Drell–Yan representation for vector-meson spin-density matrix elements in semi-inclusive electroproduction

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The polarized vector-meson production in semi-inclusive electron-nucleon scattering with longitudinally-polarized electron beam has been investigated. The Drell–Yan like representation for the spin-density matrix elements of the vector meson, that takes into account the leading radiative corrections, is derived. The calculations have been performed for two widely used reference systems: the Gottfried–Jackson and the helicity ones.

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1. The investigation of the hadron final states in semi-inclusive deep-inelastic scattering (DIS) has become of topical theoretical interest, as diffractive vector-meson production at high Q^2 (Q^2 is the negative square of the virtual-photon 4-momentum) gives information on the relative contributions of hard and soft processes as well as on vacuum-exchange dynamics [1]. Measurements of exclusive vector-meson production in ep scattering at high energy

$$e^-(k_1) + p(p_1) \to e^-(k_2) + V(p_2) + X(p_x) \qquad (1)$$

has led to considerable progress towards an understanding of diffraction in terms of QCD [2].

The measurement of spin observables gives very important information on the structure of the strong interactions [3]. The polarization of the vector meson is experimentally accessible via the decay angular distributions. In recent years, the vector-meson spin-density matrix elements in reaction (1) have been measured for the elastic electroproduction of ρ and ϕ mesons in the kinematic range $Q^2 > 2.5\ GeV^2$ [4]. The discussion of recent results on the diffractive production of the vector mesons ρ^0, ϕ and ω, reported by the H1 and ZEUS collaborations at HERA, can be found in Ref. [5]. Recently, the HERMES collaboration has found a significant ($\approx 20\%$) double-spin asymmetry in vector-meson electroproduction [6]. This result is quite intriguing since it was not expected within models of the vector-meson production processes based on convenient mesonic and pomeron exchanges.

The standard data analysis requires the account for all possible systematic uncertainties. One of the important source of such uncertainties are the electromagnetic radiative effects caused by physical processes which take place in higher orders of the perturbation theory with respect to the electromagnetic interaction. In present paper we calculate the model-independent QED radiative corrections (RC) by means of the electron structure function method [7]. Our approach is based on the covariant parametrization of the vector-meson spin 4-vector in terms of the 4-momenta of the particles in process (1) [8, 9] and use of the Drell–Yan like representation [10] in electrodynamics, which allows to sum the leading-log model-independent RC in all orders.

2. We define the cross-section of the process (1), with accounting RC, in terms of the leptonic $L_{\mu\nu}$ and hadronic $H_{\mu\nu}$ tensors contraction

$$d\sigma = \frac{\alpha^2}{4V(2\pi)^3}\frac{L_{\mu\nu}H_{\mu\nu}}{q^4}\frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2}, \qquad (2)$$

where $V = 2k_1p_1$, ε_2 (E_2) is the energy of the scattered electron (detected vector meson) and q is the 4-momentum of the virtual photon. Note that only in the Born approximation $q = k_1 - k_2$. Hadronic tensor can be expressed via hadron electromagnetic current J_μ

$$H_{\mu\nu} = (2\pi)^3 \sum_X < \tau, p_2; p_x|J_\mu(0)|p_1 > \times$$
$$\times < \tau, p_2; p_x|J_\nu(0)|p_1 >^* \delta^{(4)}(q + p_1 - p_2 - p_x), \qquad (3)$$

where p_x is the total 4-momentum of the undetected hadron system, τ is the polarization index of the vector meson and summation is done with respect to all possible states in the undetected system X. The expressions for $H_{\mu\nu}(U)$ (the spin-independent part of the hadronic tensor) and $H_{\mu\nu}(V)$ (the part of the hadronic tensor depending on the vector polarization) can be found in Ref. [9]. The expression for $H_{\mu\nu}(T)$ (the part of the hadronic tensor depending on the quadrupole-polarization tensor) is given in Ref. [11]. Thus, in general case of the longitudinally-polarized electron beam and arbitrary polarization state of the produced vector meson, the reaction (1) is characterized by 41 real structure functions. In the case of the unpolarized electron beam only symmetrical (in μ, ν indices) part of the hadronic tensor, which contains 28 structure functions (4 structure functions from $H_{\mu\nu}(U)$, 8 – from $H_{\mu\nu}(V)$ and 16 – from $H_{\mu\nu}(T)$), gives the contribution to the observables.

Let us represent the transition current $\gamma^* N \to VX$, entering to Eq. (3), in the form

$$< \tau, p_2; p_x|J_\mu(0)|p_1 >= U_\rho^{(\tau)*} < p_2, p_x|J_\mu^\rho(0)|p_1 >, \quad (4)$$

where $U_\rho^{(\tau)}$ is the polarization 4-vector of the vector meson. We can define the following tensor

$$H_{\mu\nu}^{\rho\sigma} = (2\pi)^3 \sum_X < p_2, p_x|J_\mu^\rho(0)|p_1 > \times$$
$$\times < p_2, p_x|J_\nu^\sigma(0)|p_1 >^* \delta^{(4)}(q + p_1 - p_2 - p_x), \quad (5)$$

which will be used later for the calculation of the spin-density matrix elements of the vector meson produced in the reaction (1).

The hadronic tensor $H_{\mu\nu}^{\rho\sigma}$ must be constructed using 4-momenta p_1, p_2, 4-momentum of the virtual photon q, and completely antisymmetric pseudotensor $\epsilon_{\mu\nu\lambda\delta}$. It must be gauge invariant in μ, ν indices and does not contain $p_{2\rho}$, $p_{2\sigma}$ due to the condition $p_{2\rho}U_\rho^{(\tau)} = 0$. The $P-$ and $T-$invariant form of this tensor satisfying the hermiticity condition is

$$H_{\mu\nu}^{\rho\sigma} = \frac{1}{3}\{ - g_{\rho\sigma}[g_1\tilde{g}_{\mu\nu} + g_2\tilde{p}_{1\mu}\tilde{p}_{1\nu} + g_3\tilde{p}_{2\mu}\tilde{p}_{2\nu} +$$
$$+ g_4(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_5[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] - \frac{i}{m}(\rho\sigma p_2 p_1)[g_6(\tilde{p}_1 N)_{\mu\nu} +$$
$$+ ig_7[\tilde{p}_1 N]_{\mu\nu} + g_8(\tilde{p}_2 N)_{\mu\nu} + ig_9[\tilde{p}_2 N]_{\mu\nu}] -$$
$$- \frac{i}{m}(\rho\sigma p_2 q)[g_{10}(\tilde{p}_1 N)_{\mu\nu} + ig_{11}[\tilde{p}_1 N]_{\mu\nu} + g_{12}(\tilde{p}_2 N)_{\mu\nu} +$$
$$+ ig_{13}[\tilde{p}_2 N]_{\mu\nu}] - \frac{i}{m}(\rho\sigma p_2 N)[g_{14}\tilde{g}_{\mu\nu} + g_{15}\tilde{p}_{1\mu}\tilde{p}_{1\nu} +$$
$$+ g_{16}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + g_{17}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_{18}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] +$$
$$+ [q_\rho q_\sigma - \frac{1}{3}g_{\rho\sigma}(q^2 - \frac{(p_2 q)^2}{m^2})][g_{19}\tilde{g}_{\mu\nu} + g_{20}\tilde{p}_{1\mu}\tilde{p}_{1\nu} +$$
$$+ g_{21}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + g_{22}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_{23}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] +$$
$$+ [p_{1\rho}p_{1\sigma} - \frac{1}{3}g_{\rho\sigma}(M^2 - \frac{(p_1 p_2)^2}{m^2})][g_{24}\tilde{g}_{\mu\nu} + g_{25}\tilde{p}_{1\mu}\tilde{p}_{1\nu} +$$
$$+ g_{26}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + g_{27}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_{28}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] +$$
$$+ [p_{1\rho}q_\sigma + p_{1\sigma}q_\rho - \frac{2}{3}g_{\rho\sigma}(qp_1 - \frac{p_1 p_2\ p_2 q}{m^2})][g_{29}\tilde{g}_{\mu\nu} +$$
$$+ g_{30}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + g_{31}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + g_{32}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_{33}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] +$$
$$+ (qN)_{\rho\sigma}[g_{34}(\tilde{p}_1 N)_{\mu\nu} + ig_{35}[\tilde{p}_1 N]_{\mu\nu} + g_{36}(\tilde{p}_2 N)_{\mu\nu} +$$
$$+ ig_{37}[\tilde{p}_2 N]_{\mu\nu}] + (p_1 N)_{\rho\sigma}[g_{38}(\tilde{p}_1 N)_{\mu\nu} + ig_{39}[\tilde{p}_1 N]_{\mu\nu} +$$
$$+ g_{40}(\tilde{p}_2 N)_{\mu\nu} + ig_{41}[\tilde{p}_2 N]_{\mu\nu}]\}, \quad (6)$$

$$N_\mu = \epsilon_{\mu\nu\lambda\delta}p_{1\nu}p_{2\lambda}q_\delta = (\mu p_1 p_2 q), \quad (ab)_{\mu\nu} = a_\mu b_\nu + a_\nu b_\mu,$$
$$[ab]_{\mu\nu} = a_\mu b_\nu - a_\nu b_\mu, \ \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2},$$
$$\tilde{p}_{i\mu} = p_{i\mu} - \frac{(qp_i)q_\mu}{q^2}, \quad i = 1, 2,$$

where m (M) is the vector-meson (target) mass and g_i ($i = 1-41$) are the real structure functions that describe, by model independent way, the process of the vector-meson leptoproduction by longitudinally polarized electron beam. These functions depend, in general, on four invariants which can be taken as q^2, (qp_1), (qp_2) and $(p_1 p_2)$.

The hadronic tensors $H_{\mu\nu}$ and $H_{\mu\nu}^{\rho\sigma}$ are connected by the relation $H_{\mu\nu} = \rho_{\sigma\rho}H_{\mu\nu}^{\rho\sigma}$, where $\rho_{\sigma\rho}$ is the covariant spin-density matrix for the spin-one particle which in our case is

$$\rho_{\mu\nu} = -(g_{\mu\nu} - \frac{p_{2\mu}p_{2\nu}}{m^2}) - \frac{3i}{2m}\epsilon_{\mu\nu\rho\sigma}s_\rho p_{2\sigma} + 3Q_{\mu\nu},$$
$$Q_{\mu\nu} = Q_{\nu\mu}, \ Q_{\mu\mu} = 0 \ \ p_{2\mu}Q_{\mu\nu} = 0, \quad (7)$$

where s_ρ is the 4-vector of the vector polarization of the vector meson ($s^2 = -1$, $sp_2 = 0$) and $Q_{\mu\nu}$ is its quadrupole-polarization tensor. The parametrization of the hadronic tensor $H_{\mu\nu}^{\rho\sigma}$ is chosen so that to obtain previously used hadronic tensor $H_{\mu\nu}$ [9, 11] when convoluting hadronic tensor $H_{\mu\nu}^{\rho\sigma}$ with tensor $\rho_{\sigma\rho}$. At this point we follow very close to Ref. [12] where the matrix elements of vector meson in reaction $e^+e^- \to VX$ have been studied.

The model-independent RC exhibits themselves by means of the corrections to the leptonic tensor. The expression for this tensor with account of RC can be found in Ref. [8].

The spin-density matrix of the vector meson produced in the reaction (1) can be defined as

$$\sigma^U \rho_{mn} = \frac{\alpha^2}{4V(2\pi)^3}\frac{L_{\mu\nu}H_{\mu\nu}^{\rho\sigma}U_\rho^{(m)*}U_\sigma^{(n)}}{q^4}\frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2}, \quad (8)$$

where σ^U is the differential cross-section of the reaction (1) for the case of all (except electron beam) unpolarized particles, the 4-vectors $U_\rho^{(m)}$, $m = +1, -1, 0$, characterize the production of the vector meson with definite helicity m (in its rest system). In this system the vector meson has three polarization states and its production can be completely characterized by the spin-density matrix

ρ_{mn} which may be obtained from experiment. It is convenient to discuss the angular distribution of the vector-meson decay in two widely used reference systems which differ in the choice of the spin-quantization axis (z-axis): the Gottfried–Jackson system, where the z-axis is the direction of the incident photon in the vector-meson rest system and the helicity system, where the z-axis is equal to the direction of flight of the produced vector meson in the overall $\gamma^* N$ center-of-mass system [13]. Depending upon the production mechanism, the spin of the vector meson may be aligned along the z-axis in one of these systems [14]. The system which gives the simplest description of the vector meson is then: (1) the Gottfried–Jackson system for $t-$channel helicity conservation; (2) the helicity system for s-channel helicity conservation.

Combining formulae for the cross-section (2) and spin-density matrix elements (8), and using the definitions of the radiatively corrected lepton [9] and hadron tensors, we can write the following representation for the unpolarized cross-section of the process (1) and for the spin-density matrix elements

$$\sigma^U = \varepsilon_2 E_2 \frac{d\sigma(k_1,k_2)}{d^3k_2 d^3p_2} = \\ = \iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\hat{k}_1,\hat{k}_2)}{d^3\hat{k}_2 d^3p_2}, \quad (9)$$

$$\sigma^U \rho_{mn}(k_1,k_2) = \iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\hat{\sigma}^U_B \rho^B_{mn}(\hat{k}_1,\hat{k}_2),$$

where $\hat{k}_1 = x_1 k_1,\ \hat{k}_2 = k_2/x_2$, D is the electron structure function, index B means that corresponding quantity is calculated in the Born approximation (for the details see Ref. [9]). We omit in argument of the cross-section the hadron 4-momenta. The cross-section $\hat{\sigma}^U_B = \sigma^U_B(\hat{k}_1,\hat{k}_2)$ and $\sigma^U_B(k_1,k_2)$ can be derived substituding $\delta(x-1)$ instead of both D-functions in the former equation in (9).

In theoretical calculations it is useful to parameterize the polarization state of the particle (specifically, the 4-vectors $U^{(m)}_\rho$) in terms of the particle 4-momenta [8, 9]. In our case we have four 4-momenta to express any 4-vector $U^{(m)}_\rho$ in a such way that

$$U^{(m)}_\rho = U^{(m)}_\rho(k_1,k_2,p_1,p_2). \quad (10)$$

Let us imagine for a moment that chosen parametrization on the right side of Eq. (10) is stabilized relative substitution

$$k_1 \to \hat{k}_1, \quad k_2 \to \hat{k}_2,$$
$$U^{(m)}_\rho(k_1,k_2,p_1,p_2) = U^{(m)}_\rho(\hat{k}_1,\hat{k}_2,p_1,p_2).$$

Further we will label such stabilized parametrization by the indices $\alpha = l, t$, and n. In this case the expression for the spin-density matrix elements (with account RC) is just described by the Eq. (9).

If the 4-vector $U^{(m)}_\rho$ is unstable under above substitution it can be expressed always in terms of stabilized one by means of some linear combination

$$U^{(m)}_\rho(k_1,k_2,p_1,p_2) = \\ = A^{(m)}_\alpha(k_1,k_2,p_1,p_2)U^{(\alpha)}_\rho(\hat{k}_1,\hat{k}_2,p_1,p_2). \quad (11)$$

Using the last formula we can write the master representation for the spin-density matrix elements of the produced vector meson in the process (1) in the following form

$$\sigma^U \rho_{mn}(k_1,k_2,p_1,p_2) = A^{(m)*}_\alpha A^{(n)}_\beta \times \\ \times \iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\hat{\sigma}^U_B \rho^B_{\alpha\beta}(\hat{k}_1,\hat{k}_2,p_1,p_2), \quad (12)$$

where we bear in mind the summation over indices $\alpha, \beta = l, t, n$.

This representation is the electrodynamical analogue of the well known in QCD Drell–Yan formula [10], that was applied earlier to calculate QED RC to various processes [9]. It is obvious that in the framework of the leading accuracy one needs to find the adequate parametrizations of the stabilized 4-vectors $U^{(\alpha)}_\rho$, to calculate the coefficients $A^{(m)}_\alpha$, and to derive in the Born approximation the spin-density matrix elements for a given parametrization $U^{(\alpha)}_\rho$.

3. The following set of the invariant variables completely describe the process (1)

$$z = \frac{2p_1p_2}{V}, \quad z_{1,2} = \frac{2k_{1,2}p_2}{V}, \quad y = \frac{2p_1q}{V}, \\ x = \frac{-q^2}{2p_1q}, \quad q = k_1 - k_2. \quad (13)$$

Therefore, to calculate RC we have to find the set of stabilized axes and write them in covariant form in terms of 4-momenta of the particles participating in the reaction. Further we will use the following stabilized set of the 4-vectors [9]

$$S^{(l)}_\mu = \frac{zp_{2\mu} - 2\tau_2 p_{1\mu}}{md_1},$$

$$S^{(t)}_\mu = \frac{d_1^2 k_{1\mu} + (2z_1\tau_1 - z)p_{2\mu} + (2\tau_2 - zz_1)p_{1\mu}}{d_2},$$

$$S^{(n)}_\mu = \frac{2\varepsilon_{\mu\lambda\rho\sigma}k_{1\lambda}p_{1\rho}p_{2\sigma}}{d_3},$$

$$d_1^2 = z^2 - 4\tau_1\tau_2, \quad d_2^2 = V\psi d_1^2, \quad \psi = zz_1 - \tau_2 - \tau_1 z_1^2,$$

$$d_3^2 = \psi V^3, \quad \tau_1 = \frac{M^2}{V}, \quad \tau_2 = \frac{m^2}{V}. \quad (14)$$

One can verify that the set $S_\mu^{(l,t,n)}$ remains stabilized under the scale transformation and

$$S_\mu^{(\alpha)}S_\mu^{(\beta)} = -\delta_{\alpha\beta},\ S_\mu^{(\alpha)}p_{2\mu} = 0,\ \alpha,\beta = l,t,n.$$

Any vector-meson polarization 4-vector $U_\mu^{(m)}$ corresponding to a certain helicity m can be expanded over the stabilized set of the 4-vectors $S_\mu^{(\alpha)}(\alpha = l,t,n)$. Let us define such expansion

$$U_\rho^{(m)} = A_l^{(m)}S_\rho^{(l)} + A_t^{(m)}S_\rho^{(t)} + A_n^{(m)}S_\rho^{(n)}. \qquad (15)$$

Then the spin-density matrix elements ρ_{mn} (in the helicity representation) can be expressed in terms of the spin-density matrix elements $\rho_{\alpha\beta}$ (in the representation of the stabilized set)

$$\rho_{mn} = \sum_{\alpha,\beta} A_\alpha^{(m)*}A_\beta^{(n)}\rho_{\alpha\beta}, \quad \alpha,\ \beta = l,t,n, \qquad (16)$$

where

$$\sigma^U\rho_{\alpha\beta} = \frac{\alpha^2}{4V(2\pi)^3}\frac{L_{\rho\sigma}H_{\rho\sigma}^{\mu\nu}S_\mu^{(\alpha)}S_\nu^{(\beta)}}{q^4}\frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2}.$$

The spin-density matrix elements in the Born approximation can be represented as

$$\rho_{ll} = \frac{1}{3H_1}\{H_1 + \frac{V}{12\tau_2}[H_3(3\frac{\psi_1^2}{d_1^2} - 4xy\tau_2 - (z_2-z_1)^2) + $$
$$+2d_1^2H_4 + 4\psi_1H_5]\},$$
$$\rho_{lt} = \frac{d_1V^2}{12d_2H_1}\{\frac{\psi_1}{md_1^2}(\eta_3H_3 - \frac{\eta}{4}d_1^2V^2G_3) + $$
$$+\frac{1}{m}(\eta_3H_5 - \frac{\eta}{4}d_1^2V^2G_4) + i(\eta G_2 + \eta_3H_2)\},$$
$$\rho_{ln} = \frac{V^3}{12md_1d_3H_1}\{\psi_1(\eta H_3 + \frac{\eta_3}{4}V^2G_3) + $$
$$d_1^2(\eta H_5 + \frac{\eta_3}{4}V^2G_4) + im(\eta d_1^2H_2 - \eta_3G_2)\}, \qquad (17)$$
$$\rho_{tt} = \frac{1}{3H_1}\{H_1 - \frac{\eta\eta_3}{8\psi}V^3G_3 + $$
$$+\frac{V}{12\tau_2}[H_3(3\frac{\tau_2\eta_3^2}{\psi d_1^2} - 4xy\tau_2 - (z_1-z_2)^2) - d_1^2H_4 - 2\psi_1H_5]\},$$
$$\rho_{nn} = \frac{1}{3H_1}\{H_1 + \frac{\eta\eta_3}{8\psi}V^3G_3 + $$
$$+\frac{V}{12\tau_2}[H_3(3\frac{\tau_2\eta^2}{\psi} - 4xy\tau_2 - (z_1-z_2)^2) - d_1^2H_4 - 2\psi_1H_5]\},$$
$$\rho_{tn} = \frac{V^3}{12d_2d_3H_1}\{\eta\eta_3H_3 + \frac{V^2}{4}(\eta_3^2 - d_1^2\eta^2)G_3 + $$
$$+i\frac{\psi}{m}V(d_1^2G_1 + \psi_1G_2)\},\ \rho_{nt} = \rho_{tn}^*,\ \rho_{nl} = \rho_{ln}^*,\ \rho_{tl} = \rho_{lt}^*,$$

where the following notations are used

$$H_1 = -\frac{2xy}{V}g_1 + (1-y-xy\tau_1)g_2 + (z_1z_2 - xy\tau_2)g_3 + $$
$$+(z_2 + z_1(1-y) - xyz)g_4 - \lambda\eta g_5,$$
$$H_2 = H_1(g_i \to g_{i+13}),\ H_3 = H_1(g_i \to g_{i+18}),$$
$$H_4 = H_1(g_i \to g_{i+23}),\ H_5 = H_1(g_i \to g_{i+28}),$$
$$G_1 = -\eta[(2-y)g_6 + (z_1+z_2)g_8] - \lambda(\eta_1g_7 + \eta_2g_9),$$
$$G_2 = G_1(g_i \to g_{i+4}),\ G_3 = G_1(g_i \to g_{i+28}),$$
$$G_4 = G_1(g_i \to g_{i+32}),$$
$$\eta = sign[(p_1p_2k_1k_2)]\sqrt{\frac{16}{V^4}(p_1p_2k_1k_2)^2},$$
$$(p_1p_2k_1k_2) = \epsilon_{\mu\nu\rho\sigma}p_{1\mu}p_{2\nu}k_{1\rho}k_{2\sigma},$$
$$\frac{16(p_1p_2k_1k_2)^2}{V^4} = x^2y^2(4\tau_1\tau_2 - z^2) + 2xy[z(z_2 + z_1(1-y)) - $$
$$-2z_1z_2\tau_1 - 2(1-y)\tau_2] - (z_2 - z_1(1-y))^2,$$
$$\eta_1 = y[z_2 - z_1(1-y) - xz(2-y) + 2x(z_1+z_2)\tau_1],$$
$$\eta_2 = (z_1-z_2)(z_2 - z_1(1-y)) + xyz(z_1+z_2) - 2xy(2-y)\tau_2,$$
$$\psi_1 = z(z_1-z_2) - 2y\tau_2,$$
$$\eta_3 = -yzz_1 + 2z_1(z_1-z_2)\tau_1 - \psi_1 - xy(z^2 - 4\tau_1\tau_2),$$

where the quantity λ is the degree of longitudinal polarization of the electron beam. One can see that the condition $\rho_{ll} + \rho_{tt} + \rho_{nn} = 1$ is satisfied.

Let us consider two reference systems mentioned above.

The helicity system. The z-axis is opposite the direction of the produced system X in the vector-meson rest system (i.e., equal to the direction of flight of the vector meson in the overall γ^*N c.m.s.). Then the vector-meson wave functions $U_\mu^{(m)}$ with definite helicity m can be represented as

$$U_\mu^{(0)} = \frac{1}{md}[(z + z_1 - z_2)p_{2\mu} - 2\tau_2(p_1+q)_\mu],$$
$$d^2 = (z + z_1 - z_2)^2 - 4\tau_2[y(1-x) + \tau_1],$$
$$U_\mu^{(\pm1)} = \mp\frac{1}{\sqrt{2}}(U_\mu^{(x)} \pm iU_\mu^{(y)}),\ U_\mu^{(y)} = \frac{\varepsilon_{\mu\lambda\rho\sigma}q_\lambda p_{2\rho}p_{1\sigma}}{D},$$
$$U_\mu^{(x)} = \frac{1}{D_1}(a_1p_{1\mu} + a_2p_{2\mu} + a_3q_\mu), \qquad (18)$$
$$D^2 = \frac{V^3}{4}[xy(z^2 - 4\tau_1\tau_2) + yz(z_1-z_2) - y^2\tau_2 - \tau_1(z_1-z_2)^2],$$
$$a_1 = (z_1-z_2)(z + z_1 - z_2) + 2y\tau_2(2x-1),$$

$$a_2 = yz(1-2x) - (y+2\tau_1)(z_1 - z_2),$$
$$a_3 = -z(z+z_1-z_2) + 2\tau_2(y+2\tau_1),$$
$$D_1^2 = -V\Big\{\tau_1 a_1^2 + \tau_2 a_2^2 + y a_3(a_1 - x a_3) + $$
$$+ a_2[(z_1 - z_2)a_3 + z a_1]\Big\}.$$

Then the coefficients in the expansion of the vector-meson wave function with definite helicity over the stabilized set of the 4-vectors (see Eq. (15)) can be represented as follows

$$A_l^{(0)} = -\frac{a_3}{dd_1},\ A_t^{(0)} = \frac{V\tau_2\eta_3}{mdd_2},\ A_n^{(0)} = \frac{V^2\tau_2\eta}{mdd_3}, \qquad (19)$$
$$A_l^{(\pm1)} = \pm 4\sqrt{2}\frac{mD^2}{d_1 D_1 V^3},$$
$$A_t^{(\pm1)} = \pm\frac{V}{4\sqrt{2}d_2}(2\eta_3\frac{a_3}{D_1} \pm i\eta V\frac{d_1^2}{D}),$$
$$A_n^{(\pm1)} = \pm\frac{V^2}{4\sqrt{2}d_3}(2\eta\frac{a_3}{D_1} \mp iV\frac{\eta_3}{D}).$$

The wave functions $U_\mu^{(m)}$ have the following form in the vector-meson rest system

$$U_\mu^{(0)} = (0,\mathbf{l}),\ \ U_\mu^{(x)} = (0,\mathbf{t}),\ \ U_\mu^{(y)} = (0,\mathbf{n})\ , \qquad (20)$$
$$\mathbf{l} = -\frac{\mathbf{p_x}}{|\mathbf{p_x}|},\ \mathbf{n} = \frac{\mathbf{p_x}\times\mathbf{q}}{|\mathbf{p_x}\times\mathbf{q}|},\ \mathbf{t} = \frac{[\mathbf{q}\cdot\mathbf{p_x}\mathbf{p_1} - \mathbf{p_1}\cdot\mathbf{p_x}\mathbf{q}]}{|\mathbf{q}\cdot\mathbf{p_x}\mathbf{p_1} - \mathbf{p_1}\cdot\mathbf{p_x}\mathbf{q}|},$$

where $\mathbf{q}$, $\mathbf{p_1}$ and $\mathbf{p_x}$ are the momenta of the virtual photon, the target and undetected system X, respectively. Choosing the coordinate axes as follows: $z\ ||\ \mathbf{l}$, $y\ ||\ \mathbf{n}$ and $x\ ||\ \mathbf{t}$ we get for the wave functions following expressions

$$U_\mu^{(0)} = (0,0,0,1),\ \ U_\mu^{(\pm1)} = \mp\frac{1}{\sqrt{2}}(0,1,\pm i,0). \qquad (21)$$

The Gottfried–Jackson system. In the Gottfried–Jackson system the z axis coincides with the direction of flight of the incoming photon in the vector-meson rest system. Then the vector-meson wave functions $U_\mu^{(m)}$ with definite helicity m can be represented as

$$U_\mu^{(0)} = \frac{1}{r}[2\tau_2 q_\mu - (z_1-z_2)p_{2\mu}],$$
$$r^2 = V\tau_2[4xy\tau_2 + (z_1-z_2)^2],$$
$$U_\mu^{(\pm1)} = \mp\frac{1}{\sqrt{2}}(U_\mu^{(x)} \pm iU_\mu^{(y)}),\ U_\mu^{(y)} = \frac{\varepsilon_{\mu\lambda\rho\sigma}q_\lambda p_{2\rho}p_{1\sigma}}{D},$$
$$U_\mu^{(x)} = \frac{1}{R_1}(b_1 p_{1\mu} + b_2 p_{2\mu} + b_3 q_\mu), \qquad (22)$$
$$b_1 = -(z_1-z_2)^2 - 4xy\tau_2,$$
$$b_2 = y(z_1 - z_2 + 2xz),\ \ b_3 = \psi_1,$$
$$R_1^2 =$$
$$= -V\Big\{\tau_1 b_1^2 + \tau_2 b_2^2 + b_1(yb_3 + zb_2) + b_3[(z_1-z_2)b_2 - xyb_3]\Big\}.$$

Then the coefficients in the expansion of the vector-meson wave function with definite helicity over the stabilized set of the 4-vectors (see Eq. (15)) can be represented as follows

$$A_l^{(0)} = -\frac{V\tau_2\psi_1}{mrd_1},\ \ A_t^{(0)} = -\frac{V\tau_2\eta_3}{rd_2},\ \ A_n^{(0)} = -\frac{V^2\tau_2\eta}{rd_3},$$
$$A_l^{(\pm1)} = \mp\frac{4\sqrt{2\tau_2}D^2}{d_1 R_1 V^{5/2}}, \qquad (23)$$
$$A_t^{(\pm1)} = \pm\frac{V}{2\sqrt{2}d_2}\Big\{\frac{\psi_1\eta_3}{R_1} \pm i\frac{V\eta d_1^2}{2D}\Big\},$$
$$A_n^{(\pm1)} = \pm\frac{V^2}{2\sqrt{2}d_3}(\eta\frac{b_3}{R_1} \mp i\frac{V\eta_3}{2D}).$$

In the vector-meson rest system the wave functions $U_\mu^{(m)}$ (22) have the same form as given by Eq. (20). But for the Gottfried–Jackson system the vectors $\mathbf{l},\mathbf{n},\mathbf{t}$ are

$$\mathbf{l} = \frac{\mathbf{q}}{|\mathbf{q}|},\ \ \mathbf{n} = \frac{\mathbf{p_x}\times\mathbf{q}}{|\mathbf{p_x}\times\mathbf{q}|},\ \ \mathbf{t} = \frac{[\mathbf{l}\mathbf{p_1}\cdot\mathbf{l} - \mathbf{p_1}]}{|\mathbf{l}\mathbf{p_1}\cdot\mathbf{l} - \mathbf{p_1}|}.$$

Choosing the coordinate axes as follows: $z\ ||\ \mathbf{l}$, $y\ ||\ \mathbf{n}$ and $x\ ||\ \mathbf{t}$, we get for the wave functions in the vector-meson rest system the form given by Eq. (21).

4. Let us expand the spin-density matrix elements into the terms according to the polarization state of the virtual photon. This expansion is the standard procedure when analysing the cross-section and polarization observables for the inelastic lepton scattering. The form of this expansion for the case of the longitudinally polarized electron beam is

$$\sigma_B^U\rho_{mn}^B = \rho_{mn}^U + \varepsilon\cos(2\phi)\rho_{mn}^T + \varepsilon\sin(2\phi)\rho_{mn}^{TP} +$$
$$+\varepsilon\rho_{mn}^L + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi\rho_{mn}^I +$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi\rho_{mn}^{IP} + \lambda\sqrt{1-\varepsilon^2}\rho_{mn}^{TP'} +$$
$$+\lambda\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi\rho_{mn}^{IP'} + \lambda\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi\rho_{mn}^{I'}, \qquad (24)$$

where $\varepsilon^{-1} = 1 - 2(\mathbf{q_L^2}/q^2)\tan^2(\theta_e/2)$, θ_e and $\mathbf{q}_L$ are the electron scattering angle and virtual-photon 3-momentum in laboratory system. Quantity ε represents the degree of the virtual-photon linear polarization. The angle ϕ is the angle between the electron scattering plane and plane $(\mathbf{q},\mathbf{p_2})$ (vector-meson production plane). This expansion is the consequence of the one-photon-exchange approximation, and the validity of the conservation of the electromagnetic current describing the $\gamma^* N \to VX$ transition and P-invariance of the hadron electromagnetic interaction as well. The meaning of the indices is the following: U (L) is determined by the transverse (longitudinal) current component, T is caused by the transverse current component and determine the asymmetry due to the linear polarization of the virtual photon, I is determined by the interference of the transverse and longitudinal current components, P (') means that this term is due to the vector-meson (electron beam) polarization.

For the stabilized set of the 4-vectors $S_\mu^{(\alpha)}$, where $\alpha = l, t, n$, we have in the Born approximation

$$\rho_{ll}^U = \frac{N_k}{3}\Big\{R_3 + \frac{V}{12\tau_2}[(3\frac{\psi_1^2}{d_1^2} - \psi_2)R_{21} + 2d_1^2 R_{26} + 4\psi_1 R_{31}]\Big\},$$

$$\rho_{tt}^U = L_1 + \frac{\eta_3^2 V N_k}{12\psi d_1^2} R_{21},$$

$$\rho_{nn}^U = L_1 + \frac{\eta^2 V N_k}{12\psi} R_{21}, \quad \rho_{lt}^U = \rho_2 N_k L_2,$$

$$\rho_{ln}^U = \rho_3 N_k L_2, \quad \rho_{nt}^U = \rho_1 N_k R_{21},$$

$$\rho_{\alpha\beta}^T = e_1^2 \rho_{\alpha\beta}^U (e_1 = 1, g_1 = g_{19} = g_{24} = g_{29} = 0),$$

$$\rho_{\alpha\beta}^{I'} = 2\frac{e_4\sqrt{-q^2}}{e_1 q_0}\rho_{\alpha\beta}^T (g_i \to g_{i+2}),$$

$$\rho_{ll}^{IP} = 0, \quad \rho_{tt}^{IP} = -\gamma_1 \rho_1 V d_1 P_{34}, \quad \rho_{nn}^{IP} = -\rho_{tt}^{IP},$$

$$\rho_{lt}^{IP} = -\gamma_1 \frac{\eta V d_1}{6 d_2} L_3, \quad \rho_{ln}^{IP} = \gamma_1 \frac{\eta_3 V^2}{6 d_1 d_3} L_3,$$

$$\rho_{nt}^{IP} = \gamma_1 \frac{V}{6 d_1}[-\frac{i}{m}(d_1^2 P_6 + \psi_1 P_{10}) + \frac{V}{4\psi}(\eta_3^2 - d_1^2\eta^2) P_{34}],$$

$$\rho_{\alpha\beta}^{IP'} = \rho_{\alpha\beta}^{IP}(g_i \to g_{i+1}),$$

$$\rho_{\alpha\beta}^{TP} = \rho_{\alpha\beta}^{IP}(\gamma_1 \to \gamma_2, g_i \to g_{i+2}, e_2 = 0, e_4 = 1),$$

$$\rho_{\alpha\beta}^{TP'} = \rho_{\alpha\beta}^{TP}(g_i \to g_{i+1}),$$

$$\rho_{ll}^I = \frac{e_1\gamma_1}{3e_3}\Big\{V_3 + \frac{V}{12\tau_2}[(3\frac{\psi_1^2}{d_1^2} - \psi_2)V_{21} + 2d_1^2 V_{26} + 4\psi_1 V_{31}]\Big\},$$

$$\rho_{tt}^I = L_4 + \gamma_1 \frac{\eta_3^2 V e_1}{12\psi e_3 d_1^2} V_{21}, \quad \rho_{nn}^I = L_4 + \gamma_1 \frac{\eta^2 V e_1}{12\psi e_3} V_{21},$$

$$\rho_{lt}^I = \rho_2 \gamma_1 \frac{e_1}{e_3} L_5, \quad \rho_{ln}^I = \gamma_1 \rho_3 \frac{e_1}{e_3} L_5, \quad \rho_{nt}^I = \gamma_1 \rho_1 \frac{e_1}{e_3} V_{21},$$

$$\rho_{ll}^L = \frac{\gamma_3}{3}\Big\{Q_1 + \frac{V}{12\tau_2}[(3\frac{\psi_1^2}{d_1^2} - \psi_2)Q_{19} + 2d_1^2 Q_{24} + 4\psi_1 Q_{29}]\Big\},$$

$$\rho_{tt}^L = L_6 + \gamma_3 \frac{\eta_3^2 V}{12\psi d_1^2} Q_{19}, \quad \rho_{nn}^L = L_6 + \gamma_3 \frac{\eta^2 V}{12\psi} Q_{19},$$

$$\rho_{lt}^L = \rho_2\gamma_3(imQ_{14} + \frac{\psi_1}{d_1^2}Q_{19} + Q_{29}), \tag{25}$$

$$\rho_{ln}^L = \rho_3\gamma_3(imQ_{14} + \frac{\psi_1}{d_1^2}Q_{19} + Q_{29}), \quad \rho_{nt}^L = \gamma_3\rho_1 Q_{19},$$

$$L_1 = \frac{N_k}{3}[R_3 - \frac{V}{12\tau_2}(\psi_2 R_{21} + d_1^2 R_{26} + 2\psi_1 R_{31})],$$

$$L_2 = imR_{16} + \frac{\psi_1}{d_1^2}R_{21} + R_{31},$$

$$L_3 = -iP_{10} + \frac{V^2}{4m}(\psi_1 P_{34} + d_1^2 P_{38}),$$

$$L_4 = \frac{e_1\gamma_1}{3e_3}[V_3 - \frac{V}{12\tau_2}(\psi_2 V_{21} + d_1^2 V_{26} + 2\psi_1 V_{31})],$$

$$L_5 = imV_{16} + \frac{\psi_1}{d_1^2}V_{21} + V_{31},$$

$$L_6 = \frac{\gamma_3}{3}[Q_1 - \frac{V}{12\tau_2}(\psi_2 Q_{19} + d_1^2 Q_{24} + 2\psi_1 Q_{29})],$$

$$R_i = e_1^2 g_i - 2g_{i-2}, \quad P_i = e_4 g_i + e_2 g_{i+2},$$

$$V_i = e_2 g_i + e_4 g_{i+1},$$

$$Q_i = -\frac{q_0^2}{q^2}g_i + e_4^2 g_{i+1} + 2e_2 e_4 g_{i+3} + e_2^2 g_{i+2},$$

$$\rho_1 = \frac{\eta\eta_3 V}{12 d_1\psi}, \quad \rho_2 = \frac{\eta_3 V^2 d_1}{12 m d_2}, \quad \rho_3 = \frac{\eta V^3 d_1}{12 m d_3},$$

$$\psi_2 = 4xy\tau_2 + (z_1 - z_2)^2, \quad \gamma_1 = -2e_3\frac{\sqrt{-q^2}}{q_0}N_k,$$

$$\gamma_2 = 2e_1 e_3 N_k, \quad \gamma_3 = -2\frac{q^2}{q_0^2}N_k,$$

$$N_k = \frac{\alpha^2}{4}\frac{1}{(2\pi)^3 V}\frac{1}{Q^2(1-\varepsilon)}\frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2},$$

$$e_1 = |\mathbf{p_2}|\sin\theta, \quad e_2 = |\mathbf{p_2}|\cos\theta + \frac{z_1 - z_2}{2xy}|\mathbf{q}|,$$

$$e_3 = -W|\mathbf{p_2}||\mathbf{q}|\sin\theta, \; e_4 = -\frac{W^2 - M^2 + q^2}{2q^2}|\mathbf{q}|,$$

where W is the invariant mass of the VX system.

To use the Drell–Yan representation we have to express all variables and quantities in both hands of Eqs. (24), (25) through invariant variables. The corresponding formulae read

$$W = \sqrt{V(\tau_1 + y - xy)}, \quad |\mathbf{p_2}| = \frac{d}{2}\sqrt{\frac{V}{(\tau_1 + y - xy)}},$$

$$\mathbf{q}^2 = \frac{Vy(y + 4x\tau_1)}{4(\tau_1 + y - xy)}, \quad q^2 = -xyV,$$

$$q_0 = \frac{1 - 2x}{2}\sqrt{\frac{Vy^2}{\tau_1 + y(1-x)}}, \; \varepsilon = \frac{2(1 - y - xy\tau_1)}{1 + (1-y)^2 + 2xy\tau_1},$$

$$\cos\theta = \frac{yz(1-2x) - (z_1 - z_2)(y + 2\tau_1)}{d\sqrt{y(y + 4x\tau_1)}},$$

and the azimuthal angle ϕ can be obtained from the equation

$$\sin\phi = -\frac{\eta}{d\sin\theta}\sqrt{\frac{\tau_1 + y(1-x)}{xy(1 - y - xy\tau_1)}}.$$

Note in conclusion that we obtain a rather compact formulae for the account of the leading-order RC for the process of the semi-inclusive vector-meson electroduction. The obtained results do not depend on the particular choice of the model for the investigated process. All the dynamics of the reaction under consideration is contained in the structure functions g_i.

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