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QED corrections to $\mathbf{A}(\mathbf{e}, e'B)X$ and $A(\mathbf{e}, e'\mathbf{B})X$ reactions. Case of tensor polarization

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The model–independent leading radiative corrections to polarization observables in semi–inclusive longitudinally–polarized electron–nucleus scattering with registration of a produced hadron and scattered electron in coincidence have been calculated using the Drell–Yan representation in electrodynamics. The cases of tensor–polarized target or produced hadron with tensor polarization have been considered. The exclusive process of the electrodisintegration of polarized deuteron has also been studied.

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1. During the last years much of the activity in the QCD investigation has shifted from the determination of quark distribution functions and cross–sections in leading order to the study of more detailed questions. This development went along with an increased emphasis on semi–inclusive (SI) reactions [1]. As a result of this interest some proposals to study these phenomena have been appeared in the last time. One of these is a new ELFE project [2]. ELFE, an Electron Laboratory For Europe, is a project to develop a 15 to 30 GeV, high luminosity, continuous beam electron accelerator for scattering experiments from fixed nuclear targets. The goal of this project is to explore the quark and gluon structure of matter by exclusive and SI electron scattering from nuclei. The availability of polarized electron beam and targets is of particular importance for the investigation of the internal spin structure of hadrons as the additional spin degrees of freedom allow to isolate specific quark–gluon correlators and other higher twist matrix elements [3].

Some problems of the electron–deuteron interaction can be investigated using the tensor–polarized deuterons. The tensor–polarized deuteron targets have been designed in a number of laboratories. The polarization observables due to the tensor polarization have been measured in the elastic ed-scattering [4]. The asymmetry in the reaction $\mathbf{d}(e, np)e'$ with tensor–polarized deuteron has been measured at the Novosibirsk [5].

The significance of the tensor–polarized target in deep–inelastic scattering (DIS) from the theoretical point of view has been considered in Ref. [6]. The tensor structure functions in polarized pd Drell–Yan process have been investigated in Ref. [7].

Current experiments at modern accelerators reached a new level of precision and this circumstance requires a new approach to data analysis and inclusion of all possible systematic uncertainties. One of the important source of such uncertainties is the electromagnetic radiative effects caused by physical processes which take place in higher orders of the perturbation theory with respect to the electromagnetic interaction. Earlier we calculated the radiative corrections (RC) to the polarization observables in DIS process (due to the tensor–polarized deuteron target) [8] and in SI DIS process (due to the vector–polarized target or detected hadron with vector polarization) [9][1)].

In present paper we give the covariant description of the polarization observables (due to the tensor polarization) in SI DIS of longitudinally–polarized electron beam off the tensor–polarized target (or production of tensor–polarized hadron)

$$e^-(k_1) + A(p_1) \to e^-(k_2) + B(p_2) + X(p_x) \qquad (1)$$

and we use the obtained results to calculate the model–independent QED RC by means of the electron structure function method using Drell–Yan representation [10] in electrodynamics.

We define the cross–section of the process (1), with accounting RC, in terms of the leptonic $L_{\mu\nu}$ and hadronic $H_{\mu\nu}$ tensors contraction

$$d\sigma = \frac{\alpha^2}{V(2\pi)^3}\frac{L_{\mu\nu}H_{\mu\nu}}{2q^4}\frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2}\,, \qquad (2)$$

where $V = 2k_1p_1$, ε_2 (E_2) is the energy of the scattered electron (detected particle B) and q is the 4–momentum of the virtual photon that probes the hadron block. Note that only in the Born approximation (without accounting RC) $q = k_1 - k_2$. The hadronic tensor can be ex-

[1)] Further we will use notation **I** for Ref. [9].

pressed via the hadron electromagnetic current J_μ of the $\gamma^* A \to BX$ transition (γ^* is the virtual photon)

$$H_{\mu\nu} = \sum_X \langle p_1|J_\mu(q)|p_2, X\rangle\langle X, p_2|J_\nu(-q)|p_1\rangle\delta(p_x^2 - M_x^2),$$

$$p_x = q + p_1 - p_2,$$

where p_x (M_x) is the total 4–momentum (invariant mass) of the undetected hadron system.

By definition, the model-independent RC includes the electromagnetic corrections to the leptonic piece of interaction only. Taking into account the leading contribution (terms proportional to $[\alpha\ln(Q^2/m_e^2)]^n$ in every order n of the perturbation theory), the leptonic tensor can be written in standard form by means of Drell-Yan representation in electrodynamics. This representation is defined by double integral of the contraction of two electron structure functions, which correspond to radiation of the collinear photons and e^+e^-–pairs by the initial and scattered electrons, and the Born leptonic tensor $L^B_{\mu\nu}$, that depends on the scaled electron 4–momenta. For details see **I**, Sec. 2.

Let us consider process (1) for the case of the scattering off the tensor–polarized target (for example, deuteron target). The part of the hadronic tensor, that depends on the target quadrupole polarization tensor $Q_{\rho\sigma}$, can be written as

$$H_{\mu\nu} = H_{\mu\nu\rho\sigma}Q_{\rho\sigma},$$

$$H_{\mu\nu\rho\sigma} = q_\rho q_\sigma[g_1\tilde{g}_{\mu\nu} + g_2\tilde{p}_{1\mu}\tilde{p}_{1\nu} + g_3\tilde{p}_{2\mu}\tilde{p}_{2\nu} +$$
$$+ g_4(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_5[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] + p_{2\rho}p_{2\sigma}[g_6\tilde{g}_{\mu\nu} + g_7\tilde{p}_{1\mu}\tilde{p}_{1\nu} +$$
$$+ g_8\tilde{p}_{2\mu}\tilde{p}_{2\nu} + g_9(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_{10}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] + (qp_2)_{\rho\sigma}[g_{11}\tilde{g}_{\mu\nu} +$$
$$+ g_{12}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + g_{13}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + g_{14}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ig_{15}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] +$$
$$+ (qN)_{\rho\sigma}[g_{16}(\tilde{p}_1N)_{\mu\nu} + ig_{17}[\tilde{p}_1N]_{\mu\nu} + g_{18}(\tilde{p}_2N)_{\mu\nu} +$$
$$+ ig_{19}[\tilde{p}_2N]_{\mu\nu}] + (p_2N)_{\rho\sigma}[g_{20}(\tilde{p}_1N)_{\mu\nu} +$$
$$+ ig_{21}[\tilde{p}_1N]_{\mu\nu} + g_{22}(\tilde{p}_2N)_{\mu\nu} + ig_{23}[\tilde{p}_2N]_{\mu\nu}], \quad (3)$$

$$N_\mu = \epsilon_{\mu\nu\rho\sigma}p_{1\nu}p_{2\rho}q_\sigma = (\mu p_1 p_2 q),$$

$$[ab]_{\mu\nu} = a_\mu b_\nu - a_\nu b_\mu, \quad (ab)_{\mu\nu} = a_\mu b_\nu + a_\nu b_\mu,$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \tilde{p}_{i\mu} = p_{i\mu} - \frac{(qp_i)q_\mu}{q^2}, \quad i = 1, 2,$$

where g_i ($i = 1-23$) are the hadron semi–inclusive structure functions which depend in general on four invariant variables. The set of these variables can be taken, for example, as q^2, (qp_1), (qp_2) and (p_1p_2).

We use such normalization of the tensor $Q_{\rho\sigma}$ at which the target spin–density matrix is defined as

$$\rho_{\rho\sigma} = -\frac{1}{3}\Big(g_{\rho\sigma} - \frac{p_{1\rho}p_{1\sigma}}{M^2}\Big) - \frac{i}{2M}\epsilon_{\rho\sigma\lambda\delta}W_\lambda p_{1\delta} + Q_{\rho\sigma}, \quad (4)$$

where M (W_λ) is the target mass (polarization 4–vector). The quadrupole polarization tensor satisfies conditions $Q_{\rho\sigma} = Q_{\sigma\rho}$, $Q_{\rho\rho} = Q_{\rho\sigma}p_{1\sigma} = 0$.

In general the traceless symmetrical tensor $Q_{\rho\sigma}$ has five independent components and usually is represented in the form

$$Q_{\rho\sigma} = Q^{\alpha\beta}_{\rho\sigma}R_{\alpha\beta} = Q^{ll}_{\rho\sigma}R_{ll} +$$
$$+ Q^{tt-nn}_{\rho\sigma}R_{tt-nn} + Q^{lt}_{\rho\sigma}R_{lt} + Q^{ln}_{\rho\sigma}R_{ln} + Q^{tn}_{\rho\sigma}R_{tn}. \quad (5)$$

Quantities $R_{\alpha\beta}$ on the right–hand side of Eq. (5) are the polarization degrees of the corresponding components $Q^{\alpha\beta}_{\rho\sigma}$. In the case of the polarized target these quantities are specified by the procedure of the polarized–target preparation and do not depend on reaction mechanism. On the contrary, the polarization characteristics of the final particles are determined by the reaction mechanism and their measurement requires a second scattering from another target [11].

As conserns components $Q^{\alpha\beta}_{\rho\sigma}$, they can be represented as independent bilinear combinations of three mutually orthogonal 4–vectors, P^l (longitudinal), P^t (transversal) and P^n (normal), such that $(P^i)^2 = -1$, $P^ip_1 = 0$, $i = l, t, n$, namely

$$Q^{ll}_{\rho\sigma} = P^l_\rho P^l_\sigma - \frac{1}{2}(P^t_\rho P^t_\sigma + P^n_\rho P^n_\sigma),$$
$$Q^{tt-nn}_{\rho\sigma} = \frac{1}{2}(P^t_\rho P^t_\sigma - P^n_\rho P^n_\sigma), \quad (6)$$
$$Q^{\alpha\beta}_{\rho\sigma} = P^\alpha_\rho P^\beta_\sigma + P^\beta_\rho P^\alpha_\sigma \ \text{ if } \ \alpha \neq \beta.$$

Due to the restriction on the spin–density matrix ($\mathrm{Sp}\,\rho^2 \leq (\mathrm{Sp}\,\rho)^2$), the following inequality has to be satisfied (compare with Ref. [12])

$$-\frac{W^2}{2} + \frac{3}{2}R^2_{ll} + \frac{1}{2}R^2_{tt-nn} + 2(R^2_{lt} + R^2_{ln} + R^2_{tn}) \leq \frac{2}{3}. \quad (7)$$

In order to find the dependence of the cross–section on the target tensor polarization one has to use the expansion (5) for $Q_{\rho\sigma}$ and then to contract the hadronic and leptonic tensors in Eq. (2). For covariant description of polarization phenomena with account of RC it is convenient to parametrize the 4–vectors P^i in terms of the 4–momenta of particles participating in process under consideration.

The use of polarized Born cross–section, depending on the scaled electron 4–momenta, under the integral sign in the Drell–Yan representation leads to some problem. It is clear that all components in the expansion (5) are attributes of the target only and could not depend on radiation by the initial and scattered electrons. On the other hand, it may be happened that in our theoretical calculations the 4–vectors P^i in Eq. (6), being expressed through the 4–momenta, will be changed at the scale transformation of the electron momenta: $k_{1,2} \to \hat{k}_{1,2}$. Such situation leads to the modification of $Q^{\alpha\beta}_{\rho\sigma}$, and we have to find solution in this case.

For physical reasons one can choose two different sets of the 4–vectors P^i. For the first set (P^l, P^t, P^n) the longitudinal direction in laboratory system is chosen along 3–momentum of the initial electron, the transversal one is in the plane $(\mathbf{k_1}, \mathbf{k_2})$ and the normal one is perpendicular to the electron scattering plane. Note that the choice of two arbitrary vectors from the set defines also the choice of the third one. This set remains stabilized (not changed) under the scale transformation of the electron momenta, and the corresponding form of P^i is given by Eqs. (33) and (34) in **I**. In this case there is no problem to apply the ordinary Drell–Yan representation for calculation of RC, and we have

$$\frac{d\sigma(k_1,k_2)}{d\Phi} = \iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\frac{d\sigma_B(\hat{k}_1,\hat{k}_2)}{d\hat{\Phi}}, \qquad (8)$$
$$d\Phi = dx\, dy\, dz\, dz_1\, dz_2,$$

where the notation for the kinematical variables and limits of integration is the same as in **I**. In argument of the cross–section we omit those momenta which are not affected by the scale transformation. In accordance with the expansion (5), the Born cross–section in the integrand in the right–hand side of Eq. (8) reads

$$\frac{d\sigma_B(k_1,k_2)}{d\Phi} = Z\big[P_{ll}R_{ll} + P_{tt-nn}R_{tt-nn} + P_{lt}R_{lt} + $$
$$+ P_{ln}R_{ln} + P_{tn}R_{tn}\big], \quad Z = \frac{\alpha^2 y V^6}{128\pi^2|\eta|Q^4}, \qquad (9)$$

where

$$P_{ll} = 3\eta\big[(2\tau_1 z_1 - z)R_{20} - (y+2a)R_{16}\big] +$$
$$+\frac{1}{V^2\tau_1}\big[[(y+2a)^2 - 2ab]G_1 +$$
$$+[6\tau_1 z_1(\tau_1 z_1 - z) + 2\tau_1\tau_2 + z^2]G_6 -$$
$$-2[\tau_1 z_2 + \tau_1 z_1(3a - 3b + 2) - z(y+3a)]G_{11}\big],$$
$$P_{lt} = \frac{1}{\sqrt{ab}}\big[(2\tau_1 z_1 - z)[\frac{2}{V^2}(\eta_2 G_6 + 2xybG_{11}) +$$
$$+\eta(y+2a)R_{20}] - (y+2a)[\frac{2}{V^2}(\eta_2 G_{11} +$$
$$+2xybG_1) + \eta(y+2a)R_{16}] + 2\eta\tau_1(\eta_2 R_{20} + 2xybR_{16})\big],$$
$$P_{tt-nn} = \eta(y+2a)R_{16} + \frac{2}{V^2}(xybG_1 + \eta_2 G_{11}) +$$
$$+\frac{1}{2xyb}\big[\eta(\eta_2(y+2a) + \eta_1)R_{20} + \frac{\eta_2^2 - \eta^2}{V^2}G_6\big],$$
$$P_{ln} = \frac{-1}{\sqrt{ab}}\big[(2\tau_1 z_1 - z)(\frac{2}{V^2}\eta G_6 - \eta_1 R_{20}) +$$
$$+(y+2a)(\eta_1 R_{16} - \frac{2}{V^2}\eta G_{11}) + 2\tau_1\eta^2 R_{20}\big],$$
$$P_{tn} = 2\eta_1 R_{16} - \frac{4}{V^2}\eta G_{11} -$$
$$-\frac{\tau_1}{ab}\big[\eta_2(\frac{2}{V^2}\eta G_6 - \eta_1 R_{20}) + \eta^2(y+2a)R_{20}\big],$$
$$G_i = -2\frac{xy}{V}g_i + bg_{i+1} + (z_1 z_2 - \tau_2 xy)g_{i+2} +$$
$$+[z_2 + (1-y)z_1 - xyz]g_{i+3} - \lambda\eta g_{i+4},$$
$$R_i = \frac{\eta}{2}[(2-y)g_i + (z_1+z_2)g_{i+2}] + \frac{\lambda}{2}(\eta_1 g_{i+1} + \eta_3 g_{i+3}),$$
$$a = xy\tau_1, \quad b = 1 - y - a,$$
$$\eta_1 = y[z_2 - z_1(1-y) - xz(2-y) + 2x\tau_1(z_1+z_2)],$$
$$\eta_2 = z_2 + (1-y)z_1 - xyz - 2bz_1,$$
$$\eta_3 = (z_1 - z_2)[z_2 - z_1(1-y)] - xy[2\tau_2(2-y) - z(z_1+z_2)].$$

For the second set (P^L, P^T, $P^N = P^n$) the longitudinal (L) direction is chosen along the 3–momentum $\mathbf{q}$ of the intermediate heavy photon in laboratory system and the transversal one is in the electron scattering plane. The specific form of this set is defined by Eq. (35) in **I**. At the scale transformation of the electron momenta the 4–vectors P^L and P^T begin to rotate in the electron scattering plane because direction $\mathbf{q}$ is unstable.

To compute RC in this case one needs to express these unstable 4–vectors through stabilized ones P^l and P^t. The relation between them is given by the orthogonal matrix (see Eq. (36) in **I**). The elements of this matrix have not been affected by the scale transformation and we derive the modified Drell–Yan representation that is direct analog of Eq. (30) in Ref. [8]

$$\frac{d\sigma^u(k_1,k_2)}{d\Phi} =$$
$$= X_{\alpha\beta}(k_1,k_2)\iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\frac{d\sigma_B^{\alpha\beta}(\hat{k}_1,\hat{k}_2)}{d\hat{\Phi}}, \quad (10)$$

where we use the upper index u to point that the target quadrupole polarization tensor is defined with respect to the unstable directions and the summation over indices α and β is bearing in mind. The principle moment of this modified representation is appearance of the matrix $X_{\alpha\beta}(k_1, k_2)$ before the integral that depends on non–scaled electron momenta. The elements X_{ll}, X_{tt} and X_{lt} are the same as in Eq. (31) in Ref. [8], and the rest ones have very simple form

$$X_{ln} = \cos\theta_1 R_{LN} - \sin\theta_1 R_{TN},$$
$$X_{tn} = \cos\theta_1 R_{TN} + \sin\theta_1 R_{LN}, \qquad (11)$$
$$\cos\theta_1 = \frac{y(1+2x\tau_1)}{\sqrt{y(y+4x\tau_1)}},$$

where R_{AB} are polarization degrees of corresponding components of the tensor polarization defined with respect to the directions L, T and N.

The partial cross–sections under the integral sign in the right–hand side of Eq. (10) read

$$\frac{d\sigma_B^{\alpha\beta}(k_1,k_2)}{d\Phi} = ZP_{\alpha\beta}, \qquad (12)$$

where the quantities $P_{\alpha\beta}$ are given after Eq. (9).

To obtain the Born approximation for the cross–section on the left–hand side of Eq. (10) it is enough to substitute ordinary δ–function instead of the electron structure functions in the integrand, and such procedure leads to the following result

$$\frac{d\sigma_B^u(k_1,k_2)}{d\Phi} = Z\big[P_{LL}R_{LL} + P_{TT-NN}R_{TT-NN} +$$
$$+ P_{LT}R_{LT} + P_{LN}R_{LN} + P_{TN}R_{TN}\big], \qquad (13)$$

where

$$P_{LL} = \frac{1}{\tau_1 V^2}\big[yhG_1 - 2G_{11}(2\tau_1(z_1-z_2) - yz) +$$
$$+\frac{G_6}{yh}((2\tau_1(z_1-z_2)-yz)^2 - \frac{\tau_1}{2xb}(h\eta^2 + \frac{\eta_1^2}{y}))\big],$$
$$P_{LT} = \frac{1}{h\sqrt{ab}}\big[(2\tau_1(z_1-z_2)-yz)(\frac{2\eta_1 G_6}{yV^2} + h\eta R_{20}) -$$
$$-yh(\frac{2\eta_1 G_{11}}{yV^2} + h\eta R_{16})\big],$$
$$P_{TT-NN} = \frac{1}{2xybh}\big[(\frac{\eta_1^2}{y} - h\eta^2)\frac{G_6}{V^2} + 2h\eta\eta_1 R_{20}\big],$$
$$P_{LN} = \frac{1}{y\sqrt{\tau_1 xbh}}\big[yh(\frac{2\eta G_{11}}{V^2} - \eta_1 R_{16}) -$$
$$-(2\tau_1(z_1-z_2)-yz)(\frac{2\eta G_6}{V^2} - \eta_1 R_{20})\big],$$
$$P_{TN} = -\frac{1}{xb\sqrt{yh}}\big[2\eta\eta_1\frac{G_6}{yV^2} + (h\eta^2 - \frac{\eta_1^2}{y})R_{20}\big].$$

It is easy to verify that straightforward calculation, based on the expansion of the tensor polarization by means of the set (P^L, P^T, P^N), results in the same answer.

Note that the Born cross–section (13), when the tensor polarization components are defined relative to unstable (at scale transformation) directions, is simpler as compared with cross section (9), when these components are defined with respect to the stabilized ones. The account of RC changes situation radically. In the last case it is trivial because RC does not intermix different components. Contrary, in the first case the radiation of particles by the electrons leads to mixture of the tensor–polarization components due to the rotation of P^L and P^T, therefore RC is more complicated.

3. Let us consider the tensor polarization of the detected particle B in the process (1), provided unpolarized target A. In this case every tensor–polarization degrees $R_{\alpha\beta}$ are defined by the reaction mechanism and can be measured in the second scattering (see, for example, Ref. [11], where the elastic electron–deuteron scattering with polarization transfer from electron to scattered deuteron was investigated).

Hadronic tensor $H_{\mu\nu}^{\alpha\beta}$ corresponding to the given polarization $R_{\alpha\beta}$ can be easily constructed using tensor $H_{\mu\nu}$ defined by Eq. (3). In order to do this it is enough to change expressions $Q_{\rho\sigma}p_{2\rho}p_{2\sigma}, Q_{\rho\sigma}(qp_2)_{\rho\sigma}$ and $Q_{\rho\sigma}(p_2N)_{\rho\sigma}$ in $H_{\mu\nu}$ by $V_{\rho\sigma}^{\alpha\beta}p_{1\rho}p_{1\sigma}, V_{\rho\sigma}^{\alpha\beta}(qp_1)_{\rho\sigma}$ and $V_{\rho\sigma}^{\alpha\beta}(qp_1)_{\rho\sigma}$, respectively, where $V_{\rho\sigma}^{\alpha\beta}$ is the component of the final–particle quadrupole polarization tensor $V_{\rho\sigma}$ in expansion $V_{\rho\sigma} = V_{\rho\sigma}^{\alpha\beta}R_{\alpha\beta}$. This is analog of the respective expansion for the target quadrupole polarization tensor. Besides, one needs to use another notation for the hadronic structure functions, $g_i \to f_i$, $i = 1-23$, because in general they are different.

The given tensor polarization $R_{\alpha\beta}$ of the particle B is, by definition, the ratio of the partial cross–section $d\sigma^{\alpha\beta}$, which can be derived by contraction of the leptonic tensor with $H_{\mu\nu}^{\alpha\beta}$, to the unpolarized cross–section

$$R_{\alpha\beta} = d\sigma^{\alpha\beta}/d\sigma_{(un)}. \qquad (14)$$

In the used approximation, RC to the unpolarized cross–section is defined by Eq. (19) in **I**, therefore our problem is calculation of RC to the partial cross–section. By full analogy with Eq. (6) components $V_{\rho\sigma}^{\alpha\beta}$ represent traceless independent bilinear combinations of three mutually orthogonal 4–vectors S^i which satisfy conditions $(S^i)^2 = -1$, $S^ip_2 = 0$, $i = l, t, n$. If we take the longitudinal direction, in the rest frame of the detected hadron, opposite to direction of 3–momentum $\mathbf{p}_1$ and the transversal one is in the plane $(\mathbf{k}_1, \mathbf{p}_1)$ (or in the plane $(\mathbf{k}_1, \mathbf{p}_2)$ in laboratory system), the set (S^l, S^t, S^n) remains stabilized under the scale transformation of the electron momenta. The form of this set in terms of the 4–momenta is defined by Eqs. (14) and (15) in **I**, and the Drell–Yan representation of the partial cross–section in this case reads

$$\frac{d\sigma^{\alpha\beta}(k_1,k_2)}{d\Phi} = \iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\frac{d\sigma_B^{\alpha\beta}(\hat{k}_1,\hat{k}_2)}{d\hat{\Phi}}, \quad (15)$$

$$\frac{d\sigma_B^{\alpha\beta}(k_1,k_2)}{d\Phi} = \frac{Z}{(2S_A+1)}S^{\alpha\beta},$$

$$S_{ll} = \frac{1}{V^2\tau_2}\big[2[z(z_1-z_2)-2y\tau_2]H_{11}+d_1^2H_6\big]+$$
$$+\frac{H_1}{2V^2d_1^2}\big[\frac{2}{\tau_2}[z(z_1-z_2)-2y\tau_2]^2-\frac{\eta_4^2+\eta^2d_1^2}{d_2^2}\big],$$

$$S_{lt} = \frac{1}{d_1^2d_2\sqrt{\tau_2}}\big[[z(z_1-z_2)-2y\tau_2](\frac{2}{V^2}\eta_4H_1+\eta d_1^2F_{16})+$$
$$+d_1^2(\frac{2}{V^2}\eta_4H_{11}+\eta d_1^2F_{20})\big],$$

$$S_{tt-nn} = \frac{1}{2d_2^2}(2\eta\eta_4F_{16}+\frac{\eta_4^2-\eta^2d_1^2}{V^2d_1^2}H_1),$$

$$S_{ln} = \frac{1}{d_1d_2\sqrt{\tau_2}}\big[d_1^2(\frac{2}{V^2}\eta H_{11}-\eta_4F_{20})+$$
$$+[z(z_1-z_2)-2y\tau_2](\frac{2}{V^2}\eta H_1-\eta_4F_{16})\big],$$

$$S_{tn} = \frac{1}{d_1d_2^2}\big[\frac{2}{V^2}\eta\eta_4H_1-(\eta_4^2-\eta^2d_1^2)F_{16}\big],$$

$$H_i = G_i(g_k\to f_k), \quad F_i = R_i(g_k\to f_k),$$

$$d_1^2 = z^2-4\tau_1\tau_2, \quad d_2^2 = zz_1-\tau_2-z_1^2\tau_1,$$

$$\eta_4 = 2y\tau_2-z(z_1-z_2)-xy(z^2-4\tau_1\tau_2)+z_1[2\tau_1(z_1-z_2)-yz],$$

where S_A is the target spin.

If the longitudinal direction is the same ($L=l$) but the transversal one (T) chosen in the plane ($\mathbf{q},\mathbf{p}_1$) in the rest frame of the detected hadron (or in the plane ($\mathbf{q},\mathbf{p}_2$) in laboratory system), the new set ($S^L=S^l$, S^T, S^N) has become unstable under substitution $k_{1,2}\to\hat{k}_{1,2}$. It means physically that collinear radiation changes direction of $\mathbf{q}$ and this leads to the rotation of vectors S^T and S^N around the longitudinal direction which remains unchanged. The form of the 4–vectors S^T and S^N is defined by Eq. (17) in **I**. In this case we will use the capital letters to label the tensor polarizations of the detected hadron and the respective partial cross–sections

$$R_{AB} = d\sigma^{AB}/d\sigma_{(un)}. \quad (16)$$

To compute RC to the partial cross–sections one needs, as before, to express S^T and S^N through stabilized 4–vectors S^t and S^n. The relation between these 4–vectors is given by orthogonal matrix (see Eq. (18) in **I**). Then it is necessary to find matrix Y that relates unstable $V_{\rho\sigma}^{AB}$ and stabilized $V_{\rho\sigma}^{\alpha\beta}$ components of the quadrupole polarization tensor

$$V_{\rho\sigma}^{AB} = Y_{\alpha\beta}^{AB}V_{\rho\sigma}^{\alpha\beta}. \quad (17)$$

The non–zero elements of the matrix Y are

$$\begin{gathered} Y_{ll}^{LL}=1, \quad Y_{lt}^{LT}=Y_{ln}^{LN}=\cos\theta, \\ Y_{lt}^{LN}=-Y_{ln}^{LT}=\sin\theta, \\ Y_{tn}^{TN}=Y_{tt-nn}^{TT-NN}=\cos 2\theta, \\ Y_{tt-nn}^{TN}=-4Y_{tn}^{TT-NN}=2\sin 2\theta, \\ \cos\theta=-\frac{\eta_4}{2d_2d_3}, \\ d_3^2=zy(z_1-z_2)+xyd_1^2-(z_1-z_2)^2\tau_1-y^2\tau_2. \end{gathered} \quad (18)$$

The Drell–Yan representation for the partial cross–sections has modified form

$$\frac{d\sigma^{AB}(k_1,k_2)}{d\Phi} = Y_{\alpha\beta}^{AB}(k_1,k_2)\times$$
$$\times\iint\frac{dx_1dx_2}{x_2^2}D(x_1)D(x_2)\frac{d\sigma_B^{\alpha\beta}(\hat{k}_1,\hat{k}_2)}{d\hat{\Phi}}, \quad (19)$$

where the elements of the matrix Y depend on the non–scaled electron momenta, and the Born cross–sections under the interal sign are defined by Eqs. (15).

To obtain the Born form of the partial cross–sections on the left–hand side of Eq. (19) one can use δ–function instead of D–functions under the integral sign. Such procedure gives

$$\frac{d\sigma_B^{AB}}{d\Phi}(k_1,k_2) = \frac{Z}{2S_A+1}S_{AB}, \quad (20)$$

$$S_{LL} = S_{ll},$$

$$S_{LT} = -\frac{4d_3}{V^2d_1^2\sqrt{\tau_2}}\big[(z(z_1-z_2)-2y\tau_2)H_1+d_1^2H_{11}\big],$$

$$S_{LN} = \frac{2d_3}{d_1\sqrt{\tau_2}}\big[(z(z_1-z_2)-2y\tau_2)F_{16}+d_1^2F_{20}\big],$$

$$S_{TT-NN} = \frac{2d_3^2}{d_1^2V^2}H_1, \quad S_{TN} = -4\frac{d_3^2}{d_1}F_{16}.$$

As in the case of the polarized target, the partial Born cross–sections for unstable directions are more simpler but the account of RC takes away the whole simplicity.

4. Consider now the special case of reaction (1), namely disintegration of polarized deuterons by longitudinally–polarized electrons for exclusive setup. In this case the outgoing proton is detected in coincidence with the scattered electron and the lost invariant mass is smaller than $m+m_\pi$, therefore the undetected hadronic state X consists only of a neutron.

Theoretically the problems of polarization phenomena caused by tensor–polarized target in $\mathbf{d}(\mathbf{e},e'p)n$ reaction have been investigated in a number of papers. For details see review [13]. The experimental study of the polarized exclusive deuteron desintegration is planned at future upgraded version of CEBAF at Jefferson Laboratory [14]. This will provide a test of a basic principles of our understanding of the electrodisintegration dynamics. The progress in constructing tensor–polarized deuteron target will take it feasible to study reaction at sufficiently large Q^2. In this case a direct separation of S- and D-wave contributions is possible that is important for understanding of the short–distance NN–interactions [15].

Our aim is to show how to calculate model–independent RC to considered here process. It is obvious that we have to use standard or modified Drell–Yan representation (like (8) or (10) for semi–inclusive DIS process). So, the knowledge of the respective partial Born cross–sections in terms of used invariant variables is necessary. One can derive them using δ–function in definition of the hadronic tensor $H_{\mu\nu}$ to remove the integration with respect to z_2 in Eq. (12). Therefore, for the exclusive partial cross–sections we can use Eq. (12) with substitution $d\Phi\to dxdydzdz_1$ in the left–hand side and $g_i\to h_i/V,\ z_2\to z_1+z-y(1-x)-\tau_1$ in the right–hand side, where h_i are structure functions of the deuteron electrodisintegration.

On the other hand, in the literature special kinematical variables are used. They are suitable for separation of the contributions into cross–section due to longitudinal and transversal polarizations (and their interference) of the intermediate heavy photon. These are the energy ε_2 and the polar angle θ_2 of the scattered electron in laboratory system (with z axis along $\mathbf{q}$ and x axis is in $(\mathbf{q},\mathbf{p_2})$ plane), the angle ϕ between the electron scattering plane and plane $(\mathbf{q},\mathbf{p_2})$ and the proton scattering angle θ in c.m.s. of the reaction $\gamma^*+d\to p+n$. In terms of these variables the Born cross–section looks as

$$\frac{d\sigma}{d\varepsilon_2 d\cos\theta_2 d\phi d\cos\theta}=F\Big[h_{xx}+h_{yy}+$$
$$+\varepsilon\cos 2\phi(h_{xx}-h_{yy})+\varepsilon\sin 2\phi(h_{xy}+h_{yx})-$$
$$-2\varepsilon\frac{q^2}{q_0^2}h_{zz}-\sqrt{2\varepsilon(1+\varepsilon)}\frac{\sqrt{-q^2}}{q_0}[\cos\phi(h_{xz}+h_{zx})+$$
$$+\sin\phi(h_{yz}+h_{zy})]-i\lambda\sqrt{(1-\varepsilon^2)}(h_{xy}-h_{yx})-$$
$$-i\lambda\sqrt{2\varepsilon(1-\varepsilon^2)}\frac{\sqrt{-q^2}}{q_0}[\cos\phi(h_{yz}-h_{zy})-$$
$$-\sin\phi(h_{xz}-h_{zx})]\Big], \tag{21}$$

$$F=\frac{\alpha^2}{32\pi^2}\frac{\varepsilon_2}{\varepsilon_1}\frac{|\mathbf{p}_2|}{MW}\frac{(1-\varepsilon)^{-1}}{(-q^2)},\quad \varepsilon^{-1}=1-2\frac{\mathbf{q}_L^2}{q^2}\tan^2(\frac{\theta_e}{2}),$$

where θ_e and $\mathbf{q}_L$ are the scattering angle of the electron and virtual–photon 3–momentum in laboratory system, W is the total invariant energy of $p-n$ system, q_0 and $\mathbf{p}_2$ are the virtual–photon energy and 3–momentum of the detected proton in c.m.s., and λ is the degree of the longitudinal polarization of the electron beam. Quantity ε represents the degree of the virtual–photon linear polarization. This formula is obtained in the one–photon–exchange approximation, using the conservation of the electromagnetic current describing the $\gamma^*d\to pn$ transition and P–invariance of the hadron electromagnetic interaction.

The hadronic tensor $h_{\mu\nu}$, can be derived from $H_{\mu\nu}$ by the rule

$$H_{\mu\nu}\to\frac{1}{V}h_{\mu\nu}\delta(z_2-z_1-z+y(1-x)+\tau_1),\quad g_i\to h_i, \tag{22}$$

and its components have to be written in c.m.s.

To use the Drell–Yan representation we have to express all variables and quantities in both hands of Eq. (21) through invariant variables. The corresponding formulae read

$$Fd\varepsilon_2 d\cos\theta_2 d\phi d\cos\theta=$$
$$=\frac{\alpha^2}{32\pi^2}\frac{[1+(1-y)^2+2xy\tau_1](1+2x\tau_1)}{xyV|\eta|(y+4x\tau_1)^2\sqrt{y(y+4x\tau_1)}}dx\,dy\,dz\,dz_1,$$
$$W=\sqrt{V(\tau_1+y-xy)},$$
$$|\mathbf{p}_2|=\frac{1}{2}\sqrt{V(\tau_1-4\tau_2+y-xy)},$$
$$\mathbf{q}^2=\frac{Vy(y+4x\tau_1)}{4\tau_1},$$
$$q^2=-xyV,\ q_0=\frac{1-2x}{2}\sqrt{\frac{Vy^2}{\tau_1+y(1-x)}},$$
$$\varepsilon=\frac{2(1-y-xy\tau_1)}{1+(1-y)^2+2xy\tau_1},$$
$$\cos\theta=\frac{2z-y-2\tau_1}{\sqrt{\tau_1-4\tau_2+y(1-x)}}\sqrt{\frac{\tau_1+y(1-x)}{y(y+4x\tau_1)}},$$
$$\cos\theta_2=\frac{y(1-y-2x\tau_1)}{(1-y)\sqrt{y(y+4x\tau_1)}},$$

and the azimuthal angle ϕ can be obtained from the equation

$$\sin\phi = -\frac{\eta}{\sin\theta}\frac{1}{\sqrt{xy(1-y-4xy\tau_1)(\tau_1-4\tau_2+y-xy)}}.$$

The components of the hadronic tensor on the right–hand side of Eq. (21) can be easy written taking into account that in c.m.s.

$$q_z = -p_{1z} = |\mathbf{q}|, \quad p_{2z} = |\mathbf{p}_2|\cos\theta,$$

$$p_{2x} = |\mathbf{p}_2|\sin\theta, \quad N_y = W|\mathbf{p}_2||\mathbf{q}|\sin\theta,$$

and all other components of the corresponding 3–vectors are zeros.

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