Low temperature effective electromagnetism in superfluid ^{3}He-A

J. Dziarmaga

Los Alamos National Laboratory, Theory Division T-6, Los Alamos, New Mexico 87545, USA
Instytut Fizyki Uniwersytetu Jagiellońskiego, 30-059 Kraków, Poland

Submitted 18 February 2002

A general low temperature effective action for the order parameter in the superfluid phase A of helium 3 is derived. In a symmetric case, when Fermi velocity equals transversal velocity of low energy fermionic quasiparticles, the action is the standard relativistic electromagnetic action.

PACS: 11.15.-q, 11.90.+t, 67.57.-z

1. Introduction. Quantum mechanics is not compatible with general relativity. There are well known problems with the definition of a time operator, the cosmological constant problem, the divergencies in the relativistic quantum field theory and the black hole paradoxes [1]. In recent years, as a result of interaction between the condensed matter and the high energy physics communities, a new program is emerging that may solve all these fundamental problems at once [2 – 5]. The key idea is that both general relativity and the Standard Model [2 – 5] are low energy effective field theories of an underlying condensed matter system.

By its very definition this program invalidates the time operator problem. The fundamental theory is a condensed matter system described by a "nonrelativistic" N-body Schrödinger equation in an abstract configuration space. The time operator problem is an artifact of the effective low energy relativistic theory. When the missing definition of a time operator leads to paradoxes in the effective theory, their resolution can be found at the level of the fundamental condensed matter system.

The cosmological constant or Casimir energy, when calculated within the effective relativistic quantum field theory, is divergent. It is customary to cut off this divergence at the Planck scale. Even when cut off the cosmological constant is still, by many orders of magnitude, inconsistent with observations. A fundamental condensed matter theory should, at the very least, provide a correct prescription how to make the cut off in the effective relativistic theory [3]. An example in Ref. [3] demostrates that in a condensed matter system it is even possible to have a nonzero Casimir force but at the same time an exactly vanishing cosmological constant.

The divergencies in the perturbative relativistic quantum field theory are yet another artifact of the effective low energy theory. The underlying N-body Schrödinger equation does not suffer from any divergencies.

Violation of relativity at high energies or strong fields, where the "nonrelativistic" nature of the fundamental theory shows up, allows the high energy particles to communicate over a black hole event horizon and in this way it solves the paradoxes related to the horizon [4].

The idea that relativistic fields are low energy excitations of a condensed matter system is older than the relativistic fields themselves. Maxwell derived his famous equations as a hydrodynamic description of a hypothetical ether. Later on, the Michelson-Morley experiment proved that there is no detectable motion of the Earth with respect to the ether. The fundamental condensed matter system is an ether but in a modern guise. There is an essential difference with respect to the traditional ether: now "everything", i.e. both light and fermionic matter (including the famous Michelson and Morley's experimental setup), are effective low energy bosonic and fermionic relativistic excitations. The low energy relativistic excitations cannot detect their motion with respect to the modern ether and there are no fundamental relativistic fields, otherwise we would have to deal again with the time operator and the cosmological constant problem. The condition that the low energy excitations must include relativistic fermionic quasiparticles strongly suggests that the underlying condensed matter system must contain fundamental "nonrelativistic" fermions.

Analogies between the black hole horizon and sonic horizons in a number of condensed matter systems were explored in Refs. [6]. Analogies between *fermionic* helium 3 and the standard model plus general relativity were explored in depth in Ref. [2]. See Ref. [7] for yet another example of a fermionic system with effective gauge and gravity fields. Of particular interest in the present context are two phases of the superfluid helium 3: the A phase and the planar phase [2]. In a conventional superconductor and in the B phase of helium 3 there is an energy gap Δ_0 between the Landau quasiparticles below the Fermi surface, where $p = p_F$, and those above the Fermi surface. In the A phase and the planar phase this gap has two nodes at the so called Fermi points on the Fermi surface. Order parameter includes a unit vector $\hat{\mathbf{l}}$ related to orbital angular momentum of the atoms. The two Fermi points are located at $\mathbf{p} = \pm p_F\hat{\mathbf{l}}$. Close to the Fermi point, say, $\mathbf{p} = p_F\hat{\mathbf{l}}$ the energy $e_{\mathbf{p}}$ of the fermionic Landau quasiparticles can be approximated by

$$e_{\mathbf{p}}^2 \; + \; g^{ab}(p_a - p_F l_a)(p_b - p_F l_b) \; \approx \; 0, \qquad (1)$$

where the indices a, b run over $1, 2, 3$ (or x, y, z). This spectrum is relativistic, there are low energy effective Dirac fermions in this system.

In general, the metric tensor depends on $\hat{\mathbf{l}}$,

$$g^{00} \; = \; 1 \, , -g^{ab} \; = \; c_F^2 \, l^a l^b \; + \; c_\perp^2 \, (\delta^{ab} - l^a l^b) \; , \qquad (2)$$

where c_F is an effective Fermi velocity and $c_\perp = \Delta_0/p_F$ is a transversal velocity of the fermionic quasiparticles near a Fermi point. However, in a *symmetric* case, when

$$c_\perp \; = \; c_F \; , \qquad (3)$$

the metric tensor becomes independent of $\hat{\mathbf{l}}$,

$$g^{\mu\nu} = \mathrm{diag} \left\{ 1, -c_F^2, -c_F^2, -c_F^2 \right\} \; . \qquad (4)$$

As noted in Ref. [2] the $p_F\hat{\mathbf{l}}$ in Eq.(1) can be interpreted as an electromagnetic vector potential and integration over the relativistic fermions should give an effective electromagnetic action for these gauge field. This integration over an equilibrium low temperature ensamble of fermions is a subject of the next Section. This derivation shows how an effective relativistic electrodynamics emerges from an underlying fermionic condensed matter system.

The derivation in the next Section generalizes the classic helium 3 results for $c_F \gg c_\perp$ obtained by Cross in Ref. [8]. The symmetric case $c_F = c_\perp$ is far from the real helium 3. However, it should be possible to construct an abstract symmetrized helium 3 with interactions tuned so as to have a stable phase A and $c_F = c_\perp$ at the same time. The fundamental condensed matter system does not need to be constrained by the generic properties of interactions in the electronic or atomic condensed matter systems. The aim of this paper is to better substantiate the idea [2] that the *relativistic* electrodynamics can be an effective low energy theory in a "*nonrelativistic*" fermionic condensed matter system.

2. The effective electromagnetism. *2.1. Bogolubov-Nambu space.* To describe helium 3 it is convenient to combine spin-up and spin-down fermions into a Bogolubov-Nambu spinor

$$\chi(\mathbf{x}) = \begin{pmatrix} \psi_\uparrow(\mathbf{x}) \\ \psi_\downarrow(\mathbf{x}) \\ \psi_\downarrow^\dagger(\mathbf{x}) \\ -\psi_\uparrow^\dagger(\mathbf{x}) \end{pmatrix} . \qquad (5)$$

It is understood here that $\mathbf{p} = -i\nabla$ and the nabla is applied to the $\chi(\mathbf{x})$ on the right. A mean field Hamiltonian that describes interaction of the fermionic atoms with the order parameter in the phase A of ^{3}He is given by

$$H = \frac{1}{2} \int d^3x \; \chi^\dagger(\mathbf{x}) \begin{pmatrix} +\epsilon_{\mathbf{p}} & \Delta_0^* \sigma \dfrac{p_\perp^*}{p_F} \\ \Delta_0 \sigma \dfrac{p_\perp}{p_F} & -\epsilon_{\mathbf{p}} \end{pmatrix} \chi(\mathbf{x}). \quad (6)$$

Here $\Delta_0(\mathbf{x})$ is the energy gap and p_F is the Fermi momentum. $\epsilon(\mathbf{p})$ is a quasiparticle energy, which can be approximated close to the Fermi surface by

$$\epsilon_{\mathbf{p}} \approx \frac{p^2}{2m_*} - \frac{p_F^2}{2m_*} = \frac{(p + p_F)(p - p_F)}{2m_*} \approx c_F \; (p - p_F), \qquad (7)$$

where $c_F = p_F/m_*$ is a Fermi velocity and m_* is an effective mass of Landau quasiparticles close to the Fermi surface.

$$\sigma(\mathbf{x}) \equiv d^\mu(\mathbf{x}) \sigma_\mu \qquad (8)$$

with $d^\mu d^\mu = 1$ is a 2×2 spin matrix.

$$p_\perp(\mathbf{x}) \equiv \frac{1}{2} \{ e_1^a(\mathbf{x}) + i e_2^a(\mathbf{x}), p^a \}, \qquad (9)$$

where summation runs over $a = 1, 2, 3$, and $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ satisfy

$$\hat{\mathbf{e}}_1 \hat{\mathbf{e}}_1 = 1, \quad \hat{\mathbf{e}}_2 \hat{\mathbf{e}}_2 = 1, \quad \hat{\mathbf{e}}_1 \hat{\mathbf{e}}_2 = 0, \quad \hat{\mathbf{l}} = \hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2. \qquad (10)$$

2.2 Background order parameter. We will derive an effective action for small fluctuations of the order parameter around the equilibrium order parameter

$$\Delta_0(\mathbf{x}) = \Delta_0 \in \mathcal{R}, \quad \sigma(\mathbf{x}) = \sigma_3, \qquad (11)$$

$$\hat{\mathbf{e}}_1(\mathbf{x}) = \hat{\mathbf{e}}_x, \quad \hat{\mathbf{e}}_2(\mathbf{x}) = \hat{\mathbf{e}}_y, \quad \hat{\mathbf{l}}(\mathbf{x}) = \hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_y = \hat{\mathbf{e}}_z, \qquad (12)$$

$$p_\perp = p_x + i p_y. \qquad (13)$$

With this background the Hamiltonian (6) becomes

$$H_0 = \frac{1}{2} \int d^3x \; \chi^\dagger(\mathbf{x}) \begin{pmatrix} +\epsilon_{\mathbf{p}} & \Delta_0 \sigma_3 \dfrac{p_\perp^*}{p_F} \\ \Delta_0 \sigma_3 \dfrac{p_\perp}{p_F} & -\epsilon_{\mathbf{p}} \end{pmatrix} \chi(\mathbf{x}). \qquad (14)$$

2.3. Bogolubov transformation. The Hamiltonian (14) is diagonalized by a Bogolubov transformation

$$\psi_\uparrow(\mathbf{p}) = u_\mathbf{p}\gamma_\uparrow(\mathbf{p}) + v_\mathbf{p}\gamma_\downarrow^\dagger(-\mathbf{p}), \tag{15}$$

$$\psi_\downarrow(\mathbf{p}) = u_\mathbf{p}\gamma_\downarrow(\mathbf{p}) + v_\mathbf{p}\gamma_\uparrow^\dagger(-\mathbf{p}), \tag{16}$$

where the Bogolubov coefficients $u_\mathbf{p}$ and $v_\mathbf{p}$ satisfy

$$|u_\mathbf{p}|^2 = \frac{1}{2}\left(1+\frac{\epsilon_\mathbf{p}}{e_\mathbf{p}}\right),\quad |v_\mathbf{p}|^2 = \frac{1}{2}\left(1-\frac{\epsilon_\mathbf{p}}{e_\mathbf{p}}\right), \tag{17}$$

$$2u_\mathbf{p}v_\mathbf{p} = \frac{c_\perp p_\perp}{e_\mathbf{p}},\quad e_\mathbf{p} = \left(\epsilon_\mathbf{p}^2 + c_\perp^2|p_\perp|^2\right)^{1/2}. \tag{18}$$

Here we define $c_\perp \equiv \Delta_0/p_F$. The diagonalized Hamiltonian (14) is

$$H_0 = \int d^3p\; e_\mathbf{p}\left[\gamma_\uparrow^\dagger(\mathbf{p})\gamma_\uparrow(\mathbf{p}) + \gamma_\downarrow^\dagger(\mathbf{p})\gamma_\downarrow(\mathbf{p})\right]. \tag{19}$$

Close to the Fermi point at $\mathbf{p} = \pm p_F\hat{\mathbf{l}}$ the energy squared of the quasiparticles can be approximated by Eq. (1), compare with Eqs. (7), (18), g^{ab} is a spatial part of a metric tensor

$$g^{\mu\nu} = \mathrm{diag}\left\{1, -c_\perp^2, -c_\perp^2, -c_F^2\right\}. \tag{20}$$

2.4. Small fluctuations of $\hat{\mathbf{l}}$. We add small perturbations to the background field (11), (12)

$$\hat{\mathbf{e}}_1(\mathbf{x}) = \hat{\mathbf{e}}_x + \mathbf{n}_1(\mathbf{x}),\quad \hat{\mathbf{e}}_2(\mathbf{x}) = \hat{\mathbf{e}}_y + \mathbf{n}_2(\mathbf{x}) \tag{21}$$

and define a small complex vector field

$$z_a(\mathbf{x}) \equiv n_1^a(\mathbf{x}) + i\, n_2^a(\mathbf{x}). \tag{22}$$

The Hamiltonian (6) becomes $H = H_0 + H_1 + \mathcal{O}(z^2)$ with

$$H_1 = \int d^3p\; \left[z_a^*(\mathbf{p})F_\mathbf{p}^a + \mathrm{h.c.}\right], \tag{23}$$

an interaction Hamiltonian linear in z_a. Here we use the Fourier transform

$$z_a(\mathbf{p}) = \int \frac{d^3x}{(2\pi)^3}\, e^{-i\mathbf{x}\mathbf{p}}\, z_a(\mathbf{x}), \tag{24}$$

and the operator

$$\begin{aligned} F_\mathbf{p}^a[\gamma] \equiv & -\Delta_0 \textstyle\int d^3k\; \frac{k^a}{p_F}\; \times \\ & \times \Big[\, u_{\frac{\mathbf{p}}{2}+\mathbf{k}}u_{\frac{\mathbf{p}}{2}-\mathbf{k}}\gamma_\downarrow(\frac{\mathbf{p}}{2}+\mathbf{k})\gamma_\uparrow(\frac{\mathbf{p}}{2}-\mathbf{k}) \quad + \\ & + v_{\frac{\mathbf{p}}{2}+\mathbf{k}}v_{\frac{\mathbf{p}}{2}-\mathbf{k}}\gamma_\uparrow^\dagger(-\frac{\mathbf{p}}{2}-\mathbf{k})\gamma_\downarrow^\dagger(-\frac{\mathbf{p}}{2}+\mathbf{k}) + \\ & + u_{\frac{\mathbf{p}}{2}+\mathbf{k}}v_{\frac{\mathbf{p}}{2}-\mathbf{k}}\gamma_\downarrow(\frac{\mathbf{p}}{2}+\mathbf{k})\gamma_\downarrow^\dagger(-\frac{\mathbf{p}}{2}+\mathbf{k}) + \\ & + u_{\frac{\mathbf{p}}{2}-\mathbf{k}}v_{\frac{\mathbf{p}}{2}+\mathbf{k}}\gamma_\uparrow^\dagger(-\frac{\mathbf{p}}{2}-\mathbf{k})\gamma_\uparrow(\frac{\mathbf{p}}{2}-\mathbf{k})\Big]. \end{aligned} \tag{25}$$

2.5. Second order effective action. A real (unitary) part of the second order effective action is

$$\begin{aligned} & S^{(2)}[z] = \\ & = \mathrm{Re}\,\frac{i}{2}\int dt\, dt'\, \langle \hat{T} H_1[\gamma_+(t)] H_1[\gamma_+(t')]\rangle = \\ & = \mathrm{Re}\,\frac{i}{2}\int dt\, dt' \int d^3p\, d^3p' \times \\ & \times \left[2 z_a(t,\mathbf{p})\, \langle \hat{T} F_\mathbf{p}^{\dagger a}[\gamma_+(t)] F_{\mathbf{p}'}^b[\gamma_+(t')]\rangle z_b^*(t',\mathbf{p}') + \right. \\ & + \left(z_a^*(t,\mathbf{p})\, \langle\, \hat{T}\, F_\mathbf{p}^a[\gamma_+(t)]\; F_{\mathbf{p}'}^b[\gamma_+(t')]\,\rangle\; \times \right. \\ & \left.\left. \times\, z_b^*(t',\mathbf{p}') + \mathrm{c.c.}\right)\right], \end{aligned} \tag{26}$$

where $\hat{T}$ means time ordering along the Kyeldysh contour. The interaction picture $\gamma_+(t)$'s sit on the positive (forward in time) branch of the contour. A straightforward but somewhat tedious calculation, which uses a correlator time ordered along the contour

$$\begin{aligned} & \langle \hat{T}[\gamma_+(t,\mathbf{k})\gamma_+^\dagger(t',\mathbf{k}')]\rangle = \delta(\mathbf{k}-\mathbf{k}')\; e^{-ie_\mathbf{k}(t-t')} \times \\ & \times \left[\theta(t-t')f(-\beta e_\mathbf{k}) - \theta(t'-t)f(+\beta e_\mathbf{k})\right], \end{aligned} \tag{27}$$

with $f(x) = (1+e^x)^{-1}$ and β an inverse temperature, gives an effective action

$$\begin{aligned} & S^{(2)}[z] = \int d\omega \int d^3p\; \times \\ & \times \left[z_a(\omega,\mathbf{p})\; G_1^{ab}(\omega,\mathbf{p}) z_b^*(\omega,\mathbf{p}) + \right. \\ & \left. + \left(z_a^*(\omega,\mathbf{p})\; G_2^{ab}(\omega,\mathbf{p}) z_b^*(-\omega,-\mathbf{p}) + \mathrm{c.c.}\right)\right]. \end{aligned} \tag{28}$$

The kernels are given by

$$\begin{aligned} & G_1^{ab}(\omega,\mathbf{p}) = 2\pi\Delta_0^2\, \mathrm{P.V.} \int d^3k \frac{k^a k^b}{p_F^2} \times \\ & \frac{2\sinh(\beta e_\mathbf{k})}{1+\cosh(\beta e_\mathbf{k})} \times \\ & \times \left[\frac{|u_{\mathbf{k}+\frac{\mathbf{p}}{2}}|^2\; |u_{\mathbf{k}-\frac{\mathbf{p}}{2}}|^2}{+\omega + e_{\mathbf{k}+\frac{\mathbf{p}}{2}} + e_{\mathbf{k}-\frac{\mathbf{p}}{2}}} + \frac{|v_{\mathbf{k}+\frac{\mathbf{p}}{2}}|^2\; |v_{\mathbf{k}-\frac{\mathbf{p}}{2}}|^2}{-\omega + e_{\mathbf{k}+\frac{\mathbf{p}}{2}} + e_{\mathbf{k}-\frac{\mathbf{p}}{2}}}\right] \end{aligned} \tag{29}$$

and

$$\begin{aligned} & G_2^{ab}(\omega,\mathbf{p}) = -\pi\Delta_0^2\, \mathrm{P.V.} \int d^3k\; \frac{k^a k^b}{p_F^2} \times \\ & \times \frac{2\sinh(\beta e_\mathbf{k})}{1+\cosh(\beta e_\mathbf{k})} \times (u_{\mathbf{k}+\frac{\mathbf{p}}{2}}v_{\mathbf{k}+\frac{\mathbf{p}}{2}})(u_{\mathbf{k}-\frac{\mathbf{p}}{2}}v_{\mathbf{k}-\frac{\mathbf{p}}{2}}) \times \\ & \times \left[\frac{1}{+\omega + e_{\mathbf{k}+\frac{\mathbf{p}}{2}} + e_{\mathbf{k}-\frac{\mathbf{p}}{2}}} + \frac{1}{-\omega + e_{\mathbf{k}+\frac{\mathbf{p}}{2}} + e_{\mathbf{k}-\frac{\mathbf{p}}{2}}}\right]. \end{aligned} \tag{30}$$

Here we neglect terms that are exponentially small for small temperature. In order to get a low energy effective theory, these kernels will be (gradient) expanded in powers of ω and $\mathbf{p}$.

2.6. Gradient expansion of G^{33}. A gradient expansion of G^{33} gives terms which are logarithmically divergent when $\beta \to \infty$. This divergence, localized at the Fermi points $\mathbf{k} = \pm p_F \hat{\mathbf{l}}$, can be identified as

$$G^{33}_{1,\mathrm{Log}}(\omega, \mathbf{p}) = = \frac{4\pi^2 \Delta_0^2}{3} \left[\omega^2 - \tfrac{1}{2} c_\perp^2 (p_x^2 + p_y^2) - c_F^2 (p_z^2)\right] \ln(\beta\Delta_0) \quad (31)$$

and

$$G^{33}_{2,\mathrm{Log}}(\omega, \mathbf{p}) = \frac{\pi^2 \Delta_0^2}{3} \left[c_\perp^2 p_\perp^2\right] \ln(\beta\Delta_0). \quad (32)$$

After inverse Fourier transform we obtain the logarithmically divergent part of the second order effective action

$$S^{(2)}_{\mathrm{Log}}[\mathbf{n}] = \frac{p_F^2 \ln\left(\frac{\Delta_0^2}{T^2}\right)}{24\pi^2\, c_F} \int d^4x \times \quad (33)$$

$$\left\{ \sum_{k=1,2} \left[\left(\frac{\partial n_k^3}{\partial t}\right)^2 - c_F^2 \left(\frac{\partial n_k^3}{\partial z}\right)^2 \right]^2 - c_\perp^2 \left[\partial_x n_2^3 - \partial_y n_2^3\right]^2 \right\}.$$

This action is a second order perturbative version of an action

$$S^{(2)}_{\mathrm{Log}}[\mathbf{l}] = \frac{p_F^2 \ln\left(\frac{\Delta_0^2}{T^2}\right)}{24\pi^2\, c_F} \int d^4x \times$$
$$\times \left\{ \left[\frac{\partial \mathbf{l}}{\partial t}\right]^2 - c_F^2 \left[\mathbf{l} \times (\nabla \times \mathbf{l})\right]^2 - c_\perp^2 \left[\mathbf{l}(\nabla \times \mathbf{l})\right]^2 \right\}. \quad (34)$$

Fluctuations of $\hat{\mathbf{l}}$ are not the only contribution to the logarithmically divergent part of the low energy effective action. Another contribution comes from the component of the superfluid velocity $\mathbf{v}$ which is parallel to $\hat{\mathbf{l}}$.

2.7. Small fluctuations of $\left(\hat{\mathbf{l}}\mathbf{v}\right)$. For a *uniform* stationary superfluid flow with velocity $\mathbf{v}$ and close to the Fermi surface, $p \approx p_F$, the Hamiltonian (14) becomes

$$H_0 = \frac{1}{2} \int d^3x\, \chi^\dagger(\mathbf{x}) \times \quad (35)$$

$$\times \begin{pmatrix} +\epsilon_{\mathbf{p}+m_*\mathbf{v}} + \frac{1}{2} m_* v^2 & \Delta_0 \sigma_3 \frac{(p_\perp^* + m_* v_\perp^*)}{p_F} \\ \Delta_0 \sigma_3 \frac{(p_\perp - m_* v_\perp)}{p_F} & -\epsilon_{\mathbf{p}-m_*\mathbf{v}} - \frac{1}{2} m_* v^2 \end{pmatrix} \chi(\mathbf{x}),$$

compare with Eqs. (6), (7) and use a Galilean transformation. Here $v_\perp \equiv v_x + i v_y$. We are interested in the part of the Hamiltonian (35) that is linear in $\mathbf{v}$ and we expand

$$\epsilon_{\mathbf{p}+m_*\mathbf{v}} = \epsilon_{\mathbf{p}} + \mathbf{p}\mathbf{v} + \mathcal{O}(v^2)\,. \quad (36)$$

So far $\mathbf{v}$ was constant. Now we make it space and time dependent, $\mathbf{v} = \mathbf{v}(t, \mathbf{x})$, and at the same time, to keep the Hamiltonian (35) hermitian, we make in Eq. (36) a replacement

$$\mathbf{p}\mathbf{v} \to \frac{1}{2} \{\mathbf{p}, \mathbf{v}(t, \mathbf{x})\} = \frac{1}{2} [\mathbf{p}\mathbf{v}(t, \mathbf{x})] + \mathbf{v}(t, \mathbf{x})\mathbf{p}\,. \quad (37)$$

We expand the Hamiltonian (35) to leading order in $\mathbf{v}$ using Eq. (36) and the replacement (37). In the expanded Hamiltonian we keep only terms where the operator $\mathbf{p}$ is applied to χ or $\chi^\dagger$. As the main contribution to the logarithmically divergent part of the effective action comes from near the Fermi points at $\mathbf{p} = \pm p_F \hat{\mathbf{l}}$, these terms are formally of the order of p_F. They are large as compared to terms where the operator $\mathbf{p}$ is applied to the slowly varying velocity field $\mathbf{v}$. After those last terms are neglected the interaction Hamiltonian becomes

$$H_1 \approx \frac{1}{2} \int d^3x\, \chi^\dagger(\mathbf{x}) \; [v^a(\mathbf{x}) \mathbf{p}_\mathbf{a}] \; \chi(\mathbf{x})\,. \quad (38)$$

This Hamiltonian is hermitian when we take into account that $p_a \mathbf{v}(\mathbf{x})$ is negligible as compared to $p_a \chi^{(\dagger)}$. With the definition (5) and the Bogolubov transformation (16) the Hamiltonian becomes

$$H_1 \approx \int d^3p \; \left[v_a^*(\mathbf{p})\; f^a_{\mathbf{p}} + \mathrm{h.c.}\right] \quad (39)$$

where

$$f^a_{\mathbf{p}} \equiv \int d^3k\, k_a \times$$
$$\times \left(u_{\mathbf{k}+\frac{\mathbf{p}}{2}} v^*_{\mathbf{k}-\frac{\mathbf{p}}{2}} - u_{\mathbf{k}-\frac{\mathbf{p}}{2}} v^*_{\mathbf{k}+\frac{\mathbf{p}}{2}}\right) \gamma_\downarrow(-\mathbf{k}) \gamma_\uparrow(+\mathbf{k}). \quad (40)$$

Here we neglect all mixed terms of the form $\gamma^\dagger\gamma$ which for small T give an exponentially small contribution to the effective action. The effective action is given by

$$S^{(2)}_{\mathrm{Log}}[\mathbf{v}] \approx \mathrm{Re}\frac{i}{2} \int dt dt' \int d^3p d^3p' \times$$
$$\times\, 2 v_a(t, \mathbf{p}) \langle \hat{T} f^{\dagger a}_{\mathbf{p}} [\gamma_+(t)] f^b_{\mathbf{p}'} [\gamma_+(t')] \rangle v_b^*(t', \mathbf{p}'), \quad (41)$$

compare to Eq. (26). A straightforward calculation similar to the derivation of the effective action for small fluctuations of $\hat{\mathbf{l}}$ gives

$$S^{(2)}_{\mathrm{Log}}(\mathbf{v}) = \frac{p_F^2 \ln\left(\Delta_0^2/T^2\right)}{24\pi^2} \int \sqrt{-g}\, d^4x \left(-g^{ab} \partial_a v_3 \partial_b v_3\right). \quad (42)$$

The logarithmically divergent part of the effective action (42) contains only v_3 because at the Fermi points it is only v_3 that couples to $\mathbf{p} = \pm p_F \hat{\mathbf{l}} = \pm p_F \hat{\mathbf{e}}_z$, compare Eq. (38).

2.8. The electromagnetic effective action. After identifications

$$A_0 = p_F\left(\mathbf{l}\mathbf{v}\right), \quad \mathbf{A} = p_F\hat{\mathbf{l}} \tag{43}$$

a sum of the two actions (34), (42) becomes

$$S^{(2)}_{\mathrm{Log}} = \frac{\ln\left(\Delta_0^2/T^2\right)}{12\pi^2}\int\sqrt{-g}\,d^4x\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right), \tag{44}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $g^{\mu\nu}$ is given by Eq. (20).

In the symmetric case the metric tensor (4) does not depend on $\hat{\mathbf{l}}$. A_μ uncouples from a vector potential and becomes an independent electromagnetic field.

3. Conclusion. The effective action (44) and the identifications (43) agree with the effective action and the identifications that were suggested in Ref. [2].

The effective action (44) is generalization of the classic results of Cross [8] to arbitrary $c_F/c_\perp$. This generalization makes manifest relativistic invariance present in the effective action.

I would like to thank Grisha Volovik for his contribution to this work, Pawel Mazur for his informal one audience lectures, and Diego Dalvit for his comments on an earlier draft of this manuscript.

1. S. W. Hawking, Nature **248**, 30 (1974); S. W. Hawking, Phys. Rev. **D13**, 191 (1976).
2. G. E. Volovik, Phys. Rep.**351**, 195 (2001).
3. G. E. Volovik, J. Low. Temp. Phys. **124**, 25 (2001).
4. U. R. Fisher and G. E. Volovik, Int. J. Mod. Phys. **D10**, 57 (2001); R. Laughlin and D. Pines, Proc. Natl. Acad. Sc. **97**, 28 (2000); P. O. Mazur and E. Mottola, gr-qc/0109035, to appear in Phys. Rev. Lett.; P. Huhtala and G. E. Volovik, gr-qc/0111055.
5. A. Sakharov, Soviet Physics Doklady **12**, 1040 (1968).
6. W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981); Phys. Rev. **D51**, 2827 (1995); T. Jacobson and G. E. Volovik, Phys. Rev. **D58**, 064021 (1998); L. J. Garay et al., Phys. Rev. Lett. **85**, 4643 (2000); Phys. Rev. **A63**, 023611 (2001); M. Mohazzab, J. Low. Temp. Phys. **121**, 659 (2000). G. Chapline, E. Hohlfield, R. B. Laughlin, and D. I. Santiago, Phil. Mag. **B81**, 235 (2001); U. Leonhardt, Nature **415**, 406 (2002).
7. S. C. Zhang and J. Hu, Science **294**, 823 (2001).
8. M. C. Cross, J. Low. Temp. Phys. **21**, 525 (1975).