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Cherenkov radiation and pair production by particles traversing laser beams

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It is shown that Cherenkov radiation can be observed at TESLA in electron collisions with optical laser pulses. The prospects for it to be observed at SLC, LEP, LHC and RHIC are discussed. The conclusions are compared with results for pair production.

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The problem of very high energy charged particles collisions with laser beams is widely discussed now, mostly in connection with the e^+e^--pair production (see latest references [1 – 4]) at SLC and TESLA X-ray laser facilities [5 – 7] and with some other issues of fundamental physics.

Spontaneous particle creation from vacuum induced by a strong external field was theoretically considered in many papers beginning with Refs.[8 – 10]. However, very powerful high-frequency lasers are needed for this process to be observed. First observations of this effect were done at SLC [11, 12].

Here, I would like to note that optical lasers can be used for studies of Cherenkov radiation. It is crucial for its observation that the main background process of Compton scattering does not contribute to the kinematical region of Cherenkov radiation. The principal possibility of such a process was first mentioned by V. Ritus in Ref.[13]. The results can be, in general, applied to verify our ideas about the properties of the "photon medium" in the region where new physics concepts can become essential and to measurements of beam energy and laser bunch parameters.

X-ray lasers have been proposed for use in e^+e^--pair production studies because the quanta energies are high enough to reach the threshold energy which in c.m.s. is equal to $m + 2m_e$, where m_e is the electron mass and m is the accelerated particle mass (equal to m_e at SLC, LEP, TESLA, to the proton mass at LHC and to the nucleus mass at RHIC). At the same time, for studies of Cherenkov radiation other characteristics of a laser, namely, the ratio F/F_0 of the electric field F to its "critical" value $F_0 = m_e^2/e$ or, equivalently, its peak power density S, are important. They determine the index of refraction n of the "photon medium" in laser bunches. The difference of n from 1 is proportional to the density of photons in a laser pulse, i.e., to S. Just this difference defines the threshold of Cherenkov radiation, its emission angle and intensity[1]. The parameters F/F_0 and S are higher for optical lasers than for presently available X-ray lasers. That is why they would be preferred for Cherenkov radiation studies nowadays. Besides, the energy limitations also favour optical lasers for this purpose.

The necessary conditions for Cherenkov radiation to be observed are the excess of the index of refraction n over 1, i.e.

$$\Delta n = n - 1 > 0 \tag{1}$$

and the real emission angle, given by the formula

$$\cos\theta = 1/\beta n, \tag{2}$$

where $\beta = v/c = \sqrt{1 - m^2/E^2}$, m, E are the particle mass and energy. For small values of m/E and Δn one gets

$$\theta \approx \sqrt{2\Delta n - m^2/E^2} = \sqrt{2\Delta n - \gamma^{-2}}. \tag{3}$$

Hence, the condition for the energy to exceed the threshold for Cherenkov radiation E_{Ct} is written as

$$\gamma m = E \geq E_{Ct} = \frac{m}{\sqrt{2\Delta n}} = \gamma_{Ct} m. \tag{4}$$

It is easily seen that the threshold can become very high for small Δn.

The formula (3) can be rewritten as

$$0 \leq \theta = \sqrt{\gamma^2 - \gamma_{Ct}^2}/\gamma\gamma_{Ct} \leq \frac{1}{\gamma_{Ct}} = \theta_{\max} \quad (\gamma \to \infty). \tag{5}$$

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[1] The analogous problem was considered in Ref.[14] for particles traversing the cosmic microwave background radiation. The density of relic photons is, however, extremely low and, therefore, the index of refraction is so close to 1 that the threshold energy is too high for this effect to be observable.

It is seen that the emission angles of Cherenkov radiation increase from 0 at the threshold to $\theta_{\max} = \gamma_{Ct}^{-1}$ for $\gamma \to \infty$. However, already at $E = 2E_{Ct}$ this angle is very close to $\theta_{\max}$ ($\theta(2E_{Ct}) \approx 0.866\,\theta_{\max}$).

The number of Cherenkov photons emitted by a single particle with the electric charge e in the interval of frequencies $d\omega$ from the path length dl is given by the common expression [15]

$$dN_1/d\omega dl = 2\alpha\Delta n, \tag{6}$$

where the fine structure constant $\alpha \approx 1/137$. Thus all physical characteristics of the process are determined by the value Δn. The intensity of the radiation (6) decreases with the threshold energy (4) increase:

$$\frac{dN_1}{d\omega dl} = \frac{\alpha m^2}{E_{Ct}^2} = \frac{\alpha}{\gamma_{Ct}^2}. \tag{7}$$

The value of Δn is uniquely related to the polarization operator of $\gamma-\gamma$-scattering. For high energy electrons (protons), the laser field can be considered as the constant crossed (or null) field. The refractivity index can be expressed in terms of the photon mass acquired in such a field. It has been calculated in Refs.[16, 13] and its graphical representation can be found in Ref.[17] (for other approaches see also Refs.[18 – 20]). According to the results of Refs.[16, 13], the value of Δn depends on the photon mass μ and its energy ω in a following way

$$\Delta n = -\mathrm{Re}\mu^2/2\omega^2. \tag{8}$$

The photon mass depends only on the invariant variable

$$\kappa = \frac{2\omega}{m_e}\cdot\frac{F}{F_0}. \tag{9}$$

The value of $\mathrm{Re}\mu^2$ is negative[2)] in the region about

$$0 \le \kappa \le 15 \tag{10}$$

and has a minimum at $\kappa \approx 5$ with $\mathrm{Re}\mu^2 \approx -0.2\alpha m_e^2$. According to eq. (8), the refractivity index exceeds 1 in this region, and, consequently, the Cherenkov radiation is possible at these values of κ. The perturbation theory is still applicable [13] because $\alpha\kappa^{2/3} \ll 1$.

At a fixed laser intensity, i.e., a fixed ratio F/F_0, the index of refraction does not depend on ω at low energies

$$\Delta n \approx 14\alpha F^2/45\pi F_0^2, \tag{11}$$

because $\mathrm{Re}\mu^2$ is proportional to κ^2 at small κ^2. Thus, the ratio F/F_0 defines there main features of Cherenkov radiation.

The Cherenkov threshold γ_{Ct} is also completely determined by this ratio as seen from formulas (4), (11). It is the same for electrons and protons. Therefore, the threshold energies E_{Ct} are approximately 2000 times higher for protons than for electrons. The formulas (3) and (11) show that, in principle, by measuring the angle θ one can get the energy of the particle beam and the strength of the laser field or its peak power density.

The magnitude of Δn decreases at higher values of κ and becomes negative at $\kappa > 15$ so that Cherenkov radiation is impossible there.

Even though it can again become positive at extreme energies where the hadronic channels are important, this region is completely inaccessible in collisions with laser beams.

Let us remind that the energy threshold for the e^+e^--pair production processes in high energy head-on collisions of a particle of mass m with laser quanta is given by

$$E_{th} \approx m_e(m + m_e)/\omega_L. \tag{12}$$

It depends on the energy of laser quanta ω_L and is much lower for X-ray lasers than for optical lasers. It is approximately 1000 times higher for protons than for electrons. In particular, the threshold for $\gamma\gamma$-collisions follows from (12) at $m = 0$.

The condition for Cherenkov radiation threshold to be below the pair production threshold imposes the restriction on the laser quanta energies:

$$\omega_L < \sqrt{2\Delta n}m_e(1 + \frac{m_e}{m}) = \gamma_{Ct}^{-1}m_e(1 + \frac{m_e}{m}). \tag{13}$$

The condition (13) differs for electron and proton beams only by a factor about 2 in the right hand side.

For optical and X-ray lasers, according to [2], we accept, correspondingly, $\omega_L = 1.2\,\mathrm{eV}$ (actually, it varies from 0.12 eV for CO_2-laser to 2.35 eV for Nd:glass laser) and 3.1 keV, the ratios $F/F_0 = 3\cdot10^{-4}$ and 10^{-5} or, equivalently, the peak power densities $S = 3\cdot10^{22}\,\mathrm{W/cm^2} = 5\cdot10^{16}\,\mathrm{eV^4}$ and $8\cdot10^{19}\,\mathrm{W/cm^2}$ (with a possible goal $7\cdot10^{29}\,\mathrm{W/cm^2}$).

At these parameters, the laser field can be treated as a constant crossed (or null) field because its invariants (see [13])

$$x = \frac{m_e}{\omega_L}\cdot\frac{F}{F_0}; \quad \chi = 2\gamma\frac{F}{F_0} \tag{14}$$

are large. For optical lasers with $F/F_0 = 3\cdot10^{-4}$, one gets $x = 125$, and at TESLA energies $\gamma = 10^6$, $\chi = 600$.

[2)] We consider the value for transverse polarized photons. For other polarizations it differs by a factor less than 2, and this does not change general conclusions.

Thus, the wavelength $1/\omega_L$ is much larger than the formation length m_e/eF.

Electromagnetic processes at these conditions are extremely interesting. New physics concepts can be necessary here because the effective expansion parameter [21, 22, 17] $\alpha\chi^{2/3}$ exceeds 0.5. In particular, this would indicate that the constant field allows the interaction with field quanta of the arbitrarily low energies. Radiation effects should be reconsidered.

At $F/F_0 = 3\cdot 10^{-4}$, the relations (9), (10) impose the upper limit $\omega < 12\,\text{GeV}$. Only this region of comparatively low energies is admissible for Cherenkov quanta in such strong fields.

Using these characteristics, one also concludes that the condition (13) is satisfied for optical lasers with $F/F_0 > 3\cdot 10^{-5}$ ($S > 3\cdot 10^{21}\,\text{W/cm}^2$) and not valid for the presently available X-ray lasers. To satisfy it for X-ray lasers, one must achieve the peak power density as high as $10^{27}\,\text{W/cm}^2$ which is, nevertheless, within the proclaimed goals [2].

It follows from eqs (6) and (11) that one should deal with most intensive laser fields to get higher intensity of Cherenkov radiation. Thus, in what follows, we discuss only optical lasers briefly referring to X-ray lasers for some estimates.

The numerical value of Δn for γ-quanta in the optical laser field with $F/F_0 = 3\cdot 10^{-4}$ is given by

$$\Delta n = 0.65\cdot 10^{-10}. \tag{15}$$

Therefore, the typical angles and threshold γ-factors for Cherenkov radiation are

$$\theta_{\max} = 1.14\cdot 10^{-5}; \quad \gamma_{Ct} = 8.8\cdot 10^{4}. \tag{16}$$

This implies that the energy threshold for Cherenkov radiation is exceeded at LEP2 and TESLA since it is $E^{(e)}_{Ct} = 45\,\text{GeV}$ and is close to the upper energy of SLC. Only with further increase of the laser power, it would be possible to study this process at SLC.

The pair production threshold for optical lasers is about 430 GeV. Thus SLC and LEP energies are well below it while TESLA is just close[3] to the threshold value.

For proton beams the Cherenkov radiation threshold $E^{(p)}_{Ct} = 83\,\text{TeV}$ is too high even for LHC. If the optical lasers with $F/F_0 > 4\cdot 10^{-3}$ (the peak power density $S > 5\cdot 10^{24}\,\text{W/cm}^2$) will become accessible, one can hope to observe this effect there as well. This energy is much lower than the threshold for pair production at proton accelerators which is about 400 TeV.

What concerns the X-ray laser facilities, the threshold for pair production (12) is well below the energies accessible at all high energy accelerators.

Now, let us calculate the intensity of the Cherenkov radiation for electron beams and compare it with the main background process of Compton scattering[4]. The total number of Cherenkov quanta emitted in the energy interval $d\omega$ by a particle which collides with the laser bunch of the coherent spike length L is

$$\frac{dN_{Ch}}{d\omega} = 2\alpha\Delta n L = 1.1\cdot 10^{-5} L\frac{F^2}{F_0^2}. \tag{17}$$

This is the energy distribution at low energies as given by Eqs.(6), (8). It is almost constant at low energies as demonstrated by Eq.(17) but should decrease towards the cut-off at $\kappa \approx 15$. For a fixed value of the ratio F/F_0, the energies of emitted Cherenkov quanta are proportional to κ and limited according to Eqs. (9), (10). However, already at $\kappa \approx 4$ the magnitude of Δn and, consequently, the intensity (6) are about twice lower than at $\kappa = 0$. Therefore, the effective values of ω important in the distribution can be approximately estimated according to Eq. (9) as

$$\omega_{\text{eff}} < 2m_e(F_0/F). \tag{18}$$

For the values of the ratio F/F_0 adopted above, one gets $\omega^{(o)}_{\text{eff}} < 3.5\,\text{GeV}$ for optical lasers and $\omega^{(X)}_{\text{eff}} < 100\,\text{GeV}$ for X-ray lasers. One can use Eq. (9) for an estimate of Δn in these energy regions. The threshold and effective energy of Cherenkov quanta decrease while the emission angle and the intensity of radiation increase with increase of laser fields F.

The absolute intensity can be evaluated according to the formulas (6), (17). For the coherent spike length $L \sim 1\,\text{mm}$, the number of quanta per 1 GeV is estimated as

$$dN_{Ch}/d\omega \approx 5\ \text{GeV}^{-1}. \tag{19}$$

Thus the emitted energy within the effective interval should be of the order of 30 GeV per 1 mm.

To proceed with similar estimates for Compton scattering, we consider first its kinematics. This leads to the following relation between the emission angle θ and energy ω of the scattered quantum in the laboratory system:

$$\cos\theta = \left(1 + \frac{\omega_L}{E} - \frac{2\omega_L}{\omega}\right)\left(1 - \frac{\omega_L}{E}\right)^{-1} \approx 1 - \frac{2\omega_L}{\omega}. \tag{20}$$

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[3] However, notice the rather wide spread of available wavelengths for optical lasers mentioned above.

[4] The final results are valid for any charged particles.

The precise limits imposed by this relation on the energy of emitted quanta are given by $\omega_L \leq \omega \leq E$. In the right hand side of (20) we have taken into account that the particle beam energy is much higher than the photons energies $E \gg \omega > \omega_L$. At the angles typical for Cherenkov radiation (16) the energy of the backscattered quantum obtained from Eq. (20) is equal to $\omega \approx 37$ GeV while Cherenkov radiation is much softer ($\omega_{Ch} < 3.5$ GeV) due to the cut-off imposed by the behaviour of Δn. Such "soft" photons are emitted at larger angles at Compton scattering. Therefore, there is no overlap of the kinematical regions available for Compton and Cherenkov processes. By separating the relatively soft quanta at the angle (16) one would be able to get rid of the background due to Compton processes. These processes contribute to completely different energy ranges and, therefore, can be easily disentangled.

The multiphoton processes would lead to even harder quanta in Compton scattering. In other words, they are effective at larger angles $\theta > 2\sqrt{\omega_L/\omega}$. Thus, even at high values of x usual Compton effect formulas are needed to estimate its intensity at small angles. Let us also note that these energies are much lower than the threshold for pair production (12) equal to 220 GeV for $\omega_L = 1.2$ eV.

The total energy loss due to Compton scattering is much larger than for Cherenkov radiation (19). It can be as large as 16/63 of the initial electron energy [13]. This is determined by the extremely hard backscattered quanta. The use of different lasers inevitably leads to the change of corresponding values of F/F_0 and S, and, consequently, of the threshold energy E_{Ct}. Therefore, one should be careful in estimates. The practical feasibility of observation of such an effect at high energies should be considered in close relation to the definite conditions of a particular experiment. For example, one can not use the Nd:glass laser of SLC experiments [11, 12, 23] with $F/F_0 = 2.3 \cdot 10^{-6}$ at TESLA because the Cherenkov threshold energy becomes too high. The optimum choice would be the laser with highest power density (see (17)).

At the end, let us note that, in principle, the heavy ion accelerator RHIC can be also used for pair production studies with X-ray lasers because the threshold energy according to (12) is equal to 165 GeV per nucleon that is available there. The Cherenkov radiation threshold is the same as for proton accelerators if estimated per nucleon. It has been discussed above in connection with LHC and is not reachable at RHIC.

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