportional to the "zero" exciton creation operator $Q^\dagger_0$. When the SO coupling is ignored, any state $(S_-)^N|0\rangle$ is the eigenstate independently of the r_c magnitude and of the presence of a disorder.

The SO interaction H_{SO} and the SRP field $u(\mathbf{r})$ may be accounted perturbatively as usual. Meanwhile, now the unperturbed part of the Hamiltonian involves the Coulomb interaction. For present purposes the approximation in the context of the projection onto a single Landau level is quite sufficient (c.f. Ref. [6]). We note also that the QHF state $|0\rangle$ is resistant to the SRP disorder. Such a stability is determined by the exchange energy ($\sim e^2/\kappa l_B$) which is much larger than the amplitude Δ.

There are two fundamentally different alternatives to provide the initial perturbation of spins. The first one is the perturbation of the spin system as a whole when the S number is not changed: $\Delta S = 0$, but $\Delta S_z \neq 0$. This is a Goldstone mode which presents a *quantum precession* of the vector $\mathbf{S}$ around the $\mathbf{B}$ direction. In terms of the SE representation $|\Delta S_z| = N_\phi/2 - S_z$ is the number of "zero" SEs excited in a 2DEG. Let $|\Delta S_z| = N$; the corresponding state is $(S_-)^N|0\rangle \propto \left(Q^\dagger_0\right)^N|0\rangle$, and it has the energy $N\epsilon_Z$. (Note that "zero" SEs do not interact among themselves; besides, the stability of the $(S_-)^N|0\rangle$ state with respect to the disorder is identical to that for the $|0\rangle$ state because of $u(\mathbf{r})$ and S_- commuting.) The second case of the perturbation is the $\Delta S = \Delta S_z$ type of the deviation. This does not change the symmetry and involves the excitation of "nonzero" SEs, where each SE changes the spin numbers by 1: $S \to S-1$, $S_z \to S_z - 1$. In contrast to "zero" SEs, the "nonzero" ones interact. This interaction [7] or/and the direct exciton-phonon coupling [6] govern the "nonzero" SE annihilation which should go faster than for "nonzero" SE annihilation process.

The SRP inhomogeneity does not essentially affect the SE energy and the "nonzero" annihilation. Indeed, the exciton is neutral, and the interaction with the SRP incorporates the energy $U_{\text{x-SRP}} \sim ql_B^2\Delta/\Lambda$ (the "nonzero" SE possesses the dipole momentum $el_B^2[\mathbf{q}\times\hat{z}]$, see Ref. [14]). The latter may be negative but is always smaller than the "nonzero" SE energy $\mathcal{E}_q$. If the SE would annihilate, the energy conservation condition $\mathcal{E}_q + U_{\text{x-SRP}} = 0$ is not satisfied except of negligibly rare points, where the gradient ∇u is accidentally large. Therefore, the SRP leads only to small corrections (of the order of $U_{\text{x-SRP}}/\mathcal{E}_q$) to the other mechanisms of the "nonzero" SE relaxation [6, 7].

A distinctly different process contributes to the SR in the case of the first type of the spin perturbation. This is an effective interaction of "zero" SEs among themselves arising due to the SO coupling [6]. Such an interaction does not preserve the total number of excitons N: at elementary event two "zero" excitons merge into one "nonzero" (the spin momenta change following the rule $S_z \to S_z+1$, $S \to S-1$). In other words, the SO coupling and the SRP field mix the initial $|i\rangle = \left(Q^\dagger_0\right)^N|0\rangle$ and the final $|f_q\rangle = Q^\dagger_{\mathbf{q}}\left(Q^\dagger_0\right)^{N-2}|0\rangle$ states. These "many-exciton" states have to be normalized (see the normalization factors in Refs. [6, 13]). So, the SRP plays the same role as the phonon field studied previously [6]. Now the energy conservation condition takes the form: $2\epsilon_Z \approx \mathcal{E}_q$, where the interaction of "nonzero" SE with the SRP is ignored as compared to other members of this equation. If solving this for q, we obtain $q = q_0 \equiv \sqrt{2M_n\epsilon_Z}/l_B$.

The detailed calculation of the SR rate is truly similar to that performed in Ref. [6]. Indeed, the Fourier expansion $u(\mathbf{r}) = \sum_{\mathbf{q}} \overline{u}(q)e^{i\mathbf{qr}}$ looks like the phonon field created by "frozen" (of zero frequency) phonons. Then this SRP field and the SO Hamiltonian are treated perturbatively. In so doing, it is convenient to present them in terms of the excitonic operators. We calculate the relevant matrix element between the $|i\rangle$ and $|f_q\rangle$ normalized states and obtain $\left|\mathcal{M}_{i\to f_q}\right|^2 = N^2(\alpha^2+\beta^2)|q\overline{u}(q)|^2/(\hbar\omega_c)^2N_\phi$. Finally, again with the use of the Fermi golden rule, we find after summation over all states $|f_q\rangle$ that the SR rate takes the same form as in the case of the phonon mechanism [6], $dS_z/dt = (\Delta S_z)^2/\tau_{\text{SRP}}N_\phi$, but incorporates a different time constant:

$$\tau_{\text{SRP}}^{-1} = \frac{16\pi^2(\alpha^2+\beta^2)M_n^2\epsilon_Z\overline{K}(q_0)}{\hbar^3\omega_c^2l_B^4}\,.$$

Here $\overline{K}$ stands for the Fourier component of the correlator [the equivalence $\overline{K}(q) = L^2\left|\overline{u}(q)\right|^2/4\pi^2$ has been employed]. The SR follows the low

$$\Delta S_z(t) = \frac{\Delta S_z(0)}{1+t\left|\Delta S_z(0)\right|/\tau_{\text{sr}}N_\phi}\,,$$

where $\tau_{\text{sr}}^{-1} = \tau_{\text{ph}}^{-1}+\tau_{\text{SRP}}^{-1}$, because the relaxations of both types proceed in parallel. A natural question is: *what is the ratio of the times* τ_{ph} *and* τ_{SRP}*?* If $T \lesssim \hbar c_s q_0$ (in fact, this means that $T \lesssim 1\,\text{K}$) and $B < 15$, then the SR time τ_{ph} depends weakly on T and B. In particular, at $\nu = 1$ we find that $\tau_{\text{ph}} \simeq 10\,\mu\text{s}$ [6, 7]. The ratio of interest is determined only by the Fourier component $\overline{K}(q_0)$: $\tau_{\text{ph}}/\tau_{\text{SRP}} = 0.24\pi\tau_p p_0\Delta^2\Lambda^2e^{-\Lambda^2q_0^2/4}/\hbar^2c_s$ (for $\overline{K}$ we have substituted the value calculated with help of Eq. (1)). So, for the actual parameters $\tau_{\text{ph}}/\tau_{\text{SRP}} \sim \sim 100-1000$, i.e. exactly the "disorder" time ($\tau_{\text{SRP}} \sim \sim 10^{-8}-10^{-7}\,\text{s}$) governs the breakdown of this Goldstone mode. In conclusion, we remind that *the relaxation is going non-exponentially and the actual time is increased by a factor of* $N_\phi/\Delta S_z(0)$.

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