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Entangled state generation and modulation of vacuum squeezing by classical optical fields in birefringent fibre

S. A. Podoshvedov[1)]

Department of Physics, Inha University, Incheon, 402-751 South Korea

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Evolution of the vacuum fluctuations in the two-wave mixing of the optical fields propagating in the birefringent fibre is studied. The two-wave mixing in the birefringent fibre has been suggested as a possible scheme for the entangled state generation. Our treatment to study the entangled state generation uses undepleted pump approximation and enables to trace influence of input conditions of the classical optical fields on evolution of the vacuum squeezing. We report the periodical modulation of the vacuum squeezing when input relative phase of the coherent waves varies. Measure of nonclassical correlations imposed on the generated light is calculated.

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Entanglement shared among many particles plays an important role in various schemes of quantum information processing, such as test s of Bell's inequalities and Einstein-Podolsky-Rosen "paradox" [1], quantum teleportation [2], quantum computing [3], and quantum cryptography [4]. Spontaneous parametric down-conversion (SPDC), the nonlinear optical process in which a pump is converted into pair of photons called signal and idler, is considered as main source of the strongly correlated photons [5]. Nevertheless we guess other parametric processes may be studied for the correlated photons to be generated. In the paper, we study the properties of the nonclassical light generated in the parametric two-wave mixing of the optical fields that propagate through the single-mode birefringent fibre using linearization treatment [6, 7]. Given treatment enables to allow for influence of the input conditions of the classical optical fields and the parameters of the medium on the properties of the generated state. We show that vacuum squeezing undergoes periodical modulation under change of the input relative phase of the coherent optical fields.

In order to treat with quantum noise in optical medium, it is necessary to introduce a suitable Hamiltonian comprising quantum operators. We choose the model of the two continuous waves (cw) propagating in the single-mode birefringent fibre described by the nonlinear nonlinear susceptibility $\chi^{(3)}$, which is invariant over a finite frequency range. This Hamiltonian is given by

$$H = c\eta\left(\frac{R}{6}(\hat{A}_1^{+2}\hat{A}_2^2+\hat{A}_2^{+2}\hat{A}_1^2) + \frac{R}{2}(\hat{A}_1^{+2}\hat{A}_1^2 + \hat{A}_2^{+2}\hat{A}_2^2)+ + \frac{2R}{3}\hat{A}_1^+\hat{A}_2^+\hat{A}_1\hat{A}_2\right) + \Delta k\hat{A}_2^+\hat{A}_2, \tag{1}$$

where $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_1^+$, and $\hat{A}_2^+$ are the annihilation and creation field operators orthogonally polarized along x and y directions of the fibre and obeying well-known harmonic-oscillator commutation relations $\{\hat{A}_i;\, \hat{A}_j^+\} = \delta_{ij}$, $\{\hat{A}_i;\, A_j\} = 0$, $\{\hat{A}_i^+;\, \hat{A}_j\} = 0$ $(i, j = 1, 2)$. Coefficient $R = n_2\eta\omega^2/c^2V2\varepsilon_0A_{\text{eff}}$ is the nonlinear coupling constant that characterizes strength of the nonlinear interaction in the fibre, $\omega = 2\pi c/\lambda$, λ is the wavelength of the interacting fields, $n_2 = 3.2\cdot10^{-16}\,\text{cm}^2/\text{W}$ is the nonlinear refraction index of the fibre, $\varepsilon_0 = 8.85\cdot10^{-12}\,\text{F/m}$ is the electrical constant, V is the volume of the quantized modes, $A_{\text{eff}} \cong \pi r_c^2$ is the effective area of the single-mode fibre [8], r_c is the radius of the core, and $\Delta k = k_y - k_x$ is the wave mismatch between the wave vectors. The nonlinear operator equations are derived from Heizenberg equation $i\eta\frac{d\hat{A}_i}{dt} = \{\hat{A}_i;\, H\}$ and are given by

$$\frac{d\hat{A}_1}{dz} = iR\left(\frac{1}{3}\hat{A}_1^+\hat{A}_2^2 + \left(\hat{A}_1^+\hat{A}_1 + \frac{2}{3}\hat{A}_2^+\hat{A}_2\right)\hat{A}_2\right),$$

$$\frac{d\hat{A}_2}{dz} = = iR\left(\frac{1}{3}\hat{A}_2^+\hat{A}_1^2 + \left(\frac{2}{3}\hat{A}_1^+\hat{A}_1 + \hat{A}_2^+\hat{A}_2\right)\hat{A}_2\right) + \Delta k\hat{A}_2. \tag{2}$$

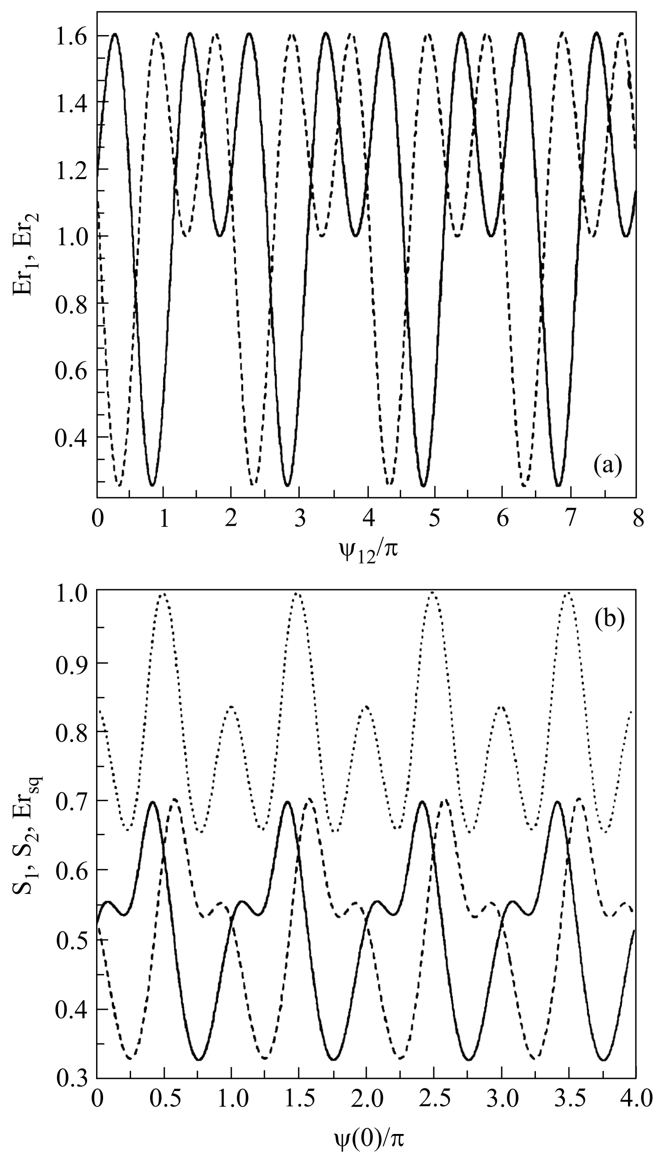
It is not clear outright whether the operator equations (2) can be solved in the terms of some analytical func-

[1)]e-mail: podoshvedov@mail.ru

tions. This question is coming up for discussion because the classical equations analogous to the quantum equations (2) are solved [9]. For this reason, one should use approximate methods in analysing the nonlinear operator equations. We use linearization treatment [6, 7] to study the evolution of the quantum fluctuations in the birefringent fibre. We consider the vacuum fluctuations as small perturbations as compared with mean values of the fields [6, 7]. The reason to consider the quantum operators like is simple. A stabilized laser, for example He–Ne laser, is known to emit almost coherent light with properties very similar to the properties of the classical field with quantum zero-point fluctuations (vacuum noise) superposed. This enables to decompose the quantum operators two parts as $\widehat{A}_1 = \langle\widehat{A}_1\rangle + \widehat{a}_1$, $\widehat{A}_2 = \langle\widehat{A}_2\rangle + \widehat{a}_2$, [6, 7], where $\langle\widehat{A}_1\rangle$, $\langle\widehat{A}_2\rangle$ are the expected values of the operators that can be obtained by averaging of the operators on the initial coherent states. The operators $\widehat{a}_1$ and $\widehat{a}_2$ describe evolution of the vacuum quantum noises in the birefringent fibre. Nonlinear equations for expectation values of the quantum operators are derived in zero order in vacuum operators if we make use of assumption conditions $\langle\widehat{A}_i^+\widehat{A}_j\widehat{A}_k\rangle = \langle\widehat{A}_i^+\rangle\langle\widehat{A}_j\rangle\langle\widehat{A}_k\rangle$ $(i,\, j = 1,\, 2)$ to be carried out. As a matter of fact this means that we use classical waves of pump. The equations in zero order in operator $\widehat{a}_i$ $(i = 1,\, 2)$ are given in the terms of quantities η and ψ by [7] ,

$$\frac{d\eta}{ds} = \tau\frac{2}{3}\eta(1-\eta)\sin(2\psi),$$
$$\frac{d\psi}{ds} = \tau\left(\frac{1}{3}(1-2\eta)\cos(2\psi) + \frac{2}{3}\eta - \frac{1}{3} + \frac{k}{2}\right), \quad (3)$$

where $\eta = |q_2|^2 = |E_2|^2/P$, $|q_1|^2 = 1 - \eta = |E_1|^2/P$, $\psi = \varphi_2 - \varphi_1$ is the difference of the phases of the normalized field amplitudes $q_1 = |q_1|\exp(i\varphi_1)$ and $q_2 = |q_2|\exp(i\varphi_2)$, $P \sim N$ is the conserved power, the quantity $N = |\langle\widehat{A}_1\rangle|^2 + |\langle A_2\rangle|^2$ is the total number of photons in the interacting fields, the quantity $\tau = L/L_{nl} = LPn_2\omega/cA_{\text{eff}}$ is the ratio of the full fibre length to the interaction one, $s = z/L$ is the dimensionless longitudinal length (s is changed from 0 up to 1) and $k = 2\Delta kL$ is the dimensionless mismatch. The quantities E_1 and E_2 are the field amplitudes module square of which is measured in units of W/cm^2. The equations (3) are the integrable Hamiltonian ones [9]. Equations for vacuum fields operators are produced when only linear terms on $\widehat{a}_1$, $\widehat{a}_2$ are kept and the conditions $\langle\widehat{A}_i^+\widehat{A}_i\rangle = \langle\widehat{A}_i^+\rangle\langle\widehat{A}_i\rangle$, $\langle\widehat{A}_i\widehat{A}_i\rangle = \langle\widehat{A}_i\rangle\langle\widehat{A}_i\rangle$, are effected. We present the equations in matrix form for vector column of the small operators $\mathbf{a} = (\widehat{a}_1,\, \widehat{a}_1^+,\, \widehat{a}_2,\, \widehat{a}_2^+)^T$ as $d\mathbf{a}/ds = i\tau G^a\mathbf{a}$, where $G^a(s)$ is the evolution matrix for the input column $\mathbf{a}(0)$ to be transformed to $\mathbf{a}(s)$.



Dependence of the quantities Er_1, Er_2 on relative normalised phase ψ_{l2}/π related to the phase of the second local oscillator (a) and vacuum squeezing S_1, S_2 and corresponding degree of the correlation Er_{sq} of the quantities S_1 and S_2 (b) on the input normalised difference of the phases $\psi(0)/\pi$. The quantities Er_1 and Er_2 are shown by solid and dash curves in the Figure (a) and S_1, S_2 and Er_{sq} are shown by solid, dash and dot curves respectively in Figure (b). The plots are made under the following conditions $r_c = 1.5\,\mu$m, $\lambda = 1.55\,\mu$m, $L = 300$ m, $k = -0.1$, $P = 0.2$ W

Solution of the matrix equation $\mathbf{a}(s) = C^a(0)$ defines coefficients of the generated entangled state that may be expressed in general case with help of density operator ρ. In private case when $C_{14}^a = C_{32}^a = C^a$ that is performed under some input conditions, the pure two-particle entangled state can be written as $|\psi\rangle = C^a_{12}/\sqrt{2}|2_1\rangle|0_2\rangle + C^a_{34}/\sqrt{2}|2_2\rangle|0_1\rangle + C^a|1_1\rangle|1_2\rangle$, where $|C^a_{12}|^2/2 + |C^a_{34}|^2/2 + |C^a|^2 = 1$. The generated state is determined by the coefficients of the matrix C^a and because it depends on input conditions of the strong classical optical fields. Because we are interested in studying quadrature operators, we introduce transformation matrix $T^{Xa} = T^{aX-1}$ of passage of the column $\mathbf{a}$ to the quadrature operator column $\mathbf{X} = (\widehat{X}_1, \widehat{Y}_1, \widehat{X}_2, \widehat{Y}_2)^T$ ($\mathbf{X} = T^{(aX)-1}\mathbf{a}$) as

$$T^{(aX)-1} = \frac{1}{2}\left|\begin{array}{cccc} \exp(-i\varphi_1) & \exp(i\varphi_1) & 0 & 0 \\ -i\exp(-i\varphi_1) & i\exp(i\varphi_1) & 0 & 0 \\ 0 & 0 & \exp(-i\varphi_2) & \exp(i\varphi_2) \\ 0 & 0 & -i\exp(-i\varphi_2) & i\exp(i\varphi_2) \end{array}\right|, \tag{4}$$

where the symbol T^{-1} means matrix reverse to the matrix T. Introduction of the vector $\mathbf{X}$ is useful since one enables to get equations for $\widehat{X}$ in such a way that the new equations comprise only quantities η and ψ incurring Eqs.(4) [6]. To show it we make use of the transformation matrix (4) and find the equation for $\mathbf{X}$ [6] ,

$$d\mathbf{X}/ds = \tau G^X \mathbf{X}, \tag{5}$$

where

$$G^x = \left|\begin{array}{cccc} -\frac{1}{3}\eta\sin(2\psi) & \frac{2}{3}\eta\cos(2\psi) & -\frac{2}{3}\sqrt{\eta(1-\eta)}\sin(2\psi) & -\frac{2}{3}\sqrt{\eta(1-\eta)}\cos(2\psi) \\ 2(1-\eta) & \frac{1}{3}\eta\sin(2\psi) & \frac{2}{3}\sqrt{\eta(1-\eta)}(2+\cos(2\psi)) & -\frac{2}{3}\sqrt{\eta(1-\eta)}\sin(2\psi) \\ \frac{2}{3}\sqrt{\eta(1-\eta)}\sin(2\psi) & -\frac{2}{3}\sqrt{\eta(1-\eta)}\cos(2\psi) & \frac{1}{3}(1-\eta)\sin(2\psi) & \frac{2}{3}(1-\eta)\cos(2\psi) \\ \frac{2}{3}\sqrt{\eta(1-\eta)}(2+\cos(2\psi)) & \frac{2}{3}\sqrt{\eta(1-\eta)}\sin(2\psi) & 2\eta & -\frac{1}{3}(1-\eta)\sin(2\psi) \end{array}\right|.$$

Components of the quadrature operator vector $\mathbf{X}_i(s)$ are found according to $\mathbf{X}_i(s) = C^X_{ij}(s)\mathbf{X}_j(0)$ [6]. Expressions for squeezing (S_1, S_2) in the generated fields are produced in [6] in the terms of the matrix elements $C^X_{ij}(s)$ that are the solutions of the equation $dC^X_{ij}/ds = \tau G^X_{ik}C^X_{ij}$ satisfying the input condition $C^X_{ij}(0) = \delta_{ij}$. As the generated state is the entangled state, we may infer quadrature-phase information about one of the interacting optical fields only by measuring the quadrature component of other optical field. To evaluate error in our inferring value of the non-measured quadrature component, knowing information only about quadrature component of other field, we introduce quantities $Er_i = V_i(s)/V_i(0)$ ($i = 1, 2$) where $V_1\langle(\widehat{X}_{l1} - \widehat{X}_{l2})^2\rangle$, $V_2\langle(\widehat{X}_{l1} - \widehat{Y}_{l2})^2\rangle$, subscript l is related to the phase of the local oscillator and the sign $\langle\rangle$ implies averaging of the quantum operators over the vacuum state. Using the results of [6], one can produce expressions for Er_1 and Er_2 in terms of the matrix elements C^X_{ij}:

$$\begin{aligned} Er_1(\psi_{l1}, \psi_{l2}) = \frac{1}{2}(&\cos^2\psi_{l1}\sum_{i=1}^{4}C^{X2}_{1i} + \sin^2\psi_{l1}\sum_{i=1}^{4}C^{X2}_{2i} + \sin(2\psi_{l1})\sum_{i=1}^{4}C^X_{1i}C^X_{2i} + \cos^2\psi_{l2}\sum_{i=1}^{4}C^{X2}_{3i} + \\ &+ \sin^2\psi_{l2}\sum_{i=1}^{4}C^{X2}_{4i} + \sin(2\psi_{l2})\sum_{i=1}^{4}C^X_{3i}C^X_{4i}) - (\cos\psi_{l1}\cos\psi_{l2}\sum_{i=1}^{4}C^X_{1i}C^X_{3i} + \sin\psi_{l1}\sin\psi_{l2}\sum_{i=1}^{4}C^X_{2i}C^X_{4i} + \\ &+ \sin\psi_{l2}\cos\psi_{l1}\sum_{i=1}^{4}C^X_{1i}C^X_{4i} + \sin\psi_{l1}\cos\psi_{l2}\sum_{i=1}^{4}C^X_{2i}C^X_{3i}), \end{aligned} \tag{6}$$

where $\psi_{l1} = \varphi_{l1} - \varphi_1(L)$ and $\psi_{l2} = \varphi_{l2} - \varphi_2(L)$ are the relative phases, φ_{l1} and φ_{l2} are the phases of the local oscillators for the first and second optical fields respectively. Measurement of the odd-ordered moments imparts phase information on the components $\mathbf{X}_i$ and because requires homodyning of the studied field with reference field known as the local oscillator with the local phase φ_{li} ($i = 1, 2$). Other quantity $Er_2(\varphi_{l1}, \varphi_{l2})$ stems from $Er_1(\varphi_{l1}, \varphi_{l2})$ according to $Er_2(\varphi_{l1}, \varphi_{l2}) = Er_1(\varphi_{l1}, \varphi_{l2} + \pi/2)$. The less the quantities Er_1, Er_2 take their values the more correlated photons are generated and, on the contrary, if the quantities Er_1, Er_2 become more of 1, one may say about generation of the non-correlated photons. The quantities Er_1 and Er_2 depends on the relative phases φ_{l1}, φ_{l2} and input parameters of the coherent optical fields, for example on $\psi(0)$, through elements of C^X_{ij}. The dependence may be manifested in periodical dependence of the quantities Er_1 and Er_2 on the input varied conditions. We performed calculations to check our idea and the results are depicted in Figure. So, Figure (a) shows dependence of Er_1 and Er_2 on φ_{l2}/π provided that φ_{l1} is chosen in such a way that the vacuum squeezing S_1 has been by this time observed. The variance of other field varies with change of the local phase φ_{l2}. Figure (b) shows dependence of the squeezing in the first S_1 and second S_2 optical fields respectively and their joint correlation function Er_{sq} on input relative $\psi(0)$ phase of the classical waves. As can be seen from Figure (b), squeezing (S_1, S_2) and correlation function Er_{sq} undergo periodical modulation with change of the input relative phase $\psi(0)$.

In conclusion, we have shown that entangled state of the light is generated in the two-wave mixing of the optical fields propagating through the single-mode birefringent fibre from the vacuum initially not containing correlations. We have found degree of the correlation of the photons generated in the birefringent fibre and discovered the possibility of modulation of the vacuum squeezing by change of the input relative phase of the coherent waves.

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