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Electric field dependence of thermal conductivity of a granular superconductor: Giant field-induced effects predicted

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The temperature and electric field dependence of electronic contribution to the thermal conductivity (TC) of a granular superconductor is considered within a 3D model of inductive Josephson junction arrays. In addition to a low-temperature maximum of zero-field TC $\kappa(T,0)$ (controlled by mutual inductance L_0 and normal state resistivity R_n), the model predicts two major effects in applied electric field: (i) *decrease* of the *linear* TC, and (ii) giant *enhancement* of the *nonlinear* (i.e. ∇T-dependent) TC with $\Delta\kappa(T,E)/\kappa(T,0)$ reaching 500% for parallel electric fields $E \simeq E_T$ ($E_T = S_0|\nabla T|$ is an "intrinsic" thermoelectric field). A possiblity of experimental observation of the predicted effects in granular superconductors is discussed.

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1. Introduction. Inspired by new possibilities offered by the cutting-edge nanotechnologies, the experimental and theoretical physics of increasingly sophisticated mesoscopic quantum devices (heavily based on Josephson junctions and their arrays) is becoming one of the most exciting and rapidly growing areas of modern science [1 – 3]. In addition to the traditional fields of expertise (such as granular superconductors [2]), Josephson junction arrays (JJAs) are actively used for testing principally novel ideas (like, e.g., topologically protected quantum bits [3]) in a bid to solve probably one of the most challenging problems in quantum computing. Though traditionally, the main emphasis in studying JJAs has been on their behavior in applied magnetic fields, since recently a special attention has been given to the so-called electric field effects (FEs) in JJs and granular superconductors [4 – 9]. The unusually strong FEs observed in bulk high-T_c superconducting (HTS) ceramics [4] (including a substantial enhancement of the critical current, reaching $\Delta I_c(E)/I_c(0) = 100\%$ for $E = 10^7\,\mathrm{V/m}$) have been attributed to a crucial modification of the original weak-links structure under the influence of very strong electric fields. This hypothesis has been corroborated by further investigations, both experimental (through observation of the correlation between the critical current behavior and type of weak links [5]) and theoretical (by studying the FEs in SNS-type structures [6] and d-wave granular superconductors [7]). Among other interesting field induced effects, one can mention the FE-based Josephson transistor [8] and Josephson analog of the *magnetoelectric effect* [9] (electric field generation of Josephson magnetic moment in zero magnetic field). At the same time, very little is known about influence of electric fields on thermal transport properties of granular superconductors. In an attempt to shed some light on this intresting and important (for potential applications) problem, in this Letter we present a theoretical study of the electric field and temperature dependence of electronic contribution to thermal conductivity (TC) κ of a granular superconductor (described by a 3D model of inductive JJAs). As we shall see below, in addition to a low-temperature maximum of zero-field TC $\kappa(T,0)$ (controlled by the mutual inductance L_0 and normal state resistivity R_n), the model predicts unusually strong (giant) field-induced effects in the behavior of *nonlinear* (i.e. ∇T-dependent) TC. In particular, the absolute values of the TC enhancement $\Delta\kappa(T,E)/\kappa(T,0)$ are estimated to reach up to 500% for relatively low (in comparison with the fields needed to observe a critical current enhancement [4, 5]) applied electric fields E matching an intrinsic thermoelectric field $E_T = S_0|\nabla T|$. The estimates of the model parameters suggest quite an optimistic possibility to observe the predicted effects in granular superconductors and JJAs.

2. The model. To adequately describe a thermodynamic behavior of a real granular superconductor for all temperatures and under a simultaneous influence of arbitrary electric field $\mathbf{E}$ and thermal gradient ∇T, we consider one of the numerous versions of the 3D JJAs models based on the following Hamiltonian

$$\mathcal{H}(t) = \mathcal{H}_T(t) + \mathcal{H}_L(t) + \mathcal{H}_E(t), \qquad (1)$$

where

$$\mathcal{H}_T(t) = \sum_{ij}^{N} J_{ij}[1 - \cos\phi_{ij}(t)] \tag{2}$$

is the well-known tunneling Hamiltonian,

$$\mathcal{H}_L(t) = \sum_{ij}^{N} \frac{\Phi_{ij}^2(t)}{2L_{ij}} \tag{3}$$

accounts for a mutual inductance L_{ij} between grains (and controls the normal state value of the thermal conductivity, see below) with $\Phi_{ij}(t) = (\hbar/2e)\phi_{ij}(t)$ being the total magnetic flux through an array, and finally

$$\mathcal{H}_E(t) = -\mathbf{p}(t)\mathbf{E} \tag{4}$$

describes electric field induced polarization contribution, where the polarization operator

$$\mathbf{p}(t) = -2e\sum_{i=1}^{N} n_i(t)\mathbf{r}_i. \tag{5}$$

Here n_i is the pair number operator, and $\mathbf{r}_i$ is the coordinate of the center of the grain.

As usual, the tunneling Hamiltonian $\mathcal{H}_T(t)$ describes a short-range interaction between N superconducting grains, arranged in a 3D lattice with coordinates $\mathbf{r}_i = (x_i, y_i, z_i)$. The grains are separated by insulating boundaries producing temperature dependent Josephson coupling $J_{ij}(T) = J_{ij}(0)F(T)$ with

$$F(T) = \frac{\Delta(T)}{\Delta(0)}\tanh\left[\frac{\Delta(T)}{2k_BT}\right] \tag{6}$$

and $J_{ij}(0) = [\Delta(0)/2](R_0/R_{ij})$ where $\Delta(T)$ is the temperature dependent gap parameter, $R_0 = h/4e^2$ is the quantum resistance, and R_{ij} is the resistance between grains in their normal state assumed [10] to vary exponentially with the distance $\mathbf{r}_{ij}$ between neighboring grains, i.e. $R_{ij}^{-1} = R_n^{-1}\exp(-r_{ij}/d)$ (where d is of the order of an average grain size).

As is well-known [2, 10], a constant electric field $\mathbf{E}$ and a thermal gradient ∇T applied to a JJA cause a time evolution of the initial phase difference $\phi_{ij}^0 = \phi_i - \phi_j$ as follows

$$\phi_{ij}(t) = \phi_{ij}^0 + \omega_{ij}(\mathbf{E}, \nabla T)t. \tag{7}$$

Here $\omega_{ij} = 2e(\mathbf{E} - \mathbf{E}_T)\mathbf{r}_{ij}/\hbar$ where $\mathbf{E}_T = S_0\nabla T$ is an "intrinsic" thermoelectric field with S_0 being a zero-field value of the Seebeck coefficient.

3. Linear thermal conductivity (Fourier law). We start our consideration by discussing the temperature behavior of the conventional (that is *linear*) thermal conductivity of a granular superconductor in arbitrary applied electric field $\mathbf{E}$ paying a special attention to its evolution with a mutual inductance L_{ij}. For simplicity, in what follows we limit our consideration to the longitudinal component of the total thermal flux $\mathbf{Q}(t)$ which is defined (in a $\mathbf{q}$-space representation) via the total energy conservation law as follows

$$\mathbf{Q}(t) \equiv \lim_{\mathbf{q}\to 0}\left[i\frac{\mathbf{q}}{\mathbf{q}^2}\dot{\mathcal{H}}_{\mathbf{q}}(t)\right], \tag{8}$$

where $\dot{\mathcal{H}}_{\mathbf{q}} = \partial\mathcal{H}_{\mathbf{q}}/\partial t$ with

$$\mathcal{H}_{\mathbf{q}}(t) = \frac{1}{v}\int d^3x e^{i\mathbf{qr}}\mathcal{H}(\mathbf{r}, t). \tag{9}$$

Here $v = 8\pi d^3$ is properly defined normalization volume, and we made a usual substitution $\frac{1}{N}\sum_{ij} A(r_{ij}, t) \to \frac{1}{v}\int d^3x A(\mathbf{r}, t)$ valid in the long-wavelength approximation ($\mathbf{q} \to 0$).

In turn, the above-introduced heat flux $\mathbf{Q}(t)$ is related to the appropriate components of the *linear* thermal conductivity (LTC) tensor $\kappa_{\alpha\beta}$ as follows (hereafter, $\{\alpha, \beta\} = x, y, z$)

$$\kappa_{\alpha\beta}(T, \mathbf{E}) \equiv -\frac{1}{V}\left[\frac{\partial\overline{\langle Q_\alpha\rangle}}{\partial(\nabla_\beta T)}\right]_{\nabla T = 0}, \tag{10}$$

where

$$\overline{< Q_\alpha >} = \frac{1}{\tau}\int_0^\tau dt\langle Q_\alpha(t)\rangle. \tag{11}$$

Here V is a sample's volume, τ is a characteristic Josephson time for the network, and $\langle ... \rangle$ denotes the thermodynamic averaging over the initial phase differences ϕ_{ij}^0

$$\langle A(\phi_{ij}^0)\rangle = \frac{1}{Z}\int_0^\pi \prod_{ij} d\phi_{ij}^0 A(\phi_{ij}^0)e^{-\beta H_0} \tag{12}$$

with an effective Hamiltonian

$$H_0[\phi_{ij}^0] = \int_0^\tau \frac{dt}{\tau}\int \frac{d^3x}{v}\mathcal{H}(\mathbf{r}, t). \tag{13}$$

Here, $\beta = 1/k_BT$, and $Z = \int_0^\pi \prod_{ij} d\phi_{ij}^0 e^{-\beta H_0}$ is the partition function. The above-defined averaging procedure allows us to study the temperature evolution of the system.

Taking into account that in JJAs [11] $L_{ij} \propto R_{ij}$, we obtain $L_{ij} = L_0\exp(r_{ij}/d)$ for the explicit r-dependence of the weak-link inductance in our model. Finally, in view of Eqs.(1)–(13), and making use of the usual "phase-number" commutation relation, $[\phi_i, n_j] = i\delta_{ij}$, we find the following analytical expression for the temperature and electric field dependence of the electronic contribution to *linear* thermal conductivity of a granular superconductor

$$\kappa_{\alpha\beta}(T,\mathbf{E}) = \kappa_0[\delta_{\alpha\beta}\eta(T,\epsilon) + \beta_L(T)\nu(T,\epsilon)f_{\alpha\beta}(\epsilon)] \quad (14)$$

where

$$f_{\alpha\beta}(\epsilon) = \frac{1}{4}\left[\delta_{\alpha\beta}A(\epsilon) - \epsilon_\alpha\epsilon_\beta B(\epsilon)\right] \quad (15)$$

with

$$A(\epsilon) = \frac{5+3\epsilon^2}{(1+\epsilon^2)^2} + \frac{3}{\epsilon}\tan^{-1}\epsilon \quad (16)$$

and

$$B(\epsilon) = \frac{3\epsilon^4+8\epsilon^2-3}{\epsilon^2(1+\epsilon^2)^3} + \frac{3}{\epsilon^3}\tan^{-1}\epsilon. \quad (17)$$

Here, $\kappa_0 = Nd^2S_0\Phi_0/VL_0$, $\beta_L(T) = 2\pi I_c(T)L_0/\Phi_0$ with $I_c(T) = (2e/\hbar)J(T)$ being the critical current (we neglect a possible field dependence of I_c because, as we shall see below, the characteristic fields where thermal conductivity exhibits most interesting behavior are much lower than those needed to produce a tangible change of the critical current [4]); $\epsilon \equiv \sqrt{\epsilon_x^2+\epsilon_y^2+\epsilon_z^2}$ with $\epsilon_\alpha = E_\alpha/E_0$, and $E_0 = \hbar/2ed\tau$ is a characteristic electric field. In turn, the above-introduced "order parameters" of the system, $\eta(T,\epsilon) \equiv \langle\phi_{ij}^0\rangle$ and $\nu(T,\epsilon) \equiv <\sin\phi_{ij}^0>$, are defined as follows

$$\eta(T,\epsilon) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\left[\frac{I_{2n+1}(\beta_E)}{I_0(\beta_E)}\right] \quad (18)$$

and

$$\nu(T,\epsilon) = \frac{\sinh\beta_E}{\beta_E I_0(\beta_E)}, \quad (19)$$

where

$$\beta_E(T,\epsilon) = \frac{\beta J(T)}{2}\left(\frac{1}{1+\epsilon^2} + \frac{1}{\epsilon}\tan^{-1}\epsilon\right). \quad (20)$$

Here $J(T) = J(0)F(T)$ with $J(0) = (\Delta_0/2)(R_0/R_n)$ and $F(T)$ given by Eq.(6); $I_n(x)$ stand for the appropriate modified Bessel functions.

3.1. *Zero-field effects.* Turning to the discussion of the obtained results, we start with a more simple zero-field case. The relevant parameters affecting the behavior of the LTC in this particular case include the mutual inductance L_0 and the normal state resistance between grains R_n. For the temperature dependence of the Josephson energy (see Eq.(6)), we used the well-known [12] approximation for the BCS gap parameter, valid for all temperatures, $\Delta(T) = \Delta(0)\tanh\left(\gamma\sqrt{(T_c-T)/T}\right)$ with $\gamma = 2.2$.

Despite a rather simplified nature of our model, it seems to quite reasonably describe the behavior of the LTC for all temperatures. Indeed, in the absence of an applied electric field ($E = 0$), the LTC is isotropic (as expected), $\kappa_{\alpha\beta}(T,0) = \delta_{\alpha\beta}\kappa_L(T,0)$ where $\kappa_L(T,0) = \kappa_0[\eta(T,0) + 2\beta_L(T)\nu(T,0)]$ vanishes at zero temperature and reaches a normal state value $\kappa_n \equiv \kappa_L(T_c,0) = (\pi/2)\kappa_0$ at $T = T_c$. Figure 1 shows the temperature dependence of the normalized LTC $\kappa_L(T,0)/\kappa_n$ for

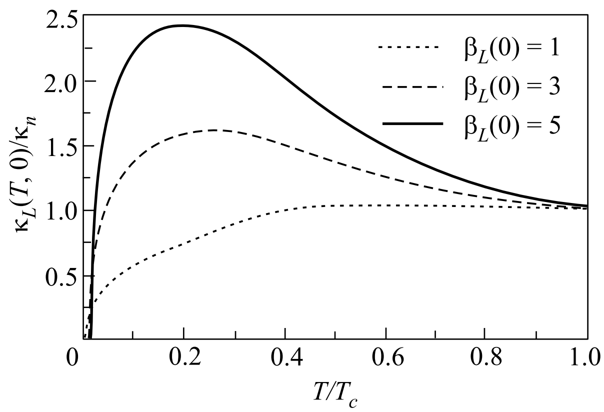

Fig.1. Temperature dependence of the zero-field *linear* thermal conductivity $\kappa_L(T,0)/\kappa_n$ for different values of the dimensionless parameter $\beta_L(0)$

different values of the dimensionless parameter $\beta_L(0) = 2\pi I_c(0)L_0/\Phi_0$. As it is clearly seen, with increasing of this parameter, the LTC evolves from a flat-like pattern (for a relatively small values of L_0) to a low-temperature maximum (for higher values of $\beta_L(0)$). Notice that the peak temperature T_p is practically insensitive to the variation of inductance parameter L_0 while being at the same time strongly influenced by resistivity R_n. Indeed, the presented here curves correspond to the resistance ratio $r_n = R_0/R_n = 1$ (a highly resistive state). It can be shown that a different choice of r_n leads to quite a tangible shifting of the maximum. Namely, the smaller is the normal resistance between grains R_n (or the better is the quality of the sample) the higher is the temperature at which the peak is developed. As a matter of fact, the peak temperature T_p is related to the so-called phase-locking temperature T_J (which marks the establishment of phase coherence between the adjacent grains in the array and always lies below a single grain superconducting temperature T_c) which is usually defined via an average (per grain) Josephson coupling energy as [13] $J(T_J, r_n) = k_BT_J$. In particular, for $T \simeq T_c$, it can be shown analytically that $T_J(r_n)$ indeed increases with r_n as $T_J(r_n)/T_c \simeq r_n/(1+r_n)$.

3.2. *Electric field effects.* Turning to the discussion of the LTC behavior in applied electric field, let us demonstrate first of all its anisotropic nature. For simplicity (but without losing generality), we assume that $\mathbf{E} = (E, 0, 0)$ and $\nabla T = (\nabla_x T, \nabla_y T, 0)$. Such a choice of the external fields allows us to consider both parallel $\kappa_{xx}(T, E)$ and perpendicular $\kappa_{yy}(T, E)$ components of the LTC corresponding to the two most interesting configurations, $\mathbf{E} \| \nabla T$ and $\mathbf{E} \perp \nabla T$, respectively. Inset in Fig.2 demonstrates the predicted electric field

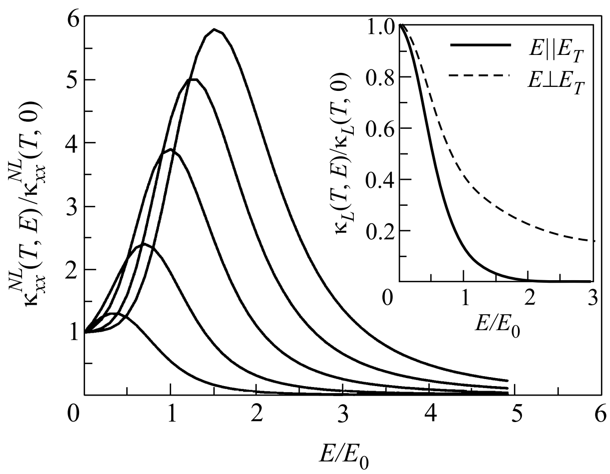

Fig.2. Electric field dependence of the *nonlinear* thermal conductivity $\kappa_{xx}^{NL}(T, E)/\kappa_{xx}^{NL}(T, 0)$ for different values of the applied thermal gradient $\epsilon_T = S_0|\nabla T|/E_0$ ($\epsilon_T = 0.2; 0.4; 0.6; 0.8; 1.0$, increasing from bottom to top). Inset: Electric field dependence of the *linear* thermal conductivity $\kappa_L(T, E)/\kappa_L(T, 0)$ for parallel ($\mathbf{E} \| \nabla T$) and perpendicular ($\mathbf{E} \perp \nabla T$) configurations

dependence of the normalized LTC $\kappa_L(T, E)/\kappa_L(T, 0)$ for both configurations taken at $T = 0.2T_c$ (with $r_n = 1$ and $\beta_L(0) = 1$). First of all, we note that both components of the LTC are *decreasing* with increasing of the field E/E_0. And secondly, the normal component κ_{yy} decreases more slowly than the parallel one κ_{xx}, suggesting thus some kind of anistropy in the system. In view of the structure of Eq.(14), the same behavior is also expected for the temperature dependence of the field-induced LTC, that is $\Delta\kappa_L(T, E)/\kappa_L(T, 0) < 0$ for all fields and temperatures. In terms of the absolute values, for $T = 0.2T_c$ and $E = E_0$, we obtain $[\Delta\kappa_L(T, E)/\kappa_L(T, 0)]_{xx} = 90\%$ and $[\Delta\kappa_L(T, E)/\kappa_L(T, 0)]_{yy} = 60\%$ for *attenuation* of LTC in applied electric field.

4. Nonlinear thermal conductivity: Giant field-induced effects. Let us turn now to the most intriguing part of this paper and consider a *nonlinear* generalization of the Fourier law and very unusual behavior of the resulting *nonlinear* thermal conductivity (NLTC) under the influence of an applied electric field. In what follows, by the NLTC we understand a ∇T-dependent thermal conductivity $\kappa_{\alpha\beta}^{NL}(T, \mathbf{E}) \equiv \kappa_{\alpha\beta}(T, \mathbf{E}; \nabla T)$ which is defined as follows

$$\kappa_{\alpha\beta}^{NL}(T, \mathbf{E}) \equiv -\frac{1}{V}\left[\frac{\partial\overline{\langle Q_\alpha\rangle}}{\partial(\nabla_\beta T)}\right]_{\nabla T \neq 0} \tag{21}$$

with $\overline{\langle Q_\alpha\rangle}$ given by Eq.(11).

Repeating the same procedure as before, we obtain finally for the relevant components of the NLTC tensor

$$\kappa_{\alpha\beta}^{NL}(T, \mathbf{E}) = \kappa_0[\delta_{\alpha\beta}\eta(T, \epsilon_{\text{eff}}) + \\ + \beta_L(T)\nu(T, \epsilon_{eff})D_{\alpha\beta}(\epsilon_{\text{eff}})], \tag{22}$$

where

$$D_{\alpha\beta}(\epsilon_{\text{eff}}) = f_{\alpha\beta}(\epsilon_{\text{eff}}) + \epsilon_T^\gamma g_{\alpha\beta\gamma}(\epsilon_{\text{eff}}) \tag{23}$$

with

$$g_{\alpha\beta\gamma}(\epsilon) = \frac{1}{8}[(\delta_{\alpha\beta}\epsilon_\gamma + \delta_{\alpha\gamma}\epsilon_\beta + \delta_{\gamma\beta}\epsilon_\alpha)B(\epsilon) + \\ + 3\epsilon_\alpha\epsilon_\beta\epsilon_\gamma C(\epsilon)] \tag{24}$$

and

$$C(\epsilon) = \frac{3 + 11\epsilon^2 - 11\epsilon^4 - 3\epsilon^6}{\epsilon^4(1+\epsilon^2)^4} - \frac{3}{\epsilon^5}\tan^{-1}\epsilon. \tag{25}$$

Here, $\epsilon_{\text{eff}}^\alpha = \epsilon^\alpha - \epsilon_T^\alpha$ where $\epsilon_\alpha = E_\alpha/E_0$ and $\epsilon_T^\alpha = E_T^\alpha/E_0$ with $E_T^\alpha = S_0\nabla_\alpha T$; other field-dependent parameters (η, ν, B and $f_{\alpha\beta}$) are the same as before but with $\epsilon \to \epsilon_{\text{eff}}$.

As expected, in the limit $E_T \to 0$ (or when $E \gg \gg E_T$), from Eq.(22) we recover all the results obtained in the previous section for the LTC. Let us see now what happens when the "intrinsic" thermoelectric field $\mathbf{E_T} = S_0\nabla T$ becomes comparable with an applied electric field $\mathbf{E}$. Figure 2 (main frame) depicts the resulting electric field dependence of the parallel component of the NLTC tensor $\kappa_{xx}^{NL}(T, E)$ for different values of the dimensionless parameter $\epsilon_T = E_T/E_0$ (the other parameters are the same as before). As it is clearly seen from this picture, in a sharp contrast with the field behavior of the previously considered *linear* TC, its *nonlinear* analog evolves with the field quite differently. Namely, NLTC strongly *increases* for small electric fields ($E < E_m$), reaches a pronounced maximum at $E = E_m = \frac{3}{2}E_T$, and eventually declines at higher fields ($E > E_m$). Furthermore, as it directly follows from the very structure of Eq.(22), a similar "reentrant-like" behavior of the *nonlinear* thermal conductivity will occur in its temperature dependence as well. Even more remarkable is the absolute value of the field-induced enhancement. According to Fig.2 (main frame), it is easy to estimate that near maximum (with $E = E_m$ and $E_T = E_0$) and for $T = 0.2T_c$, one gets $\Delta\kappa_{xx}^{NL}(T,E)/\kappa_{xx}^{NL}(T,0) \simeq 500\%$.

5. Discussion. To understand the above-obtained rather unusual results, let us take a closer look at the field-induced behavior of the Josephson voltage in our system (see Eq.(7)). Clearly, strong heat conduction requires establishment of a quasi-stationary (that is nearly zero-voltage) regime within the array. In other words, the maximum of the thermal conductivity in applied electric field should correlate with a minimum of the total voltage in the system, $V(E) \equiv (\hbar/2e)\langle\partial\phi_{ij}(t)/\partial t\rangle = V_0(\epsilon-\epsilon_T)$ where $\epsilon \equiv E/E_0$ and $V_0 = E_0 d = \hbar/2e\tau$ is a characteristic voltage. For linear TC (which is valid only for small thermal gradients with $\epsilon_T \equiv E_T/E_0 \ll 1$), the average voltage through an array $V_L(E) \simeq V_0(E/E_0)$ has a minimum at zero applied field (where LTC indeed has its maximum value, see the inset of Fig.2) while for nonlinear TC (with $\epsilon_T \simeq 1$) we have to consider the total voltage $V(E)$ which becomes minimal at $E = E_T$ (in a good agreement with the predictions for NLTC maximum which appears at $E = \frac{3}{2}E_T$, see the main frame of Fig.2).

To complete our study, let us estimate an order of magnitude of the main model parameters. Starting with applied electric fields E needed to observe the above-predicted nonlinear field effects in granular superconductors, we notice that according to Fig.2, the most interesting behavior of NLTC takes place for $E \simeq E_0$. Taking $d \simeq 10\,\mu$m and $\tau \simeq 10^{-9}$ s for typical values of the average grain size and characteristic Josephson tunneling time (valid for conventional JJs [14] and HTS ceramics [10]), we get $E_0 = \hbar/(2ed\tau) \simeq 2 \cdot 10^{-2}$ V/m for the characteristic electric field (which is surprisingly lower than the typical fields needed to observe a critical current enhancement in HTS ceramics [4, 5]). On the other hand, the maximum of NLTC occurs when this field nearly perfectly matches an "intrinsic" thermoelectric field $E_T = S_0|\nabla T|$ induced by an applied thermal gradient, that is when $E \simeq E_0 \simeq E_T$. Using $S_0 \simeq 0.5\mu$ V/K for the zero-field value of the *linear* Seebeck coefficient [10, 14], we obtain $|\nabla T|_E \simeq E_0/S_0 \simeq 4 \cdot 10^4$ K/m for the characteristic value of an applied thermal gradient.

Finally, taking as an example [15] granular aluminum films with phonon dominated heat transport (with $\kappa_{ph}(T) \simeq 2 \cdot 10^{-7}$ W/mK at $T = T_J \simeq 0.2T_c$), let us estimate the absolute value of the predicted here zero-field electronic contribution $\kappa_e(T) \equiv \kappa_L(T,0)$ at $T = 0.2T_c$. Recalling that within our model the scattering of normal electrons is due to the presence of mutual inductance between the adjacent grains L_0, and using $L_0 \simeq \mu_0 d \simeq 4\pi \cdot 10^{-12}$ H and $V \simeq Nd^2 l$ (l is a film's thickness), we obtain $\kappa_e(T = 0.2T_c) \simeq \beta_L(0) \cdot 10^{-7}$ W/mK for a rough estimate of the electronic contribution to the discussed here inductance-driven effect. Correspondingly, we get $\kappa_e(0.2T_c)/\kappa_{ph}(0.2T_c) \simeq \beta_L(0)/2$ for the ratio, where $\beta_L(0) = 2\pi I_c(0)L_0/\Phi_0$. Thus, depending mainly on the value of the critical current $I_c(0)$ and mutual inductance between adjacent grains L_0, the thermal conductivity of specially prepared granular alumina films will be dominated by either phonon (for small $\beta_L(0)$) or electronic (for large $\beta_L(0)$) contribution. Undoubtedly, the above estimates suggest quite a realistic possibility to observe the predicted non-trivial behavior of the thermal conductivity in granular superconductors and artificially prepared Josephson junction arrays. We hope that the presented here results will motivate further theoretical and experimental studies of this interesting problem.

1. *Proc. of the Conf. "Mesoscopic and Strongly Correlated Electron Systems", Chernogolovka, 1997*, Eds. V. F. Gantmakher and M. V. Feigel'man, Phys. Usp. **41**, 2 (1998); *Mesoscopic and Strongly Correlated Electron Systems-II*, Eds. M. V. Feigel'man, V. V. Ryazanov, and V. B. Timofeev, Phys. Usp. (Suppl.) **44**, 10 (2001).
2. See, e.g., a special issue on JJAs in *Studies of High Temperature Superconductors*, vol. **39**, Eds. A. Narlikar and F. Araujo-Moreira, Nova Science, New York, 2001.
3. L. B. Ioffe, M. V. Feigel'man, A. Ioselevich et al., Nature **415**, 503 (2002).
4. T. S. Orlova, B. I. Smirnov, and J. Y. Laval, Phys. Solid State **43**, 1007 (2001).
5. T. S. Orlova, B. I. Smirnov, J. Y. Laval et al., Supercond. Sci. Technol. **12**, 356 (1999).
6. A. L. Rakhmanov and A. V. Rozhkov, Physica **C267**, 233 (1996).
7. D. Dominguez, C. Wiecko, and J. V. José, Phys. Rev. Lett. **83**, 4164 (1999).
8. J. Mannhart, Supercond. Sci. Technol. **9**, 49 (1996).
9. S. A. Sergeenkov and J. V. José, Europhys. Lett. **43**, 469 (1998).
10. S. A. Sergeenkov, JETP Lett. **67**, 680 (1998).
11. A.-L. Eichenberger, J. Affolter, M. Willemin et al., Phys. Rev. Lett. **77**, 3905 (1996).
12. R. Meservey and B. B. Schwartz, in *Superconductivity*, vol.1, Ed. R. D. Parks, M. Dekker, NY, 1969, p. 117.
13. L. Leylekian, M. Ocio, L. A. Gurevich et al., JETP **85**, 1138 (1997).
14. G. I. Panaitov, V. V. Ryazanov, A. V. Ustinov et al., Phys. Lett. **A100**, 301 (1984).
15. Y. Shapira and G. Deutscher, Phys. Rev. **B30**, 166 (1984); J. Deppe and J. L. Feldman, Phys. Rev. **B50**, 6479 (1994).