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Timelapse

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We discuss the existence in an arbitrary frame of a finite time for the transformation of an initial quantum state into another e.g. in a decay. This leads to the introduction of a timelapse $\tilde{\tau}$ in analogy with the lifetime of a particle. An argument based upon the Heisenberg uncertainty principle suggests the value of $\tilde{\tau} = 1/M_0$. Consequences for the exponential decay formula and the modifications that $\tilde{\tau}$ introduces into the Breit-Wigner mass formula are described.

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The subject of this paper concerns the exponential decay law and the Breit-Wigner (BW) mass distribution formula. These formulas are a standard part of particle physics and can indeed be connected by a simple transform, as is taught in many undergraduate physics courses [1]. In recent years the validity of the BW has been demonstrated to an unprecedent degree by the LEP data and analysis upon the Z gauge particle [2]. After substantial theoretical corrections for radiative effects the predicted theoretical widht (assuming three light neutrinos) and the experimental value, *based upon a BW fit*, agree to better than one per cent. We will be interested later in the small discrpancy but our first observation is that the agreement is quit impressive. These facts are somewhat surprising because neither the exponential decay law nor the BW mass curve is predicted rigorously within Quantum Mechanics (QM). On the contrary we have precise QM objections to the former [3, 4] and only approximate derivations of the latter [5, 6]. Nor does a field theoretical treatment change the situation. To some these QM results relegate our two formulas to little more than phenomenological games. We in the other hand start from these two formulas and argue for a modification which will in part reconcile the decay law with QM, and provide an explanation for the discrpancy in the Z widht described above.

One of the implicit assumptions in particle physics is that decays occur instantaneously. However, as Einstein has taught us, istantaneity, for anything other than a point, can at best be valid in a single Lorentz frame. Thus even if, say in its rest frame, a particle decayed instantaneously, a general observer would find different times for decays at different points within the wave packet. Of course, this could only be determined in a statistical sense since the wave function is not an observable. Thus, in general, there will exist times during which the quantum state is neither the initial nor the final state but a *linear combination* of both which tends towards the later with increasing time. We shall call a measure of this time interval the "timelapse".

There is only one exception to the above observations, a measurement process may involve (ideally) the localization in space and time of a particle. This collapse of the wave function is instantaneous for all observers (it defines the corresponding event in each frame) and is fundamentally irreversible since the creation of a particle at a given space time point cannot instantaneously inflate to finite space regions without violating the limiting velocity of light. The collapse of the wave function is a subject of great interest in itself but will not concern us further in this paper.

Why cannot we avoid the discussion of timelapse by considering particle or state creation to be a delta function in space and time? Firstly because in many practical problems we know this not to be the case, such as for a particle *trapped* within a potential well e.g. a muon within a muonic atomic state. Secondly because we often know or desire to study particles which approximate energy-momentum eigenstates and this implies *large* spatial dimensions.

An earlier introduction of a type of timelapse is contained in the book of Jackson [7]. In one of the classical derivations of essentially quantum effects, Jackson introduces the "formation time" of the electron in nuclear beta decay. Arguing that the outgoing energetic electron could be considered to have been accelerated from rest to its final velocity over a finite formation time, he calculates the induced spectrum of radiation by the electron, the *inner bremsstrahlung*. Jackson also notes that the same effect would result if the charge were *created*

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over the same time interval. Invoking the uncertainty principle, he evaluates this time interval Δt as

$$\Delta t \sim 1/E, \tag{1}$$

where E is the electron's energy. Now, while acknowledging precedence for the idea of a formation time to Jackson, his approach is significantly different from ours. The use of the particles energy in Eq. (1) implies that different particles take different times for acceleration. Of course, the antineutrino does not contribute to the inner bremsstrahlung and the heavy nucleon contributions are negligible, so this may appear an academic question. However, to us, it is obvious that the same timelapse must occur for each of the particles in the final state, independent of their final energy or momentum. It is not conceivable that the outgoing electron has been created with certainty close to one while the antineutrino is perhaps to all extents still to be created. We shall take care to define for *each decay* a common timelapse that depends at most upon the kinematics of the initial system in a preferential Lorentz frame. Of course, one must also guarantee that the timelapse of the outgoing state coincides with that of the vanishing incoming state.

Is the timelapse a function of the spatial localization of the quantum state? We believe not. There *is* a Δt directly connected to Δx but this is what we might call "passage time". Consider a single particle with a sufficiently large Δx to be an approximate $(E, p, 0, 0)$ eigenstate (for simplicity we take its momentum to be along the x axis). Using the uncertainty principle $\Delta x \Delta p \sim 1$ and the Einstein relation $E^2 = p^2 + M^2$, we find

$$\Delta x \to \Delta p \to \Delta E = p\Delta p/E \sim v/\Delta x$$

hence

$$\Delta t \sim \Delta x/v, \tag{2}$$

where v is the velocity of the particle. Thus Δt is for our wave packet a measure of the time it takes to pass a given $y - z$ plane, hence the name passage time. This Δt obviously has nothing to do with a timelapse, and indeed becomes infinite in the particle rest frame. On the other hand, we expect, from the approximate validity of the exponential decay law, that timelapse must be small compared to the lifetime of a particle in any frame, see below.

We believe that timelapse must be a close relative to lifetime. As for lifetime, we will define it in the rest frame of an initial single particle state, or more precisely the frame in which the average velocity is null. In analogy with the lifetime τ, we shall denote the timelapse for a process by $\tilde{\tau}$. What does the Heisenberg uncertainty principle tell us? Well, we have excluded a connection to ΔE and we can also exclude the half width ΔM for a decay particle since this is reserved for τ

$$\tau = 1/\Delta M. \tag{3}$$

This leaves us with essentially only one choice

$$\tilde{\tau} = 1/M_0 \tag{4}$$

for a decaying particle with central mass M_0.

Now a decay of a composite particle such as a J/ψ may be considered an annihilation and/or interaction at the quark level. Thus, timelapses for interactions should also be defined. The natural choice for $\tilde{\tau}$ in these cases is

$$\tilde{\tau} = 1/E_{CM}, \tag{5}$$

where E_{CM} is the center of mass energy. Equation (5) would automatically include Eq. (4) if it where not for the fact that a given decay may occur at a mass diverse from M_0 due to the existence of mass curves. To reconcile the two, we should modify Eq. (4) to read

$$\tilde{\tau} = 1/M, \tag{6}$$

but in the subsequent applications, in this paper we will employ Eq. (4) for simplicity.

How does the existence of a $\tilde{\tau}$ modify the exponential decay law? This can easily be derived after assuming a given analytic form for a state during timelapse. For simplicity and in analogy with the original decay law, we shall assume this to be an exponential form. Exponential decreasing $\exp[-t/\tilde{\tau}]$ for the incoming state and its complement, $1 - \exp[-t/\tilde{\tau}]$, for the corresponding outgoing state. Thus when we consider an ensemble of $N(t)$ particles with a given lifetime τ and timelapse $\tilde{\tau}$ we must divide them into two classes: N_u, the number of undecayed particles, and N_d, those that have begun the decay process but have still a residual probability of being found in a measurement of $N(t)$. N_d would be zero if instantaneous decay ($\tilde{\tau} = 0$) were valid.

The differential equation that governs $N_u(t)$ is the standard one

$$dN_u(t) = \frac{1}{\tau} N_u(t) dt \tag{7}$$

while that for $N_d(t)$ must allow for a source term proportional to $dN_u(t)$,

$$dN_d(t) = -\frac{1}{\tilde{\tau}} N_d(t) dt - dN_u(t) \tag{8}$$

the negative sign in front of the last term is the correct one since $dN_u(t) < 0$ for $dt > 0$.

Now solving these coupled equations and using as initial condition $N_d(0) = 0$, we find

$$N(t) = \frac{N(0)}{\tau - \tilde{\tau}} \left[\tau \exp\left(-\frac{t}{\tau}\right) - \tilde{\tau} \exp\left(-\frac{t}{\tilde{\tau}}\right)\right]. \quad (9)$$

This is the modification of the standard exponential decay law that our timelapse $\tilde{\tau}$ introduces. Note that for $\tilde{\tau} = 0$ we obtain the single exponential form and the limit $\tau = 0$ yields an exponential decay with lifetime $\tilde{\tau}$. Whence $\tilde{\tau} = 1/M_0$ sets a lower limit to the effective lifetime and hence an upper limit to the half width.

A simple calculation yields for the effective lifetime τ_{eff}

$$\tau_{\text{eff}} = (\tau^3 - \tilde{\tau}^3)/(\tau^2 - \tilde{\tau}^2). \quad (10)$$

For $\tilde{\tau} \ll \tau$, we can write

$$\tau_{\text{eff}} = \tau[1 + \epsilon^2 + O(\epsilon^3)], \quad (11)$$

where $\epsilon = \tilde{\tau}/\tau$. However, we note that the decay rate Γ remains connected to τ, $\Gamma = 1/\tau$. This follows from our assumption that $\tilde{\tau} \sim 1/M_0$ and hence is *independent* of any interaction coupling constants in contrast to Γ and τ which obviously are directly dependent.

Another, very interesting, observation is that, for very small t, Eq. (9) has no linear term in t. Indeed for $t \ll \tilde{\tau}, \tau$

$$N(t) = N(0)\left[1 - \frac{t^2}{2\tau\tilde{\tau}} + O(t^3)\right]. \quad (12)$$

This reconciles, for short times, our modified decay law (no longer a single exponential) with basic quantum mechanical arguments [8–10] which have led, amongst other things, to the so called quantum Zeno effect [11, 12]. This at least in principle allows $\tilde{\tau}$ to be calculated, from Eq. (12) and the quantum mechanical result $P(t) = 1 - t^2(\Delta H)^2 + \ldots$ [13], specifically $\tilde{\tau} = 1/[2\tau(\Delta H)^2] = \Gamma/[2(\Delta H)^2]$.

In Fig.1, we show the modification of $|\psi(t)|^2 \equiv P(t)$ for various ϵ values. In this plot the increase of the effective lifetime is evident, as is the annulment of $dP(t)/dt$ (insert) for $t \to 0$, source of the quantum Zeno effect. The direct measurement of $|\psi(t)|^2$ is often possible (e.g. in muon decay). However, for muons $\epsilon \sim 3 \cdot 10^{-16}$ so that no effect due to $\tilde{\tau}$ could ever be detected.

Let us now calculate the modification in the standard BW mass formula produced by Eq. (9). We have

$$|\psi(t)| \propto \exp\left(-\frac{t}{2\tau}\right)\left\{1 - \epsilon \exp\left[\frac{t(\epsilon - 1)}{\epsilon\tau}\right]\right\}^{1/2}.$$

Hence,

$$\chi(x) = \int_0^\infty dt \exp\left(i\frac{x}{2\tau}t\right)|\psi(t)| \propto$$

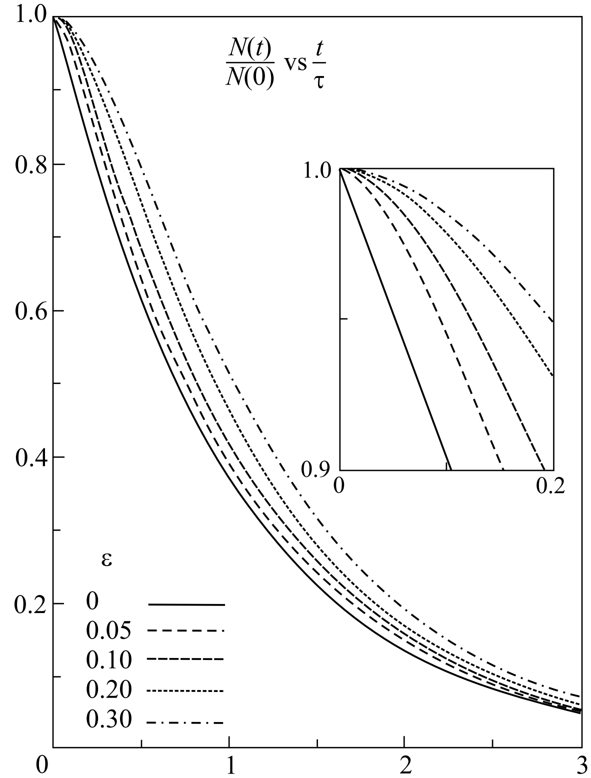

Fig.1. The decay law $N(t)/N(0)$ versus t/τ for various values of ϵ

$$\propto \frac{1}{x+i} - \sum_{n=1}^{\infty} \frac{n!!}{2^n n!} \frac{\epsilon^n}{x + ia_n}$$

where $x = 2\tau(M - M_0)$ and $a_n = 1 + 2n(1-\epsilon)/\epsilon$. Treating ϵ as a small quantity, we may perform an analytic calculation of $|\chi(x)|^2$ to lowest order in ϵ. We find for our modified Breit-Wigner (MBW):

$$\text{MBW} \equiv |\chi(x)|^2 = BW\left\{1 - \epsilon\frac{x^2 + a_1}{x^2 + a_1^2} + \right.$$

$$\left. + \frac{\epsilon^2}{4}\left[\frac{7}{2} + \frac{x^2+1}{x^2+a_1^2} - \frac{x^2+a_2}{x^2+a_2^2}\right] + O(\epsilon^3)\right\} \quad (13)$$

where $\text{BW} = 2\tau M_0/\{[\pi/2 + \arctan(2\tau M_0)](x^2+1)\}$ is the standard BW.

Now ϵ is so small for almost all weak or electromagnetic decays that one might think to pass directly to the strong decays in the search of evidence for a MBW. However, of all the weak processes a special role is played by the decays of the heavy intermediate vector bosons W and Z. These have widths of *several* GeV and thus correspond to ϵ of a few %. Furthermore, the data upon the Z is particularly precise with errors in M_z and Γ_z of order 10^{-5}. The LEP data have yielded such precise results that there is even a two standard deviation from theory in Γ_z. This is expressed by two equivalent numbers [2]

$$\Gamma_{\text{inv}} = -2.7^{+1.7}_{-1.5}\,\text{MeV}.$$

and/or

$$N_\nu = 2.9841 \pm 0.0083.$$

Now for the Z we can apply the small ϵ formula given above since $\epsilon_Z = 2.7\%$. From this formula (see also Fig.2 below) we readily see that.

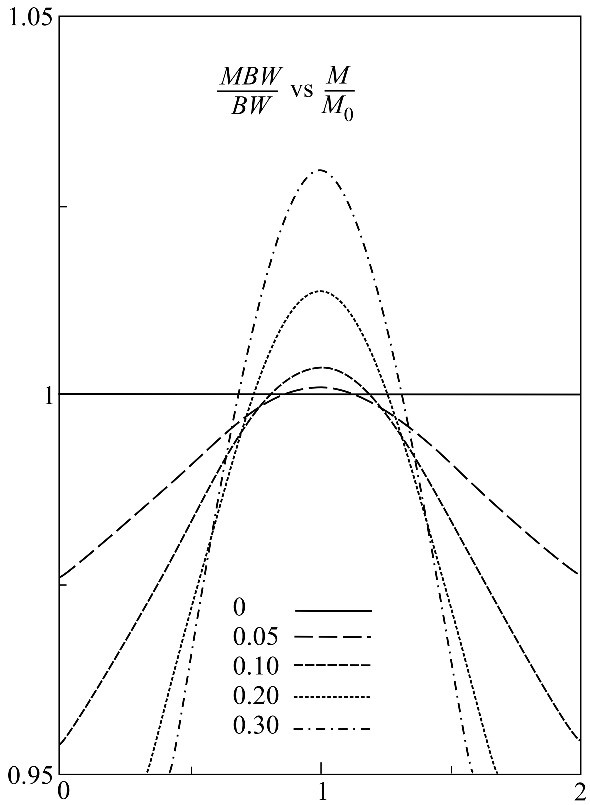

Fig.2.Numerical calculation of the ratio of the modified and standard BW mass formulas MBW/BW versus M/M_0 for different values of ϵ

1) The maximum modification to the *underlying* BW ($\tilde{\tau} = 0$) is at $M = M_0$ i.e. at the peak. This effect is an increase of the peak value by $1 + 3\epsilon^2/8$. Note, however, that this underlying BW must not be confused with the *best fit* BW need to the data in the presence of a non negligible $\tilde{\tau}$.

2) The halfwidth of the underlying BW and the MBW are almost the same for small ϵ. That of the MBW is reduced by a factor of order ϵ^3 and not ϵ^2 as might have been expected from Eq. (11) for the lifetime modification.

The smallest errors of the curve are around the peak value $M = M_0$. In a fit with a BW to data in accord with our modified curve one would be inclined to raise the peak value with consequently the same decrease in percentage of the halfwidth. Hence, we expect the *fitted* BW to yield a width lower than ours by, at most, the factor $1 - 3\epsilon^2/8$. Hence our estimate of Γ_{inv} is

$$-0.7\,\text{MeV} \leq \Gamma_{\text{inv}} < 0$$

less than 1/4 of the measured central value. It is amusing to note that using the result $\Gamma_{\text{eff}} = (1 + \epsilon^2)\Gamma$ one might have expected an effect on Γ_{inv} in good agreement with the experimental value.

The modified BW can also be calculated numerically for any ϵ and in Fig.2 we show the ratio of this with the underlying BW for various ϵ values. The width is indeed reduced. From Fig.2, we see that, for $\epsilon \leq 0.3$ and such that $1.4 > M/M_0 > 0.6$, the main modification indeed occurs at $M = M_0$ and is an increase of the order of a few % or less. This means that no significant evidence for the existence of $\tilde{\tau}$ from mass curves is possible until the individual errors of the data points are of this order or better.

Obviously in looking for evidence for our modified BW we are led to consider the largest ϵ values available. This means particles with strong interaction decays. For example the $\rho(770)$ where $\epsilon \sim 20\%$. However, the best data points for the ρ [14] are somewhat dated and are not yet precise enough to yield evidence for a $\tilde{\tau}$.

It is natural to extend the concept of timelapse, from the realm of decays to interactions in general. We have already anticipated that in theses cases $\tilde{\tau} = I/E_{CM}$. However what is $\tilde{\tau}$ to be compared with. What plays here the role of τ? The only thing available is $\sqrt{\sigma}$ the square root of the cross-section. For numerical comparisons, we recall that $\sqrt{60\,\text{mb}} \sim 10^{-23}\,\text{s}$ while $1\,\text{Gev} \sim 6.6 \cdot 10^{-25}\,\text{s}$.

Finally, we wish to discuss briefly our stimulus for this investigation, which seems at first sight far removed from the content of this paper. In *oscillation studies* some authors insist that a single time interval is involved. The argument is essentially that both the creation of say a flavor neutrino and its detection, possible as a different flavor, occur at fixed times. Now, as we explained in our introduction, such a situation, *instantaneous creation*, could at most be valid in a unique Lorentz frame which improbably coincides with the laboratory. However, the existence of an intrinsic timelapse $\tilde{\tau}$ would imply that instantaneous creation is a myth *in any frame*. In practice this means that in interference studies we must deal with multiple times in a similar way that the *slippage* of interfering wave packets obliges us to consider multiple distance intervals between creation and observation.

In conclusion, we have argued that in an arbitrary frame a wave packet will take a finite time to "grow" to, or decay from, its full normalized value. This encouraged us to postulate the existence in the preferential center of mass frame of an intrinsic timelapse $\tilde{\tau}$ in analogy with τ. Such an assumption leads to a modification of the decay formula and consequently of the BW mass formula. We have also suggested that $\tilde{\tau} = 1/E_{CM}$ on the basis of the Heisenberg uncertainty principle. In practice the modification of the decay formula is not experimentally detectable. However, it has, as an aside, reconciled for small times the decay law with basic quantum mechanical arguments (at least within the hypothesis of an exponential dependence upon $\tilde{\tau}$). The BW mass formula is a more practical tool for detecting a $\tilde{\tau}$. Comparing the fits to the data upon p decay suggest that with improved experiments (precision of the order of 10^{-3}) we could distinguish between the standard and modified BW. Note that with $\epsilon = \Delta M/M_0$ the modified version has no extra free parameters. We may simply compare the best χ^2 fits of both to the data. At the moment the most promising source for evidence of a $\tilde{\tau}$ appears to be in the Z decay. Timelapse provides a justification for the existence of a negative $\Gamma_{\rm inv}$. But we must remember that this is experimentally only a two sigma effect.

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