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Magnetic field rectifiers using semicircular Josephson junctions

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A novel method for rectifying alternating magnetic fields is demonstrated using fluxons in semicircular Josephson junctions. An external magnetic field applied parallel to the dielectric barrier of the semicircular junction has opposite polarities at the ends of the junction and supports penetration of opposite polarity fluxons into the junction in the presence of a constant dc bias. When the direction of the field is reversed, flux penetration is not possible and flux-free state exists in the junction. Thus effective rectification of an alternating magnetic field can be achieved in semicircular Josephson junctions. This unique phenomenon is specific to this geometry and can be employed in rf SQUID magnetometers.

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Two superconductors separated by a thin oxide layer is called a Josephson junction (JJ) which allows the superconducting Cooper pairs to tunnel through the barrier [1]. Long JJ's offer the possibility of studying solitons that account for the magnetic flux-quanta (fluxon) moving along the tunnel barrier [2]. A fluxon is basically a quantum of magnetic field which can be used for transmission of information or can be an object based on which certain Josephson devices such as flux-flow oscillators [3], voltage rectifiers [4], logic gates [5], etc. can be implemented. Fluxons can be trapped in the junction either during the normal-superconducting transition or by applying an external magnetic field into the junction. In the superconducting state only fluxons or antifluxons can exist in the junction [6] and they are driven by the Lorentz force associated with a dc current. Different geometries are proposed for JJ to study the fluxon dynamics and among them, annular geometries offer the advantage of reflectionless motion of fluxons and is extensively studied both theoretically and experimentally [7]. In the absence of an external magnetic field, trapped fluxons cannot escape from a linear junction and they make successive reflections at the edges of the junction [8]. Progressive fluxon motion in JJ is associated with a dc voltage which can be detected across the junction.

Josephson junctions are best transducers which can convert magnetic energy into electrical energy. They are widely used in SQUID magnetometers [9], SIS mixers [10] and in voltage standard applications [11]. Recently, fluxon based voltage rectifiers [4, 12] have attracted much attention due to the fact that they can find important applications in Josephson digital devices [13]. Various geometries and external conditions are investigated towards this end [14]. The influence of an artificially created ratchet potential on fluxon dynamics in nonuniform JJ have been studied and voltage rectification properties of these JJs are demonstrated in recent papers [15]. The net unidirectional motion exhibited by a particle in a ratchet potential is the key factor which is employed in rectifying bias currents. However, the working of all these voltage rectifiers critically depend on the ratchet potential and we cannot expect stable performance from these devices as ratchet potentials are highly sensitive to external perturbations. Amplitude ranges of rectification is limited in these devices and the rectified output in these devices does not have a linear relationship with the input. In all these works, rectification properties are studied using alternating bias currents and effective means of rectification of alternating magnetic fields are not discussed.

In this letter, we demonstrate a novel method to construct fluxon based diodes for rectifying harmonically oscillating magnetic fields. Investigations on a dc biased semicircular JJ placed in an alternating magnetic field applied parallel to the plane of the dielectric barrier shows that the junction supports flux flow only in alternate half cycles of the field. The flux linked with the edges of the junction has opposite polarities and support penetration of fluxons and antifluxons simultaneously from opposite ends of the junction under a constant dc bias. When the direction of the field is reversed, flux penetration is not possible and flux-free state exists in the junction. Thus, with this geometry, effective rectification of oscillating fields can be achieved. This is a unique phenomenon associated with the semicircular junctions. In this letter, we first formulate the model equations corresponding to the proposed junction with an applied alternating magnetic field and study the be-

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haviour of the junction in static fields and then in time varying fields to demonstrate rectification properties.

Theoretical model. A JJ with a semicircular geometry is considered with an external harmonically varying magnetic field applied parallel to the dielectric barrier of uniform thickness (see Fig.1). The external field is applied in such a way that it is directed radially at the

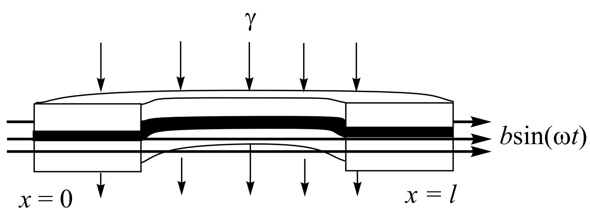


Fig.1. A sketch of the semicircular JJ with the applied field $b\sin(\omega t)$ parallel to the plane of the dielectric barrier of uniform thickness (not drawn to scales)

left-end ($x = 0$) of the junction. The field interacts with the interior as well as through the boundaries of the junction and the flux linked with the junction can be expressed as $d\varphi(x) = \varepsilon\,(\bar{H}(t)\cdot\bar{n}) = \varepsilon\, H\sin(\omega t)\cos(kx)dx$ [16]. Where H is the strength of the applied magnetic field, $\bar{n}$ is the unit vector normal to the plane of the junction, ε is the coupling factor which links the external field with the junction, $k = \pi/l$ is the geometrical constant which defines the magnetic field inside the semicircular junction, the spatial coordinate x is normalized to λ_J (Josephson penetration depth) and l ($\gg \lambda_J$) is the normalized length of the junction. The time t is normalized to the inverse plasma frequency, $\omega_0 = \bar{c}/\lambda_J$, $\bar{c}$ is the maximum velocity of the electromagnetic waves in the junction and ω is normalized frequency of the oscillating field. Therefore the induced current in the junction due to the applied field is $d\varphi(x)/dx = \varepsilon H\ \sin(\omega t)\cos(kx)$. This current term gives a net zero value over the length of the junction and therefore circulate in closed form across the junction. Thus the effect of an external field in the junction is to induce spatially varying currents. Thus a semicircular JJ under a time varying magnetic field with a dc bias is modelled with the general perturbed sine-Gordon (SG) partial differential equation [16,17]

$$\varphi_{tt} - \varphi_{xx} + \sin\varphi = -\alpha\varphi_t + b\sin(\omega t)\sin(kx) - \gamma \quad (1)$$

where $\varphi(x,t)$ is the superconducting phase difference between the electrodes of the junction, α is the dissipation parameter due to quasiparticle current, $b = \varepsilon\ H\ k$ and γ is the normalized amplitude of the dc bias. The boundary conditions of the junction can be obtained from the induced current term $d\varphi(x)/dx = \varepsilon H\ \sin(\omega t)\cos(kx)$ as

$$\varphi_x(0,t) = \frac{b}{k}\sin(\omega t); \quad \varphi_x(l,t) = -\frac{b}{k}\sin(\omega t). \qquad (2)$$

These boundary conditions are consistent with the fact that the effective field linked with the junction has opposite polarities at the ends. For sufficiently higher positive values of γ in Eq. (1), fluxons can enter the junction from $x = 0$ and antifluxons can enter the junction from $x = l$ (right-end) and they can move in opposite directions. The propagation of the fluxons in one direction and antifluxons in opposite direction produce a nonzero voltage across the junction. As the boundary conditions are not reflective, after a transitory motion, fluxons and antifluxons are exited from the junction. When the direction of the field is reversed, fluxon (or antifluxon) penetration become impossible and flux-free state exists in the junction.

Eq. (1) with boundary conditions (Eq. (2)) represents a semicircular JJ in an alternating magnetic field. In the absence of perturbations ($\alpha = b = \gamma = 0$), Eq. (1) becomes the SG equation which is a conservative, nonlinear dispersive wave equation that supports special solutions called solitons. A SG soliton is a localized wave with particle-like properties and is analytically described by the formula [17]

$$\varphi(x,t) = 4\tan^{-1}\left[\exp\left\{\frac{\sigma(x-x_0)}{\sqrt{1-u^2}}\right\}\right] \qquad (3)$$

where $x_0 = ut + x_0'$ is the location of the soliton with u as the velocity of the soliton and x_0' as the initial location center. $\sigma = \pm 1$, is the polarity of the soliton. A long JJ could support the resonant propagation of fluxons trapped in the junction, the fluxon being a 2π jump in the phase difference φ across the insulating barrier which separates the two superconductors. There are two possible orientations for the flux. A quantum of flux with $\sigma = +1$ is called a fluxon and that with $\sigma = -1$ is called as an antifluxon. A moving fluxon is accompanied by a voltage pulse φ_t which can be detected across the junction.

To get some information on the fluxon dynamics, we first determine the potential induced by the external field inside the junction and then find energy change associated with a moving fluxon in the junction. Lagrangian density of Eq. (1) with $\alpha = \gamma = 0$ is

$$\mathbf{L} = \left\{\frac{\varphi_t^2}{2} - \frac{1}{2}\left(\varphi_x - \frac{b}{k}\sin(\omega t)\cos(kx)\right)^2 - (1-\cos\varphi)\right\}. \qquad (4)$$

Therefore the corresponding potential energy density is (second term of the above equation)

$$\mathbf{U}(x,t) = \frac{1}{2}\left\{\varphi_x^2 - \frac{2b}{k}\sin(\omega t)\cos(kx)\varphi_x + \right. \left. + \left(\frac{b}{k}\sin(\omega t)\cos(kx)\right)^2\right\}. \quad (5)$$

The first term is independent of the applied field and the third term is independent of the flux motion in the junction. Therefore the change in the potential due to the combined effect of the applied field and the flux motion in the junction can be determined from the second term as :

$$U(x,t) = -\frac{b}{k}\int_{-\infty}^{+\infty} \varphi_x \; \sin(\omega t)\cos(kx)\,dx. \quad (6)$$

Substituting Eq.(3) in (6) and integrating, we get

$$U(x_0,t) = -2bl\,\mathrm{sec}\,h\left(\frac{\pi^2}{2l}\sqrt{1-u^2}\right)\sin(\omega t)\cos(kx_0). \quad (7)$$

For long junctions and at relativistic velocities, $u \sim 1$, Eq. (7) becomes

$$U(x_0,t) = -C\sin(\omega t)\cos(kx_0) \quad (8)$$

where $C = 2bl$ is a constant. Eq. (8) shows that the potential is oscillating at the frequency of the applied field. This oscillating potential controls the flux flow inside the junction and helps in the rectification of the field.

Energy of the unperturbed SG system is

$$H^{SG} = \int_0^l \left[\frac{1}{2}(\varphi_t^2 + \varphi_x^2) + 1 - \cos\varphi\right]dx. \quad (9)$$

Perturbational parameters modulate the velocity of the solitons and may cause to dissipate energy. The rate of dissipation is calculated by computing

$$\frac{d}{dt}(H^P) = [\varphi_x\varphi_t]_0^l + + \int_0^l [-\alpha\varphi_t^2 + (b\sin(\omega t)\sin(kx) - \gamma)\varphi_t]\,dx \quad (10)$$

where the first term on the right side account for the boundary conditions. From Eq. (3), we get $\varphi_t = -u\varphi_x$ and from Eq. (2), we get $\varphi_x^2(0,t) = \varphi_x^2(l,t)$ (symmetric boundary conditions). Substituting these expressions, we find that the first term in the right hand side of the above equation vanishes – a symmetric boundary condition does not change average energy value of a fluxon. Inserting Eq. (3) in Eqs. (9) and (10) and following perturbative analysis [17], we get

$$(1-u^2)^{-3/2}\frac{du}{dt} = -\alpha\frac{u}{\sqrt{1-u^2}} - - \frac{\pi}{4}\{b\,\mathrm{sec}\,h[\frac{\pi^2\sqrt{1-u^2}}{2l}]\sin(\omega t)\sin(kx_0) - \gamma\}. \quad (11)$$

This expression describes the effect of perturbations on the fluxon velocity. In the above equation, the first term in the right-hand side represents the energy dissipation due to internal damping, second term account for the energy change associated with the external field and the third term represents the input power from the bias current.

The effects of a dc current on the fluxon dynamics in the presence of the external field is studied using Eq. (11). Zero-voltage state exists in the junction (flux-free state) when the dc bias is below a threshold value. By variation of the soliton position x_0, from Eq. (11), we find the largest possible bias current of zero-voltage state ($u = 0$) to be [18]

$$\gamma_1 = b\,\mathrm{sec}\,h(\pi^2/2l) \quad (12)$$

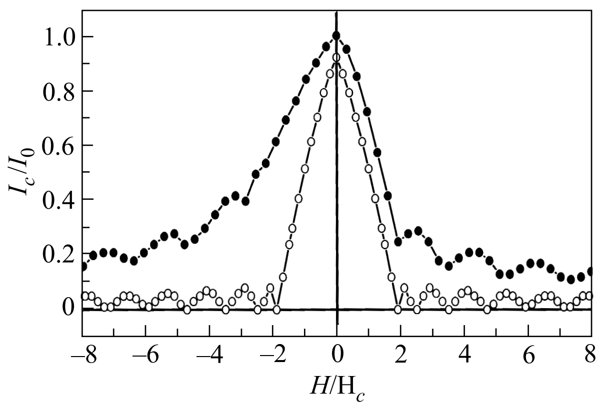
This is the threshold value of the applied bias, below which flux propagation is not possible in the junction. This threshold value depends on the magnetic field and is directly proportional to the field.

Numerical methods. To solve Eq. (1) with boundary conditions given by Eq. (2), we use an explicit method treating φ_{xx} with a five point, φ_{tt} with a three point and φ_t with a two point finite-difference method. A time step of 0.0125 and a space step of 0.025 is used for the discretization. Details of the simulation can be found in Ref. [18]. After the simulation of the phase dynamics for a transient time, we calculate the average voltage V at the load over a period of the field T as

$$\langle V\rangle = \frac{1}{T}\int_0^T \varphi_t(l)\,dt = \frac{\varphi(T) - \varphi(0)}{T}.$$

The average velocity attained by the fluxons during the transit can be calculated from the average voltage using the relation $\langle u\rangle = (l/2\pi)\langle V\rangle = (l/2\pi)\langle\varphi_t\rangle$. Thus the mean voltage in the junction is proportional to the average velocity of the fluxons. Voltage rectification can be clearly demonstrated using the time domain snapshots of the instantaneous voltages, $v(t) = \varphi_t(l,t)$, in the junction. These voltage pulses are measured at intervals of $t = 4$ time units and to get smooth curves we additionally averaged the pulses in that intervals. Time period of the ac signals are taken much larger than the typical response time of the system. In the following simulations we assumed the dissipation parameter $\alpha = 0.1$.

Properties of the junction under a static field. It is important in practical applications to know the behaviour of the junction under a static magnetic field especially the dependance of critical current (I_c) on the applied field (H) [19]. In weak static magnetic fields, long JJs behave like weak superconductors and show the Meissner effect. In this regime the critical current decreases linearly with the external field. This behaviour exists up to a critical field H_c. At this critical field, magnetic flux in the form of fluxons can overcome the edge barrier effects and can penetrate the junction [20]. For long JJs the first critical field is $H_c = \Phi_0/\pi\Lambda\lambda_J$, where Λ is the effective magnetic thickness of the junction and $\Phi_0 = h/2e = 2.064\cdot10^{-15}$ Wb is the flux quantum. The dependance of I_c (normalized to maximum Josephson current I_0) on a static magnetic field (H/H_c) applied to a semicircular JJ of $l = 10$is shown in Fig.2 (solid circles). For comparison, critical current versus magnetic



Fi.2. Normalized critical current (I_c/I_0) versus static magnetic field (H/H_c) of a semicircular JJ (•) and that of a rectangular JJ (○)

field pattern of a standard rectangular JJ is presented (open circles). In positive magnetic fields, $I_c(H)$ pattern in semicircular JJ shows that static fluxons can exists in the junction and a minimum critical current is required to induce flux motion in the junction. In negative fields, the junction behave differently and the critical current pattern is displaced and indicates that higher critical currents are required to induce flux motion in the junction.

In the absence of an external field ($b = 0$), fluxon dynamics in semicircular JJ is same as that in any ordinary rectangular junction. When an external static magnetic field is applied, due to the opposite polarities of the flux linked with the ends of the junction, opposite polarity fluxons can enter the junction from the ends in a properly biased state. In the junction, fluxons and antifluxons move in opposite directions under the influence of the dc bias. When they reach the ends of the junction after a transit, they are exited from the junction due to the non-reflecting boundary conditions. When the direction of the external field is reversed, fluxon penetration is not possible due to the repulsive Lorentz force of the dc bias and zero voltage corresponding to the flux-free state exists in the junction. This typical characteristics demonstrates that semicircular junctions offer the advantage of rectifying alternating magnetic fields. The rectification property of the junction can be observed from Fig.3, where we plot the IVC of a semicircular

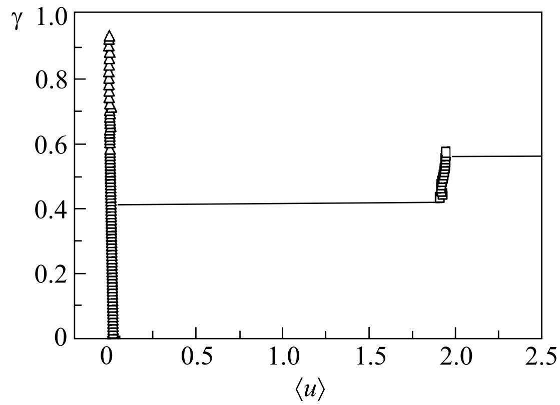


Fig.3. Applied dc bias γ versus the average normalized velocity $\langle u\rangle = -(l/2\pi)\langle\varphi_t\rangle$ of a semicircular junction of $l = 20$ with a static field $b = +0.15$ (□) and with $b = -0.15$ (△)

junction of length $l = 20$ with a static magnetic field of strength $b = +0.15$ (squares) and with $b = -0.15$ (triangles). To get these plots, we calculate average voltage for a current γ, then the current γ is increased in steps of $\delta\gamma = 0.01$ to calculate the voltage at the next point of the IVC. We use a distribution of the phases and their derivatives achieved in the previous point of the IVC as the initial distribution for the following point. For positive values of b, flux propagation is possible and we get finite average voltage across the junction. In this regime the junction behaves as a forward biased diode. At small positive bias values, a fluxon-antifluxon pair is found to be taking part in dynamics and the dynamics of this pair gives an average velocity of $\langle u\rangle \simeq 2$. The pair executes stable dynamics in a range of bias values. At higher values of the bias, large number of fluxons enter the junction resulting in a switch to high voltage states. For negative values of b, flux penetration and propagation is not possible and zero voltage state exists in the junction which is equivalent to the reverse biased state of a diode.

IVC in rf fields. An oscillating magnetic field is applied parallel to the dielectric barrier of the junction with a constant dc bias. In the positive half cycles of the applied field, flux penetration and propagation is possible and finite voltages are observed across the junction. In the negative half cycles of the field, fluxons (or antifluxons) cannot enter the junction due to the repulsive Lorentz force, and zero voltage exists in the junction. Simulations are started with $\varphi = 0$ on a junction of $l = 10$. Fig.4 shows the IVC of the junction for different values of the oscillating field amplitudes and at a

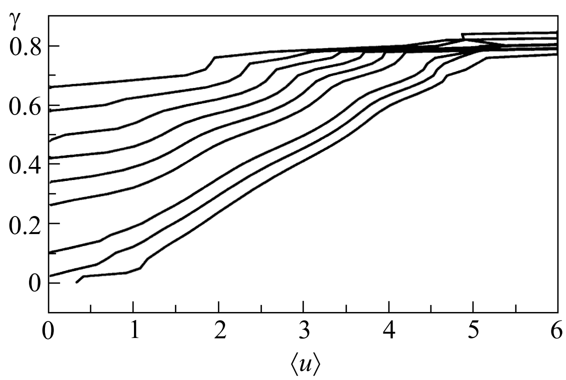


Fig.4. Applied dc bias γ versus the average normalized velocity $< u >$ for different values of the applied rf field. The parameters are $l = 10$ and $\omega = 0.1$. The applied field strength increases from the top to the bottom curve from 0.50 to 1.50 in steps of 0.1

constant frequency ($\omega = 0.1$). In the figure, applied magnetic field is increasing from the top to the bottom curve in the range 0.50 to 1.50 in steps of 0.1. At lower magnetic fields, critical currents for fluxon penetration is large and the critical current gradually decreases on increasing the field strength.

Rectification of alternating fields. To demonstrate the rectification properties of the junction, we show a series of plots showing the time domain snapshots of voltage pulse forms $v(t)$ as a function of time t. The magnitude of the field should be sufficiently large to introduce fluxons into the junction. At small magnetic fields, fluxons cannot enter the junction and zero voltage exists. For sufficiently higher amplitudes (e.g. $b = 1.0$), fluxon penetration is possible in the positive half cycles and we get finite voltage in the junction (see Fig.5). Rectification takes place in the following way. In the first half (positive part) of the alternating field, fluxons enter from the left-end and antifluxons enter from the right-end and they move in opposite directions under the influence of the dc bias. The motion of fluxons in opposite directions produces a finite voltage across the junction. During the second half (negative part) of the magnetic field, antifluxon (or fluxon) penetration is not possible due to the repulsive Lorentz force and zero voltage (flux-free state) exists in the junction. Thus effective rectification of the field can be achieved in semicircular Josephson junctions. The number of fluxons taking part

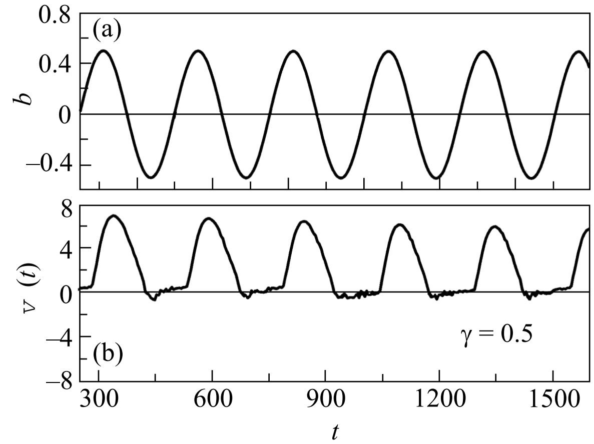


Fig.5. Rectification of a rf field with $\gamma = 0.5$ on a junction of $l = 10$. (a) Applied field of amplitude $b = 1.0$ and frequency $\omega = 0.05$ (b) Output pulse form $v(t)$ as a function of time t

in the dynamics (and therefore the output voltage) can be controlled by controlling the strength of the magnetic field.

In Fig.6 we plot the average velocity (averaged over a period of the field) as a function of the magnitude of

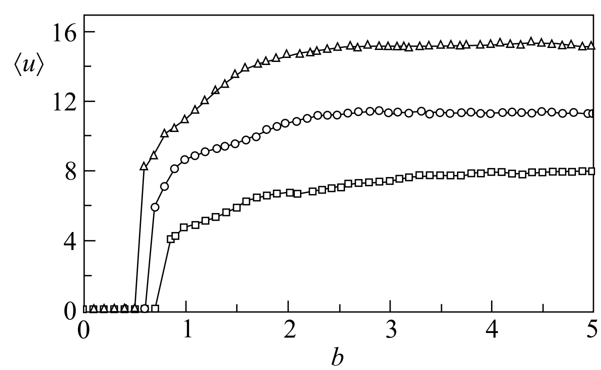


Fig.6. Magnetic field amplitude b versus average velocity $\langle u \rangle$ for different junctions. The parameters are $\omega = 0.05$, $\gamma = 0.5$, $l = 10$ (□), $l = 15$ (○) and $l = 20$ (△)

the field for different length of the junctions. A constant dc bias is applied to the junction in order to maintain flux motion in alternate half cycles. In the figure average voltage increases from zero and then increases linearly at higher values of the external field. Thus this device gives output which is linearly proportional to the input.

By reversing the dc bias (i.e., γ to $-\gamma$), positive part of the alternating field can be suppressed. In this case, fluxons cannot enter the junction during positive half cycles of the field due to the repulsive Lorentz force while flux penetration and propagation is possible in the negative half cycles. In Fig.7 we show rectification in a

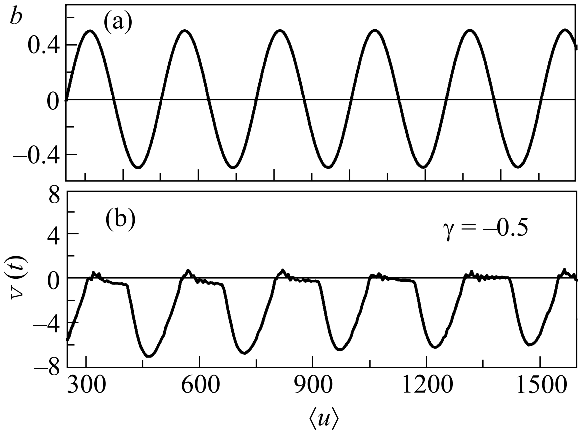


Fig.7. Rectification on a junction of l = 10 with $\gamma = -0.5$. (a) Applied field of amplitude $b = 1.0$ and frequency $\omega = 0.05$ (b) Output pulse form showing negative pulses

junction of $l = 20$ with $\gamma = -0.5$. Thus by choosing appropriate dc bias, either positive part or negative part of the alternating field can be rectified.

In conclusion, this letter contains theoretical predictions of rectification of harmonically oscillating rf fields using fluxons in semicircular JJs. This device may find important applications in sub-millimeter radio wave astronomy, SQUID magnetometers, SIS mixers, etc. The main advantages of the proposed diode are (i) very simple to fabricate, (ii) output of the device is linearly proportional to the applied field, (iii) flux motion takes place only in alternate half cycles so that heating and energy losses associated with flux motion can be reduced and (iv) independent of external perturbations. In the proposed JJ diode, velocity of a fluxon is proportional to the voltage and a nonzero average velocity over a period of the rf field means rectification of the field. By properly selecting junction parameters and the dc bias, it is possible to rectify fields in different amplitude and frequency ranges. This geometry can also be used for rectification of ac currents with the help of a static magnetic field.

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