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On depolarization of ultracold neutrons in traps

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Mechanisms of depolarization of ultracold neutrons in traps are considered when they reflect from the trap's wall. One is due to neutron spin-flip elastic or quasielastic incoherent scattering on protons of surface hydrogen contaminations. According to the second one significant depolarization may take place because of sudden change of neutron trajectory at reflection from the wall when neutron move even at large adiabaticity parameters in nonuniform magnetic field.

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1. Investigation of angular correlation coefficients in the neutron beta decay is important for determining the fundamental coupling constants of weak interactions [1]. It is important to improve the accuracy of these measurements with the aim to search for new physics beyond the Standard Model with increased sensitivity [2]. One of serious difficulties in increasing precision of these measurements is reliable determination with high accuracy the polarization of decaying neutrons.

Significant progress was proposed [3] to be reached due to application of polarized ultracold neutrons (UCN) [4] stored in closed volumes or flowing through decay region. Neutrons transmitted through high enough magnetic potential barrier must have perfect 100% polarization in the direction opposite to magnetic field. First experimental data on neutron depolarization in traps were published in [5]. It was found that surprisingly large depolarization ($\sim 10^{-5}$ per UCN collision with the wall) was observed for all tested surface materials, and it was shown in special experiment that this value of depolarization practically did not depend on the UCN loss coefficient, the latter being changed by additional depositing water layer on the cooled surface. It was stated also that depolarization due to large gradient of magnetic field was impossible, which was confirmed by numerical evaluations.

The objection of this paper is to consider possible mechanisms of depolarization UCN in traps not having in mind exact quantitative interpretation of existing experimental data.

2. The obvious and trivial reason of UCN depolarization in traps is neutron spin-flip scattering on hydrogen. The most perspective materials for UCN chambers: Be, C, glass, fluoropolymers do not contain nuclei with significant spin-flip cross section. UCN upscattering on hydrogen contaminations of the trap surface is the usual and the main reason of abnormally large losses UCN from the traps [4]. Extrapolated to thermal point UCN upscattering cross section on bound proton of room temperature sample is $\sigma_{\mathrm{ups}} \simeq (4-7)b \cdot 2200/v\,(\mathrm{m/s})$ [6], depending on chemical bond of hydrogen atom (v – is UCN velocity). Elastic (or quasielastic in the case of diffusing protons or/and presence of hyperfine splitting of hydrogen atom) spin-flip scattering cross section $\sigma_{el} \simeq \frac{2}{3} \cdot 80b$. Therefore the ratio of spin-flip to UCN-hydrogen upscattering loss probabilities is $\sim (1.5-3) \cdot 10^{-2}$. In the trap with hydrogen upscattering UCN loss probability $\sim 4 \cdot 10^{-5}$ (rather good figure) spin-flip probability is $\sim 10^{-6}$, which is not quite satisfactory for neutron decay correlation coefficient.

Energy splitting of magnetic sub-levels in paramagnetic atoms is usually much larger than UCN energy. Therefore after spin-flip paramagnetic scattering neutrons must leave storage chamber. Anyway cross section of paramagnetic scattering is of the order 1 b, probability of spin-flip at reflection from the wall being $\sim 10^{-7}$ even in the case when all atoms of the wall are paramagnetic. It is hardly possible for typical wall's materials.

3. Effect of magnetic field inhomogeneities on spin relaxation of neutral particles with magnetic moment was considered in [7, 8]. It was assumed in these works that the applied magnetic field is a superposition of a weak spatially varying field upon a much stronger homogeneous field, and the problem was solved with application of perturbation method. According to [3] it is not the case when UCN are polarized by transmission through magnetic potential barrier and then are stored in experimental chamber. Fluctuations of magnetic field for slow neutron moving in the chamber are not random and small in this case, but adiabaticity parameter –

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the ratio of Larmor frequency to frequency of rotation of magnetic field in the neutron reference frame – may be large enough, so that spin relaxation probability for freely moving neutron must be exponentially small. It will be shown here that nevertheless depolarization may be significant because time derivative of magnetic field is discontinuous as long as the wall collision takes place in a time small compared to period of Larmor precession.

Let the neutral particle with magnetic moment (in particular ultracold neutron) moves in nonuniform magnetic field inside the trap, reflecting from the trap walls. We place the neutron reference frame in the reflection point so that x-axis is along magnetic field line.

With constant $H_x \neq 0$, $H_y = 0$, $H_z = t\dot{H}$, $\dot{H} > 0$ and following the paper by V. V. Vladimirsky [9] we have equation for spin wave function:

$$i\dot{\psi} + \omega\sigma_x\psi + at\sigma_z\psi = 0 \tag{1}$$

where $\omega = \mu H_x/\hbar$, $a = \mu\dot{H}/\hbar$, μ is magnetic moment of neutron.

Equations for each of the spinor components look as follows:

$$\ddot{\phi} + (\omega^2 - ia + a^2t^2) = 0, \quad \ddot{\chi} + (\omega^2 + ia + a^2t^2)\chi = 0. \tag{2}$$

Substituting $\phi = e^{-z/2}u$, $\chi = e^{-z/2}v$, $z = -iat^2$, and introducing $\alpha = \omega^2/4ia$ transforms (2) to confluent hypergeometric equations

$$\begin{gathered} zu'' + (\frac{1}{2} - z)u' - \alpha u = 0, \\ zv'' + (\frac{1}{2} - z)v' - (\alpha + \frac{1}{2})v = 0. \end{gathered} \tag{3}$$

Solutions of these equations are

$$\phi(t) = e^{iat^2/2}\Big[c_1 F(\alpha, 1/2, -iat^2) + + c_2 i\omega t F(\alpha + 1/2, 3/2, -iat^2)\Big], \tag{4}$$

and

$$\chi(t) = e^{iat^2/2}\Big[c_1 i\omega t F(\alpha + 1, 3/2, -iat^2) + + c_2 F(\alpha + 1/2, 1/2, -iat^2)\Big], \tag{5}$$

where $F(\alpha, \gamma, z)$ are Kummer functions.

Asymptotics at $t \to \pm\infty$ are

$$\phi(t) = \sqrt{\pi}e^{-\pi\omega^2/8a}\Big[\frac{c_1}{\Gamma(1/2 - \alpha)} + + \frac{i\omega}{2\sqrt{ia}}\frac{c_2}{\Gamma(1-\alpha)}\frac{t}{|t|}\Big](at^2)^{-\alpha}e^{iat^2/2} \tag{6}$$

and

$$\chi(t) = \sqrt{\pi}e^{-\pi\omega^2/8a}\Big[\frac{i\omega}{2\sqrt{-ia}}\frac{c_1}{\Gamma(1+\alpha)}\frac{t}{|t|} + + \frac{c_2}{\Gamma(1/2+\alpha)}\Big](at^2)^{\alpha}e^{-iat^2/2}. \tag{7}$$

If z-component of magnetic field changes according $H_z = -t\dot{H}$, $\dot{H} > 0$, equations and solutions for spin components interchange.

At reflection from the surface at $t = 0$ particle suddenly changes its trajectory and then follows along the with different value of gradient of z-component of magnetic field. The condition for this transition being "sudden" is fulfilled if reflection time is much smaller than any characteristic time parameter of the problem: $t_{\text{refl}} \ll T_{\text{Larmor}}$. It will be shown below that typical value for ultracold neutrons $t_{\text{refl}} \approx 10^{-9}$ s, which is sufficient for any practical case.

Now we match the solutions with different values of a at $t = 0$ corresponding to different trajectories of particle in magnetic field before and after reflection from the wall. When the signs of dH_z/dt are the same matching means $\phi(\alpha)_{t=0} = \phi(\alpha')_{t=0}$, when the signs are different we use $\phi(\alpha)_{t=0} = \chi(\alpha')_{t=0}$. We put $\chi(-\infty) = 0$, $|\phi(-\infty)|^2 = 1$ (initially particle is polarized opposite to z-axis) and obtain expressions for $|\phi(+\infty)|^2$:

$$|\phi(+\infty)|^2 = \pi^2 e^{-\frac{\pi\omega^2}{4}(\frac{1}{a}+\frac{1}{a'})} \times \times\Big|\frac{1}{\Gamma(1/2+\alpha)\Gamma(1/2-\alpha')} - - \frac{\omega^2}{4\sqrt{aa'}}\frac{1}{\Gamma(1+\alpha)\Gamma(1-\alpha')}\Big|^2 \tag{8}$$

for the cases when the signs of change of $H_z(t)$ component of magnetic field are the same before and after particle reflection from the wall, and

$$|\phi(+\infty)|^2 = \pi^2 e^{-\frac{\pi\omega^2}{4}(\frac{1}{a}+\frac{1}{a'})} \times \times\Big|\frac{1}{\Gamma(1/2+\alpha)\Gamma(1/2+\alpha')} - - \frac{i\omega^2}{4\sqrt{aa'}}\frac{1}{\Gamma(1+\alpha)\Gamma(1+\alpha')}\Big|^2 \tag{9}$$

when these signs are different e.g. particle returns after reflection from the wall to region of magnetic field of the same sign of H_z. α' in these expressions corresponds to particle motion after reflection.

When in Eq. (8) $\alpha = \alpha'$ we get $|\phi(+\infty)|^2 = e^{-\pi\omega^2/a}$, which corresponds to result of [9] for probability of spin reverse when particle moves freely without change its trajectory. For ultracold neutrons adiabaticity parameter $|\alpha| \gg 1$, $|\phi(+\infty)|^2 \to 0$ (exponentially small) practically in all cases – neutron spin follows the direction of magnetic field. Calculation according to formulas (8) and (9) show that it is not the case when particle reflects from the wall. Probability of spin reverse in respect to magnetic field increases drastically. Fig.1 shows results of computation of this probability for different cases of reflection as a function of $|\alpha^{'}|$.

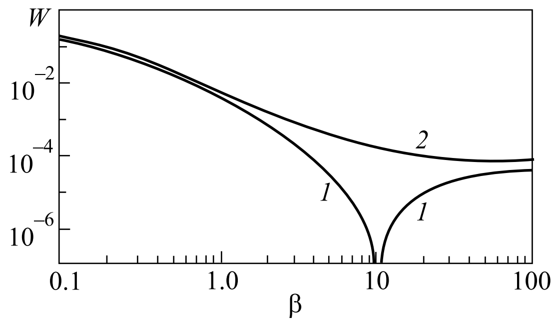

Fig.1. Probability of UCN spin reverse at reflection from the trap's wall as a function of adiabaticity parameter $\beta = |\alpha^{'}|$ after reflection (before reflection $\beta = |\alpha| = 10$): *1* – signs of time dependence of $H_z(t)$ are the same before and after reflection, *2* – these signs are opposite

In practice it is difficult to calculate probability according to Eqn. (8) and (9) for $|\alpha| \gg 1$. Using asymptotic formulas for Gamma-function it is possible for these cases to obtain for Eq. (8)

$$|\phi(+\infty)|^2 \simeq \frac{9}{4}(\xi - \xi^{'})^2, \tag{10}$$

and for Eq. (9)

$$|\phi(+\infty)|^2 \simeq 1 - 11 \cdot (\xi^2 + \xi^{'2}) - 18\xi\xi^{'} \tag{11}$$

where $\xi = 1/24|\alpha|$, $\xi^{'} = 1/24|\alpha^{'}|$.

Fig.2 shows the results of calculations according these formulas in terms of probability of spin reverse as a function of gradient of H_z. Neutron velocity along z-axis was taken 300 cm/s, $H_x = 5$ Oe.

4. Interaction time of particle reflecting from potential wall may be calculated in the spirit of Baz' [10, 11] idea introducing fictitious infinitesimal magnetic field in the interaction region and calculating the rotation angle of magnetic moment of particle in result of interaction. Reflection time in our case is determined as a ratio of rotation angle to Larmor frequency of precession of magnetic moment in this magnetic field. Let incident wave polarized in x-direction reflects from half-space $z \geq 0$, in which magnetic field is directed along z-axis. Schrödinger equation is:

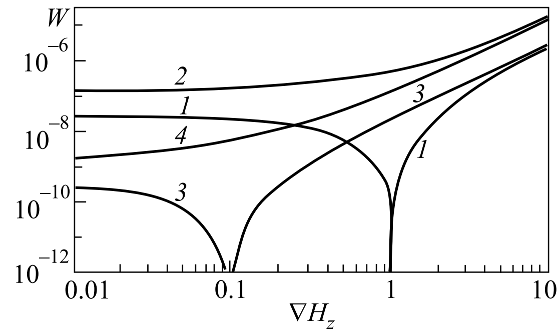

Fig.2. Probability of UCN spin reverse at reflection from the trap's wall as a function of dH_z/dz after reflection: before reflection $dH_z/dz = 0.1$ Oe/cm, *1* – signs of time dependence of $H_z(t)$ are the same before and after reflection, *2* – these signs are opposite; before reflection $dH_z/dz = 1$ Oe/cm, *3* – signs of time dependence of $H_z(t)$ are the same before and after reflection, *4* – these signs are opposite; in both cases $v_z = 300$ cm/s, $H_x = 5$ Oe

$$(\Delta + u_0 - \frac{2m}{\hbar^2}\mu H\sigma_z)\psi = k^2\psi, \tag{12}$$

where $u_0 = \hbar^2 U_0/2m$, U_0 is the height of potential wall, k is incident wave vector, H is the value of magnetic field in half-space $z \geq 0$.

The incident wave is

$$\psi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} e^{i(\boldsymbol{\kappa}\boldsymbol{\rho}+k_z z)}. \tag{13}$$

Reflected wave is

$$\psi = e^{i\varphi_0}\left[\begin{pmatrix}1\\1\end{pmatrix} + i\Delta\varphi\sigma_z\begin{pmatrix}1\\1\end{pmatrix}\right] e^{i(\boldsymbol{\kappa}\boldsymbol{\rho}-k_z z)}. \tag{14}$$

It follows[10] that

$$\Delta\varphi = \frac{d\varphi}{du_0}\Big(\frac{2m}{\hbar^2}\mu H\Big) \tag{15}$$

and as it is easy to show that

$$\frac{d\varphi}{du_0} = \frac{k_z}{u_0 K}, \tag{16}$$

where $K = \sqrt{u_0 - k_z^2}$, we have from $\omega = 2\mu H/\hbar$ and $\omega m k_z/\hbar u_0 K = \omega\Delta t$ reflection time

$$\Delta t = \frac{\hbar}{U_0}\frac{k_z}{K} \sim \frac{\hbar}{U_0}, \tag{17}$$

which at $U_0 \sim 10^{-7}$ eV gives $\Delta t \sim 10^{-9}$ s.

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