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Hot electron production in plasmas illuminated by intense lasers

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Electron-ion collisions in strong electromagnetic fields, whether non-relativistic or ultra-relativistic, can lead to the acceleration of electrons to high energies. The production efficiency and the Joule heating rate are calculated. Experimental verification of theoretical predictions, including the power law scaling, is presented.

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Recent experiments with petawatt laser plasmas revealed interesting and unpredictable phenomena [1, 2]. A large number of fast electrons with energies up to several tens of MeV was detected. The estimated energy of these electrons was up to ten per cent of the pump laser energy. On the other hand, the plasma temperature was of the order of hundreds of eV, only weakly dependent on laser intensity, but significantly dependent on the pump pulse duration. The number of these hot electrons was dependent on the laser intensity, and the angular distribution function of these electrons was very wide. It seems difficult to imagine at all these results are consequences of any plasmas wave turbulence. Moreover, a resonant wake process such as might be used for deliberate acceleration of electrons, would exhibit strong directionality n the accelerated electrons. Thus, the review paper [1] considers the electron distribution phenomena rather puzzling. In this work, we point out that many of the important features of strong laser-plasma interactions and particularly hot electrons production can be interpreted as consequences of electron-ion collisions.

However, traditional models of electron-ion collisions in strong laser fields, based on the small-angle scattering approximation [3], i. e. under the assumption that quivering electrons pass near ions along straight lines, cannot explain the existing experimental results. An alternative description of Coulomb collisions, taking into account the substantial acceleration of particles during the scattering process, was proposed [4]. The application of the proposed model to the description of hot electron production provided by electron-ion collisions and the comparison with experimental data from [1, 2] constitute the major emphasis of the present work.

The paper is organized as follows. First, we discuss the applicability conditions and the main parameter for the model being used. We show that, for relativistic levels of laser intensities, these effects are very important. We give estimates for the "energy spectrum" of hot electrons (named so after Ref. [1]) directly formed by electron-ion collisions, obtaining a power tail distribution. We estimate the total number of hot electrons production from unit volume per unit time, We calculate as well the heating rate of the background plasma. Finally, we compare the experimental data [1, 2] with our theoretical predictions and show good agreement between the two.

Let us note first the range of laser emission parameters, where the present model is suitable. In further expressions, the electron temperature T is in eV, the intensity P is in 10^{18} W/cm^2, frequency ω is in 10^{15} Hz, density n is in 10^{18} cm^{-3}, and all other values are given in CGS units.

The plasma is assumed cold in comparison with the oscillatory energy so that

$$v \ll v_{\rm osc} = \frac{eE}{m\omega} \Leftrightarrow T \ll 6.7 \cdot 10^5 \cdot \frac{P}{\omega^2}. \qquad (1)$$

This condition is satisfied easily and remains true practically for all plasmas interacting with short intensive laser pulses, especially in the first stage of the experiment, preceding the Joule heating.

Second, the laser field intensity must be large enough for the characteristic spatial scale of scattering $b_{\rm osc}$ to be small compared to the radius of oscillations $r_{\rm osc}$:

$$b_{\rm osc} = \frac{e^2 Z}{v_{\rm osc} p_{\rm osc}} \ll r_{\rm osc} = \frac{v_{\rm osc}}{\omega} \Leftrightarrow \omega \ll 110 \cdot P^{3/8}, \qquad (2)$$

where $p_{\rm osc} = eE/\omega$ is the oscillatory electron momentum. This parameter range was first introduced in [4]. It has never been considered in conventional theories of electron-ion collisions, but it exhibits useful physical limits. It can be written as a limit on the ion Coulomb field potential energy at the distance of the oscillations radius $r_{\rm osc}$, which must be small compared to the oscillatory energy $mv_{\rm osc}^2$. In other words, the dimensionless parameter

$$\Omega = \left(\frac{b_{\rm osc}}{r_{\rm osc}}\right)^{\frac{1}{4}} \approx \left\{ \begin{array}{ll} \dfrac{1}{110}\dfrac{\omega}{P^{3/8}}, & p_{\rm osc} \ll mc \\ \dfrac{1}{170}\dfrac{\sqrt{\omega}}{P^{1/8}}, & p_{\rm osc} \gg mc \end{array} \right\} \ll 1 \qquad (3)$$

needs to be small. This parameter appears naturally when the test particle motion equation is put into dimensionless form. In particular, in non-relativistic approximation, for the field with the linear polarization along $\mathbf{z}_0$ axis this equation can be rewritten as

$$m\ddot{\mathbf{R}} = -\frac{\mathbf{R}}{R^3} + \cos\Omega t \cdot \mathbf{z}_0. \qquad (4)$$

Here the time is normalized to Ω/ω and distance to the characteristic scale

$$r_E = \sqrt{r_{\rm osc} b_{\rm osc}} = \sqrt{eZ/E}. \qquad (5)$$

Note, that radius r_E is equal to the distance from the ion, at which the amplitude of the laser field becomes of the order of the amplitude of the ion Coulomb field [4]. In terms of r_E, the smallness of the parameter Ω is equivalent to the fact that the radius of the sphere surrounding the ion, inside of which the Coulomb field dominates, is less than the radius of electron oscillations. Moreover, this scale appears naturally when the acceleration due to the ion during the scattering process is considered (see below).

Thus, the only one parameter Ω determines the structure of the 7-dimensional phase space of equation (4). In absence of the external field ($\Omega \to \infty$) particle motion is regular and well-known from the solution of the Rutherford problem [6]. Finite value of Ω results in formation of a stochastic layer in the vicinity of separatrix curves, but, as long as $\Omega \gg 1$, its volume remains exponentially small.

As the field amplitude increases (which corresponds to the decrease of Ω), the stochastic layer broadens and, at $\Omega \le 1$, occupies the whole region $|p| \le p_{\rm osc}$ in momentum space. Even in this case, the description of the electron dynamics is possible under the approximation of regular trajectories but only under the condition that those are highly energetic particles ($p \gg p_{\rm osc}$), which contribute mostly to the collision integral. However, we are primarily interested in the opposite limit of small thermal velocities (1), when particles dynamics is stochastic, since usually this is exactly the regime realized in experiments.

In order to describe particle scattering in presence of the strong laser field, let us make use of the fact that the collision process proceeds in two stages [4]. In the beginning, particles are just attracted to the ion with the essential changing of the impact parameters, i.e., the variation of the test particle density and momentum direction occurs at practically constant kinetic energy of the drift motion. Also the electron bunching happens at first stage, so that the wave phase at momentum of "hard" collision is the same for different electrons and ions. Secondly the "hard" collision occurs (which is actually last collision), accompanied by substantial change of electron momentum and electron departure from the Coulomb center, and, at this stage, scattering at large angles with a corresponding large energy exchange is possible.

It is enough to find the particle density $n(\boldsymbol{\rho}, t)$ before hard collision (i.e. the density in small vicinity of the ion) for deriving the probability density of collision with impact parameter $\boldsymbol{\rho}$ over time $W(\boldsymbol{\rho}, t) = vn(\boldsymbol{\rho}, t)d^2\rho$. To obtain the particle density $n(\mathbf{r}, t)$ prior to the last hard collision, one can use both the results of numerical simulation and the results of analytical analysis [4]. In both cases, the dependence $n(\mathbf{r}, t)$ is a singular periodical function of t:

$$n(\mathbf{r}, t) = n_e \frac{a}{\rho} \sum_{n=-\infty}^{\infty} \delta(\omega t - (n + \tfrac{1}{2})\pi). \qquad (6)$$

Here $\rho = \sqrt{x^2 + y^2}$ is the transverse electron coordinate (impact parameter) before the hard collision, $a(\mathbf{v}) \ge b_v = e^2 Z/mv^2$ is a coefficient describing the efficiency of particles attraction to the ion and depending on the direction of the initial velocity $\mathbf{v}$ relatively to $\mathbf{v}_{\rm osc}$. It is important to emphasize that this dependence on drift velocity direction is weak [4]. Thus, for the major fraction of test particles we have quasi-isotropic scattering, so that, we can use the expression (6) for further estimates. It is also important to note that the obtained singularity of the probability function occurs independently from the wave polarization and intensity. In particular, it can be shown that the same estimate is appropriate for ultra-relativistic intensities as well.

The distribution (6) describes electrons which have experienced strong attraction to the ion. Previously, such particles were called "representative" electrons [4]. Note, that for most of such particles one can consider the scattering of the total velocity, $\mathbf{V} = \mathbf{v} + \mathbf{v}_{\rm osc}(t)$ as a small-angle scattering.

The hard collision can be described by the relations from the Rutherford problem solution [6]. With smallness of the drift velocity (1) taken into account, momentum variation here is determined by the oscillatory momentum value at the collision moment and by the impact parameter ρ:

$$\Delta p \underset{\Delta p \ll p_{\rm osc}}{\approx} 2p_{\rm osc}\frac{b_{\rm osc}}{\rho}, \quad b_{\rm osc} = \frac{e^2 Z}{p_{\rm osc} v_{\rm osc}}. \qquad (7)$$

It is supposed in (6) that collisions occur only when the oscillatory velocity reaches its maximum (it is the effect of bunching which provides the latter [4, 5]) and the collision is momentary. The latter condition implies the upper limitation on the impact parameter:

$$\rho/v_{\rm osc} \ll \pi/\omega_o \Leftrightarrow \rho \ll r_{\rm osc}. \qquad (8)$$

Otherwise, for such large impact parameters velocity variation during scattering process is substantial and Rutherford's formulas (7) are not applicable. However, this limitation is not important, since energy variation ΔW of such far particles in strong fields ($b_{\rm osc} \ll r_{\rm osc}$) is small compared to the oscillatory energy:

$$\frac{2m\Delta W}{p_{\rm osc}^2} \underset{\rho > r_{\rm osc}}{\leq} \left(\frac{b_{\rm osc}}{r_{\rm osc}}\right)^2 \ll 1. \qquad (9)$$

Eq. (7) allows to find the relation between the density function on the impact parameters (6) and the distribution of hot particle production rate on momentum per unit volume and unit time:

$$g(p) = v n_i n(\rho) \frac{\rho}{p}\frac{d\rho}{dp} = n_i n(\rho)\cdot v\frac{4b_{\rm osc}^2 p_{\rm osc}^2}{p^4}. \qquad (10)$$

Using the density distribution (6) one gets finally:

$$g(p) = 4 n_i n_e p_{\rm osc}^2 \frac{v a b_{\rm osc}}{p_{\rm osc} p^3}. \qquad (11)$$

Note that the dependence of the hot electron distribution on momentum has universal law $\sim 1/p^3$ for any (relativistic and non-relativistic) energies of particles.

From the relation between the kinetic energy and the particle momentum

$$w = \sqrt{p^2c^2 + m^2c^4} - mc^2 \approx \begin{cases} p^2/2m, & p \ll mc \\ cp, & p \gg mc \end{cases}$$

(m is the rest mass of electron) it is easy to find the particles distribution on energy for non-relativistic case $w \ll mc^2$

$$g(w) = 8\pi n_i n_e m p_{\rm osc}^2 \frac{v a b_{\rm osc}}{p_{\rm osc}(2mw)^{\frac{3}{2}}} \qquad (12)$$

and the relativistic one $w \gg mc^2$

$$g(w) = 8\pi n_i n_e p_{\rm osc}^2 \frac{v c a b_{\rm osc}}{p_{\rm osc} w^2}. \qquad (13)$$

We will insert here the dimensional estimate for particles density $dn(w)/dt = \int_w^\infty g(w)dw$ with energies exceeding some limit in relativistic case for the period of field, supposing $w, cp_{\rm osc} \gg mc^2$:

$$\frac{dn(w)}{dt}[\text{cm}^{-3}\cdot\text{s}^{-1}] \approx \frac{10^{25} n_e^2 Z}{\sqrt{T}w}. \qquad (14)$$

In this relation, particle energy w is measured in MeV and other quantities are measured in units specified in (1). Note, that this density does not depend on laser intensity. However the total number of hot electrons depends on pump intensity due to the larger interaction volume with laser intensities.

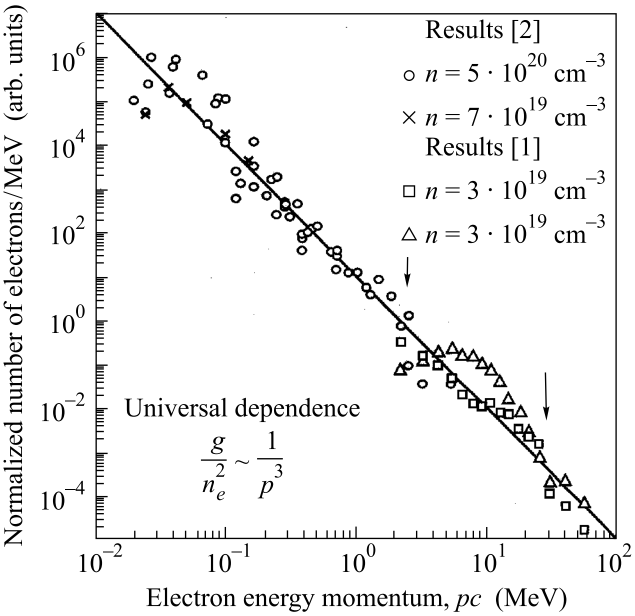
In particular, considering number of particles with energy higher than 1 MeV for plasma[1)] with density 10^{19} cm^{-3} and volume $300 \times 20 \times 20$ μm at the pulse duration of 10 ps, one gets that the hot electron number must be of the order of $10^9 Z$ particles, where $Z \geq 10$ is the charge of ions in plasma, which coincides well with the number of particles, 10^{10} to 10^{11}, measured experimentally. Another comparison with experimental data that one can perform is to observe that the number of hot electrons produced by collisions must be proportional to the square of the plasma density. We compared this result with the data taken from Ref. [2] and found good agreement between the theory and the experiment.

In experiments [1, 2], it is the distribution of particles scattered off in the same direction that is measured, i.e., the distribution function over momentum $g(p)$ as found in (11). Superposing the theoretical dependence (11) on the experimental data points one can see good coincidence between the two (Figure). Note that in Figure, we combined four different series of experimental measurements [1, 2].

Figure represents further evidence of the collisional effects on hot electrons. The collisional heating gives a natural upper limit to the momentum (and, correspondingly, the energy), which particles may get. That is the doubled oscillatory momentum $2p_{\rm osc}$, which, in conditions of the experiment [2] (Figure), corresponds to an energy of about 2 MeV. We see, indeed, the abrupt decrease of the hot particles number for energies higher than 2 MeV. Similar results were obtained also in [1][2)].

[1)] These data correspond to experiment [1].

[2)] We should note that much more energetic particles (with energies up to $p_{\rm osc}^2/m$ or $p_{\rm osc}^3/m^2c$) can be produced in result of electron-ion collisions in ultra-relativistic case. While the distribution law (11) is applicable only for electrons with energy less than oscillatory one $p_{\rm osc}c \gg mc^2$. The momentum and angular distribution law of such ultra-energetic particles is different from (11). Probably exactly these electrons has been seen at distribution tails in experiments [1, 7].



The comparison between the experimental results (Figure from [1, 2]) and the theoretical (solid line, (11)) dependence of hot electrons distribution on "energy momenta" pc. Arrows show the "cut off" effect

It is important to emphasize that what is shown in Figure is the dependence on the "energy momentum" (electron kinetic momentum times the speed of light, pc), but not on the actual energy. Those would be identical only in the case of ultra-relativistic particles in [1] only. The possibility of this interpretation of experimental results might be connected with the fact that magnetic scintillator used for the measurements, in fact, measures the distribution function of the particle momentum rather than the particle energy[3)].

Further comparisons with the experimental data can be performed by analyzing the heating rate $Q = \int g(w)wdw$, which is easy to calculate using the particle energy distribution (12). By substituting $a = b_v$, one gets the expression for the heating rate:

- in non-relativistic case $w, cp_{\text{osc}} \ll mc^2$

$$Q \simeq 4\pi n_i n_e m v_{\text{osc}}^2 \cdot v a b_{\text{osc}}; \quad (15)$$

- in the ultra-relativistic case $cp_{\text{osc}} \gg mc^2$

$$Q \simeq 4\pi n_i n_e mc^2 \cdot c b_c^2 \frac{c}{v}. \quad (16)$$

[3)] Taking this into account and also considering the dependence on plasma density and the "cut off" effect, we might conjecture that the power law shown in [2] was aberrant due to a calibration mistake.

Here $r_a = v/\omega$ is the adiabaticity radius, i.e., the distance over which the incident particles with the impact parameter exceeding r_a have adiabatically small energy variation; $b_c = e^2 Z/mc^2$ is the Rutherford radius, which, if estimated with for $Z = 1$ and for electrons velocity equal to the speed of light c, matches the classical electron radius.

Expression (16) must be supplemented by an additional term, describing the contribution of ultra-relativistic electrons with distribution law (13). This term will, obviously, coincide with (16), at least approximately, but for a more precise calculation, a more detailed description of collisions is necessary. That description would need to take into account the radiation losses and quantum effects taking place in the case when the momentum variation becomes significant.

Note that the heating rate in ultra-relativistic case (16) does not depend on the pumping field amplitude. The estimate of the heating rate per unit volume,

$$Q[\text{eV} \cdot \text{cm}^{-3}\text{s}^{-1}] = 10^{13} \frac{nZ}{\sqrt{T}} \quad (17)$$

allows to estimate the plasma temperature (kinetic energy) after pulse passing. In particular, for pulse with ultra-relativistic intensity and duration of 1 ps (which corresponds to the conditions of the experiment [1]), the electron temperature is of the order of hundreds of eV. That is exactly the order of the temperature (200-600 eV) observed in the experiment [1].

The results represented above were obtained using the pair collisions approximation, when the probability of the simultaneous collisions of three and more particles is assumed negligible. The condition of this approximation is the smallness of the interaction volume $nV_{int} \ll 1$. Usually (without field) the interaction volume is estimated as $V_{int} = b_v^3$, giving

$$n b_v^3 \ll 1 \Longleftrightarrow n r_D^3 \gg 1, \quad (18)$$

where $r_D = \sqrt{4\pi e^2 n / m v_T^2}$ is the Debye radius. In strong fields the interaction volume is $V_{\text{int}} \approx \sigma_{\text{eff}} r_{\text{osc}}$ ($\sigma_{eff} = \pi b_v b_{\text{osc}}$ is effective collisional cross-section [4]), which leads to the mild requirement

$$\frac{r_E}{r_D} = 3.67 \cdot 10^{-3} \frac{\sqrt{ZT}}{\sqrt{n}\sqrt[4]{P}} \ll 1. \quad (19)$$

But this condition, obviously, can be derived using different approaches. Indeed, the new scale, r_E, that appears as the particle attraction is taken into account, is the distance to the ion (multiplied by the factor $\sqrt{2\pi} r_E$ – see [4]), at which a particle moving near the ion with an oscillation velocity hits the ion after a single oscillation. The effect of attraction will not be "washed off" by neighboring particles if this scale is less than the Debye shielding radius r_D. Hence one again comes to the condition (19). One more simple condition can be considered. This is influence absent of external ions on the dynamics of hard collision. The volume of hard collision is $V_{\text{hard}} = 2\pi r_E^2 r_{\text{osc}}$. So the condition is $nV_{\text{hard}} \ll 1$ or

$$2\pi r_E^2 r_{\text{osc}} n = \frac{Z}{2}\omega_{pl}^2/\omega^2 \ll 1 \qquad (20)$$

ordinary condition of transparent plasmas. Both conditions (19), (20) are simple to be fulfilled.

To summarize, in considering the two types of particles being scattered (6), we derived an expression for the effective collision frequency and the hot particles energy distribution, which agree well with experimental data. Moreover, taking into account the "representative" electrons (the singular part of (6)) is necessary for an adequate explanation of the experimental results.

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