

Polarized triplet production by circularly polarized photons

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A process of the pair production by a circularly polarized photon in the field of unpolarized atomic electron has been considered in the Weizaecker-Williams approximation. The degree of longitudinal polarization of positron and electron has been calculated. An exclusive cross-section as well as a spectral distribution are obtained. We estimate the accuracy of our calculations at the level of a few percent. We show the identity of the positron polarization for considered process and for process of pair production in the screened Coulomb field of nucleus.

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In this paper we consider the process $\gamma(k) + e^-(p) \rightarrow e^-(q_-) + e^+(q_+) + e^-(p')$ in the high-energy limit when polarization states of initial photon and production particles e^+ , e^- are helical. The differential cross-section of electron-positron pair photoproduction on a free electron in the Born approximation is described by eight Feynman diagrams (FD) [1] which shown in Fig.1 and Fig.2. In the high-energy limit (for photon energy

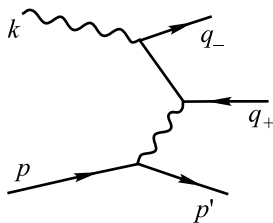


Fig.1. Types of relevant Feynman diagrams

54 MeV at Lab system) only subset of two Bethe-Heitler (BH) diagrams (see Fig.1) are relevant at the level of accuracy at $m^2/s \sim 10^{-2}$, m is electron mass and s is $s = 2kp = 2m\omega$, where ω is the photon energy at the laboratory frame, whereas the contributions we examine is order of $s/m^2 \sim 200 \gg 1$. Non-logarithmic terms we also don't consider because of their smallness $\sim 10^{-1}$ in comparison with logarithmic terms $2 \ln(s/m^2) \sim 10$. Further calculations will be performed in Weizaecker-Williams (WW) approximation [2].

The differential cross-section of triplet photoproduction in the Born approximation for the unpolarized case was calculated numerically in [3, 4] by using Monte-

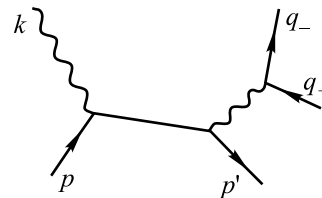


Fig.2. Types of irrelevant Feynman diagrams

Carlo simulation of all 8 FD contribution. The closed analytical expression is very cumbersome and was first obtained in a complete form in works [5]. A detailed analysis of the expressions of Haug's work reveals that the interference terms of the BH matrix elements with the other three gauge-invariant subsets (which take into account the bremsstrahlung mechanism of pair creation and Fermi statistics for fermions) turn out to be of the order of some percent for $s > 50 - 60 m^2$.

The differential cross-section for electron-positron pair production by linearly polarized photons was derived in a series of papers [6–8] (see also [9] and references therein). In Ref. [10] was performed a Monte-Carlo simulation of the process under consideration, in which all eight lowest order diagrams can be numerically treated without approximation. There it was shown that one might consider only the two leading graphs in a wide range of photon energies from 50 to 550 MeV. Note that this observation was made earlier for the unpolarized case in works of Kopylov et al. [3] and Haug [5] (who presented his results in explicit analytical form).

The process of the polarized pair production by a polarized photon in the screened Coulomb field of nucleus have been considered in the high energy limit in

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the works [11, 12]. Here the degree of longitudinal polarization of electron has been calculated.

From the paper [3, 5] it follows that: 1. The contribution of FD (see Fig.2) as well as interference it's amplitude with amplitude of (see Fig.1) can be neglected within accuracy 3% compared with contribution of (see Fig.1) already starting from photon energies $\omega > 30$ MeV in laboratory frame; 2. One can use the asymptotic formulae at very high photon energies of contribution of Bethe-Haitler FD (see Fig.1) within accuracy 5% starting from energies of order $\omega > 100$ MeV. Taking into account that unpolarized cross-section dominates the ones depending on particle polarization we estimate accuracy of WW (logarithmical approximation) on the level of 10%.

Our paper organized as follows. Using the Sudakov technique we calculate the differential cross-section and the degree of longitudinal polarization of electron and positron in the process of pair production by circularly polarized photon on electron. In conclusion we discuss the different schemes of production of longitudinally polarized positrons in experimental set-ups. The method described here is one of the most perspective one. It's our motivation for investigation. In Appendix we give in Born approximation the differential cross-section of triplet production process for the case when polarization states of all particles are helical by using of crossing-transformation of corresponding differential cross-section for Möller bremsstrahlung process in the ultrarelativistic (massless) limit. Corresponding formulae for the degree of longitudinal polarization of positron in terms of kinematical invariant and helicity of initial photon is given for a case of high energy large angle scattering.

The cross-section of the process. The cross-section of the process is proved in the next form:

$$d\sigma = \frac{(4\pi\alpha)^3}{4(2\pi)^5 s} \Sigma M^2 \frac{d^3 q_-}{2\varepsilon_-} \frac{d^3 q_+}{2\varepsilon_+} \frac{d^3 p'}{2\varepsilon'} \times \delta^4(p + k - q_+ - q_- - p'), \quad (1)$$

where p' , ε' is four-momentum and energy of scattered electron; q_{\pm} and ε_{\pm} are four-momentum and energy of electron and positron correspondingly. We accept here the Sudakov picture [2] of the peripheral process. Let us determine the basic vectors in Sudakov representation of kinematics: vector of momentum transfer $q = p - p'$ and the light-like vector $\tilde{p} = p - k \frac{m^2}{s}$, a $\tilde{p}^2 = 0$ (k is 4-momentum of impact photon). The Sudakov representation of four-momenta are the next:

$$q = \alpha\tilde{p} + \beta k + q_{\perp}, \quad q_{\pm} = \alpha_{\pm}\tilde{p} + \beta_{\pm}k + q_{\perp\pm}, \\ a_{\perp}k = a_{\perp}\tilde{p} = 0. \quad (2)$$

The quantities β_{\pm} can be interpreted as the energy fraction of pair components of photon energy, $\beta_+ + \beta_- = 1$. The conservation law of transversal momenta is $q_{\perp} = q_{\perp+} + q_{\perp-}$. Momentum components along \tilde{p} is small. Using the mass-shell conditions of pair components and the scattered electron leads to:

$$s\alpha_{\pm} = \rho_{\pm}/\beta_{\pm}, \quad (3)$$

$$s\beta(1 - \alpha) = -\mathbf{q}^2 - m^2\alpha, \quad (4)$$

$$\rho_{\pm} = \mathbf{q}_{\perp\pm}^2 + m^2, \quad (5)$$

where we use the two-dimensional Euclidean vectors $q_{\perp}^2 = -\mathbf{q}^2$, $q_{\perp\pm}^2 = -\mathbf{q}_{\perp\pm}^2$. The Sudakov parameter α can be related with the invariant mass of created pair

$$s_1 = (q_+ + q_-)^2, \quad s\alpha = s_1 + \mathbf{q}^2. \quad (6)$$

Using conditions of mass-shell and smallness of Sudakov parameter α the momentum transfer to the target square can be written in the next form:

$$q^2 = -\mathbf{q}^2 - m^2 \left(s_1/s \right)^2. \quad (7)$$

We see that $q^2 < 0$, is negative and has non-zero magnitude. Using the Sudakov representation of momentum phase volume of the particle

$$d^4 q = \frac{s}{2} d\alpha d\beta d^2 q_{\perp} \quad (8)$$

we perform the phase volume of the final state:

$$d\Gamma = \frac{d^3 q_-}{2\varepsilon_-} \frac{d^3 q_+}{2\varepsilon_+} \frac{d^3 p'}{2\varepsilon'} \delta^4(p + k - q_+ - q_- - p') = \\ = \frac{1}{2s} \frac{d\beta_-}{\beta_- \beta_+} d^2 q_- d^2 q_+. \quad (9)$$

Let us now consider the matrix element:

$$M = \bar{u}(p') \gamma^{\mu} u(p) \frac{1}{q^2} e_{\rho}(k) \bar{u}(q_-) O^{\rho\nu} v(q_+) g_{\mu\nu}.$$

By using the Gribov's decomposition of metric tensor and omitting the terms of order m^2/s compared of the terms of order of unity

$$g_{\mu\nu} = g_{\mu\nu}^{\perp} + \frac{2}{s} \left(\tilde{p}_{\mu} k_{\nu} + \tilde{p}_{\nu} k_{\mu} \right) \approx \frac{2}{s} \tilde{p}_{\nu} k_{\mu}, \quad (10)$$

one can obtain the matrix element in the next form:

$$M = \frac{-2sN_p}{\mathbf{q}^2 + m^2(s_1/s)^2} e_{\rho}(k) \bar{u}(q_-) V^{\rho} v(q_+), \quad (11)$$

$$V^{\rho} = \frac{1}{s} O^{\rho\nu} \tilde{p}_{\nu}, \quad N_p = \frac{1}{s} \bar{u}(p') \hat{k} u(p), \quad |N_p| = 1.$$

Using the mass-shell conditions we can express the matrix four-vector V_{ρ} as:

$$V_\rho = \beta_+ \beta_- \left(\frac{1}{\rho_-} - \frac{1}{\rho_+} \right) \gamma_\rho + \frac{\beta_+}{\rho_+} \hat{p} \hat{q} \gamma_\rho - \frac{\beta_-}{\rho_-} \gamma_\rho \hat{q} \hat{p}. \quad (12)$$

One can see that the quantity V_ρ is proportional to $|q_\perp|$ at small $|q_\perp|$. WW approximation corresponds to the logarithmical enhancement factor:

$$\int \frac{d\mathbf{q}^2 \mathbf{q}^2}{(\mathbf{q}^2 + m^2 (s_\pm/s)^2)^2} \approx 2 \ln \frac{s}{s_1}. \quad (13)$$

The polarization matrix of density $\tau_\pm^{\delta\pm}$ for e^\pm particles have the next form:

$$\tau_\pm^{\delta\pm} = \frac{1}{2} (\hat{q}_\pm \mp m) (1 - \delta_\pm \gamma^5 \hat{s}_\pm), \quad \gamma^5 \hat{s}_\pm \tau_\pm^{\delta\pm} = \delta_\pm \tau_\pm^{\delta\pm}. \quad (14)$$

We will express the particle spin vectors s_\pm in terms of the 4-momenta q_\pm and \vec{p} ,

$$s_\pm = \frac{\mp q_\pm}{m} \pm \frac{2m\vec{p}}{s\beta_\pm}, \quad s_\pm q_\pm = 0, \quad s_\pm^2 = -1. \quad (15)$$

The circular-polarization vector e^λ of a photon with 4-momentum k is conveniently defined by using the 4-vectors q_+ , q_- and k [13]:

$$e_\mu^\lambda = \frac{(q_- k)(q_+)_\mu - (q_+ k)(q_-)_\mu + i\lambda \varepsilon_{\mu\nu\rho\sigma} q_+^\nu q_-^\rho k^\sigma}{\sqrt{2z}},$$

$$z = -((q_- k)q_+ - (q_+ k)q_-)^2 = \frac{\rho^2(\rho - m^2)}{4\beta_+^2\beta_-^2}.$$

Performing the angular averaging on d^2q and extracting the WW-factor we write down the differential cross-sections in the form (in WW-approximation we can put $\rho_+ = \rho_- = \rho$):

$$\begin{aligned} \frac{d\sigma}{d\mathbf{q}_\perp^2 d\beta_-} &= \frac{2\alpha^3}{\rho^2} \ln \frac{s}{m^2} \left[1 - 2\beta_+ \beta_- + \frac{4m^2}{\rho^2} (\rho - m^2) \beta_+ \beta_- + \right. \\ &+ \lambda \delta_+ \left(\beta_+ - \beta_- + \frac{4m^2}{\rho^2} (\rho - m^2) \beta_- \right) + \\ &+ \lambda \delta_- \left(\beta_+ - \beta_- - \frac{4m^2}{\rho^2} (\rho - m^2) \beta_+ \right) + \\ &+ \delta_+ \delta_- \left(-6\beta_+ \beta_- + \frac{\rho - m^2}{m^2} - \right. \\ &\left. - \frac{4m^2}{\rho^2} (\rho - m^2) (1 - 3\beta_+ \beta_-) \right) \left. \right]. \quad (16) \end{aligned}$$

In the case when initial electron is being polarized formulae given above isn't changed. The corresponding effects are of order m^2/s . Performing the summation over the polarization of electron in (16) we will get the differential cross-sections for creating polarized positron:

$$\begin{aligned} \frac{d\sigma^+}{d\mathbf{q}_\perp^2 d\beta_-} &= \\ &= \frac{4\alpha^3}{\rho^2} \ln \frac{s}{m^2} \left[1 - 2\beta_+ \beta_- + \frac{4m^2}{\rho^2} (\rho - m^2) \beta_+ \beta_- + \right. \\ &\left. + \lambda \delta_+ \left(\beta_+ - \beta_- + \frac{4m^2}{\rho^2} (\rho - m^2) \beta_- \right) \right] \quad (17) \end{aligned}$$

and analogously expression for cross-section of creating polarized electron:

$$\begin{aligned} \frac{d\sigma^-}{d\mathbf{q}_\perp^2 d\beta_-} &= \\ &= \frac{4\alpha^3}{\rho^2} \ln \frac{s}{m^2} \left[1 - 2\beta_+ \beta_- + \frac{4m^2}{\rho^2} (\rho - m^2) \beta_+ \beta_- + \right. \\ &\left. + \lambda \delta_- \left(\beta_+ - \beta_- - \frac{4m^2}{\rho^2} (\rho - m^2) \beta_+ \right) \right]. \quad (18) \end{aligned}$$

From expressions (17) and (18) we will have the degree of longitudinal polarization of electron (positron) when polarization of positron (electron) no registered:

$$\delta_\pm^f = \lambda \frac{\beta_+ - \beta_- \pm (4m^2/\rho^2)(\rho - m^2)\beta_\mp}{1 - 2\beta_+ \beta_- + (4m^2/\rho^2)(\rho - m^2)\beta_+ \beta_-}. \quad (19)$$

The result (21) is in agreement with one given in more general form in [12] (see formulae (19.8)). Performing the integration over transversal momenta of pair we obtain for the spectral distribution:

$$\frac{d\sigma^+}{d\beta_-} = 4\alpha r_0^2 \ln \frac{s}{m^2} \left[1 - \frac{4}{3}\beta_+ \beta_- + \lambda \delta_+ \left(1 - \frac{4}{3}\beta_- \right) \right]. \quad (20)$$

Here we use in WW-approximation $|\mathbf{q}_+| = |\mathbf{q}_-|$, $\rho = \rho_+ = \rho_- = \mathbf{q}_\perp^2 + m^2$, λ is the degree of circular polarization if the initial photon. The degree of longitudinal polarization of created e^\pm particles have a form:

$$\delta_\pm^f = \lambda \left(1 - \frac{4}{3}\beta_\mp \right) / \left(1 - \frac{4}{3}\beta_+ \beta_- \right). \quad (21)$$

We see from (21) that in the limit $\beta_+ \rightarrow 1$ ($\beta_- \rightarrow 1$) the degree of longitudinal polarization of positron (electron) equal to degree of circularly polarization of initial photon (see Fig.3).

Let's compare the result obtained with calculations of longitudinal positron polarization from pair production process in the screened Coulomb field of a nucleus [11] with the same accuracy as before we may write (see [14]):

$$\delta_+^f \approx \frac{\frac{4}{3}\beta_+ - \frac{1}{3}}{(\beta_+^2 + \beta_-^2) + \frac{2}{3}\beta_+ \beta_-} \lambda. \quad (22)$$

Having in mind that $\beta_- = 1 - \beta_+$, one may obtain from (21) the same result.

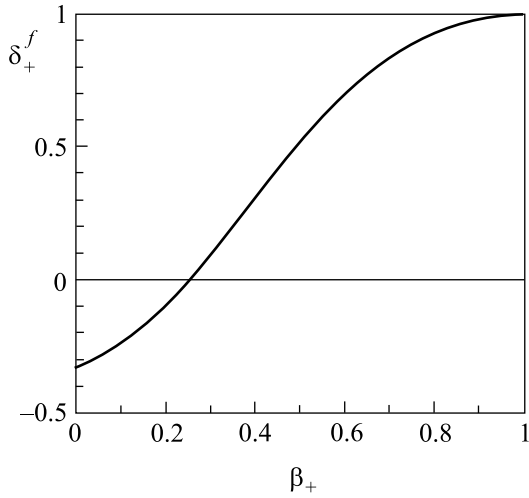


Fig.3. The degree of longitudinal polarization of positron δ_+^f (21) versus of β_+

A few schemes to create polarized positrons for electron-positron colliders were proposed [15]. Longitudinally polarized positrons are produced in the pair production process by circularly polarized photon with energy > 10 MeV. A circularly polarized γ -beam is formed due to undulator radiation of electrons with energy $E \sim 10^2$ GeV in a helical undulator [16] or due to Compton backscattering process of the circularly polarized laser photons on the electron beam with energy > 1 GeV [17]. In both cases there was considered the process of pair production in the screened coulomb field of nucleus only as a source of polarized positrons. The process of triplet production was considering as background process [14]. The obtained results allow to develop a correct Monte-Carlo code in order to receive the positron polarization for the real experimental situation taking into account both processes of pair production in amorphous target [18].

In the recent experiment [19] authors measured the circular polarization of γ -quanta with energy > 20 MeV using the magnetized iron polarimeter. The thickness of iron substance was too big (7 cm), so the correct estimation of the analyzing power may be done by a Monte-Carlo technique. In this case the polarization characteristics of all reaction particles should be took into account and our results cover the existing lack.

Appendix

Differential cross section for triplet production process one can easily get with help of crossing transformation of expression for square amplitudes of Möller bremsstrahlung process

$$e^-(p_1) + e^-(p_2) \rightarrow e^-(p_3) + e^-(p_4) + \gamma(k), \quad (23)$$

which corresponds eight Feynman diagrams. In papers [13] was calculated the differential cross section for this reaction in the case when all fermions were massless ($p_i^2 = 0$, where $i = 1, 2, 3, 4$) with taking into account polarization of initial electrons and emitted photon. Let us introduce invariant variables:

$$\begin{aligned} s_1 &= (p_1 + p_2)^2, \quad t_1 = (p_1 - p_3)^2, \quad u_1 = (p_1 - p_4)^2, \\ s_2 &= (p_3 + p_4)^2, \quad t_2 = (p_2 - p_4)^2, \quad u_2 = (p_2 - p_3)^2, \end{aligned} \quad (24)$$

and helicities δ_1 and δ_2 for initial electrons with momentum p_1 and p_2 respectively, λ for helicity of emitted photon. The differential cross section of reaction (23) in the case of helically polarized initial electrons and photon has the next form [13]:

$$d\sigma_M = \frac{\alpha^3}{2\pi^2 s} A_M W_M d\Gamma_M, \quad A_M = \frac{A_{MB}}{t_1 t_2 u_1 u_2}, \quad (25)$$

$$d\Gamma_M = \frac{d^3 \mathbf{p}_3}{2p_{30}} \frac{d^3 \mathbf{p}_4}{2p_{40}} \frac{d^3 \mathbf{k}}{2\omega} \delta^4(p_1 + p_2 - p_3 - p_4 - k).$$

Expressions for A_{MB} and W_M have forms [13]:

$$\begin{aligned} A_{MB} &= (1 + \delta_1 \delta_2) [(1 + \delta_1 \lambda) s_1 s_2 s_2^2 + (1 - \delta_1 \lambda) s_1 s_2 s_1^2] + \\ &+ (1 - \delta_1 \delta_2) [(1 + \delta_2 \lambda) (t_1 t_2 t_1^2 + u_1 u_2 u_1^2) + \\ &+ (1 - \delta_2 \lambda) (t_1 t_2 t_2^2 + u_1 u_2 u_2^2)], \end{aligned} \quad (26)$$

$$W_M = - \left(\frac{p_1}{p_1 k} + \frac{p_2}{p_2 k} - \frac{p_3}{p_3 k} - \frac{p_4}{p_4 k} \right)^2. \quad (27)$$

Photoproduction diagrams of triplet

$$\gamma(k) + e^-(p) \rightarrow e^-(q_-) + e^+(q_+) + e^-(p') \quad (28)$$

are different from ones for Möller bremsstrahlung process (23) with exchange

$$\begin{aligned} p_1 &\rightarrow p, \quad p_2 \rightarrow -q_+, \quad p_3 \rightarrow p', \quad p_4 \rightarrow q_-, \\ k &\rightarrow -k, \quad \lambda \rightarrow -\lambda, \quad \delta_1 \rightarrow \delta_1, \quad \delta_2 \rightarrow -\delta_+. \end{aligned} \quad (29)$$

Both processes (23) and (28) are two crossing canals with the same (generalized) reaction. After the replacement (29) of invariant variables (24) they takes the next form:

$$\begin{aligned} s_1 &= (p - q_+)^2, \quad t_1 = (p - p')^2, \quad u_1 = (p - q_-)^2, \\ s_2 &= (p' + q_-)^2, \quad t_2 = (q_+ + q_-)^2, \quad u_2 = (p' + q_+)^2. \end{aligned} \quad (30)$$

Since square amplitude for process (28) after replacement of variables (29) is like to one for process (23), then one can obtain after initial electron polarization summing in the reaction (28) from (25) the next formulae for triplet production with taking into account polarization of initial photon and scattered positron:

$$d\sigma_{tr} = \frac{\alpha^3}{2\pi^2 s} A_{tr} W_{tr} d\Gamma, \quad (31)$$

$$A_{tr} = \frac{A_{MB}^{tr}}{t_1 t_2 u_1 u_2}, A_{MB}^{tr} = A_{tr}^0 + \delta_+ \lambda A_{tr}^1,$$

$$A_{tr}^0 = s_1 s_2 (s_1^2 + s_2^2) + t_1 t_2 (t_1^2 + t_2^2) + u_1 u_2 (u_1^2 + u_2^2),$$

$$A_{tr}^1 = -s_1 s_2 (s_1^2 - s_2^2) + t_1 t_2 (t_1^2 - t_2^2) + u_1 u_2 (u_1^2 - u_2^2),$$

$$W_{tr} = - \left(\frac{p}{pk} + \frac{q_+}{q_+k} - \frac{p'}{p'k} - \frac{q_-}{q_-k} \right)^2, \quad (32)$$

were $s = (p+k)^2$ and $d\Gamma$ are defined by expression (9). Then for the degree of longitudinal polarization of produced positron $\delta_+^f = A_{tr}^1/A_{tr}^0$ we will have expression:

$$\delta_+^f = \lambda \frac{-s_1 s_2 (s_1^2 - s_2^2) + t_1 t_2 (t_1^2 - t_2^2) + u_1 u_2 (u_1^2 - u_2^2)}{s_1 s_2 (s_1^2 + s_2^2) + t_1 t_2 (t_1^2 + t_2^2) + u_1 u_2 (u_1^2 + u_2^2)}.$$

This formulae is valid only in the case when invariant mass square of any particle pair is large with comparison to electron mass square.

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